

# Scientific report on the ESF exploratory workshop EW07-185

## The Relevance of Mathematics Education: integrating multiple perspectives

Cambridge, United Kingdom, 7 – 10 January 2008

Convened by

Paul Andrews<sup>1</sup>, Paul Conway<sup>2</sup> and Markku Hannula<sup>3</sup>

1 Faculty of Education, University of Cambridge

2 Department of Education, University College Cork

3 Department of Applied Sciences of Education, University of Helsinki

### 1. Executive Summary

The outcomes of recent international measures of mathematical competence such as TIMSS and PISA have not only created considerable interest among politicians and policy makers but highlighted the development of students' procedural, conceptual and problem solving skills as essential outcomes of the educational process. However, it is generally accepted within the mathematics education research community that such measures rarely expose students' understanding of and competence with mathematics in ways helpful to researchers and teachers.

It is also generally acknowledged that mathematics is neither value or culture free; mathematical learning is a function not only of obvious factors like the opportunities teachers present their students but also of differences in, for example, curricula, systemic expectations of the processes of education, didactic traditions, teacher education programmes, student prior knowledge and, significantly, participant beliefs.

Most of these dimensions have been researched at the national or systemic level and some at the comparative or cross-national level. However, despite a growing awareness of the impact of culture on all aspects of education, there has been little systematic comparative research of the interactions of, say, learners' mathematical cognition, teachers' practices and participant beliefs. These formed the basis of the exploratory workshop.

#### Student beliefs

In respect of student beliefs it is acknowledged that mathematical proficiency necessitates a productive disposition embracing a range of beliefs about the nature of mathematics, mathematical knowledge, motivation, learning and teaching. However, relatively little beliefs-related research has been undertaken in cross-cultural contexts.

#### Teacher beliefs

Available evidence suggests that teachers' beliefs about mathematics and its teaching are strong influences on classroom practice. However, most studies have focused on narrowly defined belief components in single educational systems rather than attempting a comprehensive and comparative examination of belief structures.

#### Teacher practices

Research suggests not only that although teachers' actions clearly impact on achievement but also that their behaviour identifies them closely with educational systems in which they

operate. However, the extent to which this view reflects a consensus is variable and further comparative research is necessary.

### Student achievement

Recent research on, for example, students' misconceptions, overgeneralisations and intuitive rules provides not only insights into the ways in which learners make sense of mathematics but also warrants the development of targeted interventions. However, despite clear advantages over large scale tests of achievement, little comparative research has yet been undertaken.

### Combining the above

Although any one of the four themes discussed above is worthy of isolated study, particularly from a comparative perspective, it is our longer term objective to move beyond loosely warranted links and conjectured causal effects to expose the relationships between student beliefs, student achievement, teacher beliefs and teacher actions. In particular, no studies have examined the relationship between learner beliefs and mathematical cognition as described above. Teacher actions have been implicated in student achievement, particularly in respect of procedural competence, however, the relationship between teacher actions and achievement, as reflected in the development of intuitive rules or overgeneralisations, has not been examined. Neither has the relationship between teacher actions and student beliefs although there is some evidence that certain teacher actions impact on the formation of certain forms of student beliefs.

Thus, the exploratory workshop has enabled colleagues, each of whom is expert in at least one of the four fields discussed above, to share perspectives and discuss the development of frameworks for a timely pan-European collaboration.

## 2. Scientific content

The exploratory workshop brought together colleagues from fifteen European countries, each of whom has expertise in at least one of the areas mentioned above. The intention was to familiarise ourselves with the ways in which these factors have been researched with a view to identifying and initiating potential collaborations within the remit of the ESF's funding arrangements. The underlying assumption was that mathematics teaching and learning are not only complex enterprises but also that little research acknowledgement has been made of the interactions between the various components discussed above. The following summarises the discussions undertaken during the two days of the Workshop. The presentations on the first day were intended to allow the group to explore the concepts likely to underpin any subsequent project. There were four sets and these are summarised below.

The first theme addressed current research on student cognition and was introduced by means of presentations from Pessia Tsamir, Wim van Dooren and Lucia Mason. All three alerted colleagues to the ways in which students both acquire mathematical knowledge and, importantly, fail to acquire mathematical knowledge in the ways intended by their teachers, particularly in respect of problem solving. This second aspect led us to an awareness that the terminology in the area of students' failure to acquire their teachers' intended mathematical knowledge – and here we intend mathematical knowledge to be a placeholder for any intended learning outcome – is contested. It seems to us, whether we are discussing misconceptions as defined by Hart and her colleagues, intuitive rules as defined by Tsamir and her colleagues or the illusion of linearity as defined by Van Dooren and his colleagues, that whenever students engage with mathematics they bring to bear some sense of intuition and that this may be way forward for resolving the terminological problem. In so doing it is our hope to move beyond tests of achievement which, as many critics have commented, have

not addressed particularly well the culturally located nature of all educational practice. Our proposed alternative to the assessment of mathematical understanding will provide considerable insights into the ways in which different systemic expectations and didactic traditions impact on learning.

We see the learning of mathematics as a specific case of cognitive development and our ambition is to find a deeper understanding of this development than is achieved in previous international comparative studies. Mathematics is not for us just a ‘placeholder’, which could be replaced by any subject. Instead, we see mathematics as a subject whose specific nature allows understanding of specific learning processes that have proven to be difficult. These include access and application of previously learnt knowledge in problem solving situations where surface characteristics of the task do not provide obvious clues as well as learning issues that are counter-intuitive. We believe that deeper understanding of cross-cultural differences in such issues will shed more light on the reasons behind cross-national differences in earlier international achievement tests.

The second theme focused on teachers’ mathematics-related beliefs and was introduced by Joao Pedro da Ponte, Paul Andrews and Zsolt Lavica. They alerted colleagues to the complex nature of teacher beliefs and their frequently inconsistent impact on teacher actions and learner achievement. Most studies have focused on teachers within a single cultural context and many of these, particularly those deriving from North America and Australasia, have attempted to develop, particularly in respect of the acquisition of the skills of problem solving, instruments for characterising *reform* or *constructivist* beliefs as compared with *traditional* or *transmissive* beliefs. Clearly, such terms are not beyond dispute and, particularly when we consider the outcomes of the few extant comparative studies of school teachers’ beliefs about mathematics and its teaching, it will be important that any instrument we develop acknowledges that different educational systems have different systemic priorities which may or may not reflect the notions of *reform* curricula. It was interesting to observe that despite substantial differences in the beliefs held by teachers in different countries, academic mathematicians, probably due to the nature of their work and their mobility, share many beliefs about mathematics and its teaching. Interestingly, survey studies have tended to highlight, partly due to their focus on *reform* classrooms mentioned above, similar outcomes while qualitatively focused studies have highlighted a greater diversity of beliefs which vary in the extent to which they align transparently with the belief dichotomies identified by the surveys. Importantly, in respect of our project, any attempt to integrate existing instruments will need to acknowledge that beliefs are culturally and experientially determined and, in their manifestation, allude to classroom norms and practices, and the socialisation of teachers and learners.

We are well aware of the often-repeated results that what teachers say about their teaching may differ significantly from the actual teacher behaviour in class. We see this as an indication of the contextuality of beliefs. Although the relationship between teacher beliefs and their actions is not straightforward, we believe that differences in teacher beliefs are related in different classroom behaviour and hence also different classroom norms. However, any research on teacher beliefs needs to be complemented with classroom observation.

The third theme, students’ mathematics-related beliefs, was introduced by Markku Hannula, Guenter Toerner and Peter Op’t Eynde and alerted colleagues not only to the range of constructs examined in the literature but the ongoing issue of definition – the mathematics education community has still to agree operational definitions and, in particular, the debate concerning the relationship between belief and knowledge remains vibrant. There is agreement, in general, that the affective domain comprises attitudes, emotions and beliefs

although the interactions between psychological, social and physiological constructs remain complex and ill-determined. Importantly, from a research perspective, there may be advantages in retaining ambiguity rather than trying to formalise definitions. Some of the domains in which beliefs have been examined, frequently from the perspective of problem solving, include epistemological – and interesting issue in this respect concerns beliefs about mathematics as distinct from school mathematics – goal orientation (mastery, performance or ego-defensive), self-efficacy or confidence, intrinsic or extrinsic motivation, and so on. However, too many large scale studies have effectively ignored the affective domain and its powerful mediating impact on learning. Methodological approaches were generally confined to survey and interview studies with the latter either informing the development of the former or being used to confirm it. Importantly, not only has little cross cultural work been undertaken in respect of students' mathematics-related belief systems but few extant instruments have been evaluated cross-culturally with much work still has to be done in terms of clarifying the constructs and examining their interactions, and the influence of context in the creation and manifestation of beliefs.

The fourth session, led by Paul Conway, Teresa Assude and José Carrillo, offered insights into research on teacher actions. We were alerted to the significance of teacher identity, and the manner of its formation, in the ways in which teachers think and act within their respective professional contexts. Also, the mediating impact of identity and beliefs on the ways in which the intended curriculum is implemented is an under researched area although there is increasing research indicating the existence of cultural scripts characteristic of the didactic practices of teachers within a given system. Several recurrent themes emerged. One concerned the relationship between teacher and student and whether learning was, essentially, a passive or active process. A second concerned the nature of the tasks teachers present their learners and the opportunities embedded within them. A third concerned the mental images that teachers construct of their lessons and the ways in which such images are constructed and reified within the classroom. A fourth concerned the ways in which lessons are analysed in respect of their episodic nature – it seems clear that this allows for straightforward and manageable analysis. Importantly, a key element in our understanding of teachers' actions lies in the interactions of their subject knowledge, however it is defined, their beliefs about teaching and learning, their pedagogic content knowledge in all its forms and their relationship with problem solving both as poser and solver.

In summary, the four sets of presentations, while each offered something distinctive, comprised many similarities. The cultural and contextual location of beliefs and practices and their systemic nature appeared common to all presentations. The relative lack of cross-cultural work in any of the four domains and the need to define constructs and agree terms remains an issue. Lastly, the nature of mathematics, mathematical knowledge – construed as different from school mathematical knowledge - and mathematical problem solving – itself a contested expression – permeated all four sets of presentations.

The second day began, after some group summaries from the previous day, with two presentations to focus attention on comparative research methods from Paul Andrews and Pauline Vos. Both reiterated issues identified the previous day. In particular was the need to ensure that any instruments developed for project use satisfied the need for conceptual, linguistic and measurement equivalence. Also, unlike many large scale tests of achievement, it was probably wise to adopt a bottom up rather than top down approach to instrument development not only to facilitate participants' inclusion but also to guarantee participants being able to make operational any definitions. During the previous day a number of colleagues had expressed an interest in including qualitative measures as complements to the more obvious need for survey instruments which were discussed at some length. In particular,

colleagues were alerted to issues of sampling and associated questions concerning target groups and whether the project was likely to be construed as longitudinal or cross-sectional.

The remainder of the day was structured to allow colleagues, in both whole and small group contexts, to discuss the implications of the above. One of the main concerns raised by a number of colleagues, and likely to impact significantly on the project's development, related to problems of classroom observation, particularly in respect of video studies, and not only teachers' reluctance to expose themselves in so public a manner but also the need to gain informed consent from participants. This was a particularly important discussion as few colleagues dissented from the view that videotaped lessons would be a powerful and very convincing form of data. In related ways, some time was given to a consideration of the form that videotaping should take were it to be adopted as a means of data collection. Comparisons were made with the random samples of the TIMSS video studies, the sequences of lessons on undefined topics of the Learner's Perspective Study or the sequences of lessons on defined topics of the Mathematics Education Traditions of Europe study.

### 3. Contribution to the future direction of the field

The consensus of the meeting was to be that a project of the form described below would lead to significant gains in our understanding of the processes of mathematical learning. Significantly, such insights would be of considerable interest to policy-makers not least because of their potential to explain aspects of achievement as measured by TIMSS and PISA. The proposed project, for which funding will be sought, would comprise main and supplementary studies. A key aspect of colleagues' thinking was that the first year would be spent with small groups of colleagues, already expert in the particular domain, working on the development of instruments synthesised from those already available.

#### Main study

Students' mathematics-related beliefs: A subgroup of interested colleagues would, drawing on available instruments, develop an instrument which, satisfying the need to acknowledge the varying cultural contexts, would tap into a number of key belief constructs. The precise constructs have yet to be decided although epistemological and ontological beliefs, motivational, goal-orientation and self-efficacy beliefs would be likely contenders.

Students' mathematical cognition: A small group of colleagues would, drawing on extant material, develop an instrument for assessing learners' cognition by examining the nature and extent of their intuitions. The precise intuitions are to be decided although in the work of project colleagues and, for example, Kath Hart there is much of interest which should allow us to avoid the problems of curriculum content experienced by other studies. However, any decisions will acknowledge the curricular traditions of project countries to ensure parity of participation.

Student samples: The discussion during the day considered whether the study should be cross-sectional or longitudinal. It was decided that a cross-sectional study of two age groups would essentially overcome the need for a longitudinal study and provide a more economical rationale for a funding application. Also, a lengthy discussion on appropriate age groups in relation to different educational system's traditions and practices concluded that the only appropriate age groups for study would be grade 4, as this reflected the final year in which all students were taught by generalists, and grade 8, as this was the final year in which all students in project countries would still study compulsory mathematics.

It was agreed that the study would focus on educational systems rather than nations. This would allow, for example, the autonomous regions of a number of project countries, of which Spain, Germany and Belgium are examples, to focus on a region and avoid the problematic of sampling across regions. Within a system, it was also agreed that we would attempt to create representative samples by accounting for school type, geographical diversity and so on. It was agreed that we would survey the whole cohort in any school sampled.

Teacher beliefs: It was agreed that a small group of colleagues would examine the extant literature with a view to constructing an instrument for determining, in cross-culturally valid ways, teachers' beliefs concerning mathematics, school mathematics and mathematics didactics. The precise constructs have yet to be determined although it was felt that an emphasis on *reform* classrooms, as in much of the literature, would be inappropriate in such a diverse community, particularly as the notion of reform may be alien to some colleagues and anathema in respect of their culturally located culturally determined curricular and didactic traditions.

Teachers would be surveyed in schools representative of their educational systems and all teachers who teach a particular cohort – grade 4 or grade 8 – would be surveyed.

#### Complementary study

It was agreed that a supplementary qualitative study should be undertaken. This would include videotaped lessons taught by volunteer teachers and teacher and student interviews. Precise details in respect of this aspect of the study are still to be clarified. For example, would we tape a sequence of lessons rather than just one? How many sequences would be expected in each country? Our discussion seemed to converge on the conclusion that videotaped sequences of lessons with volunteers identified from the survey instruments with a focus on problem solving and an analysis of the tasks presented to learners would be appropriate, particularly as guaranteeing topic equivalence across, potentially, almost twenty countries, would be problematic. However, we did not go beyond this and so colleagues' input in this aspect of the work will be particularly welcome.

#### 4. Workshop programme

Monday 7 January 2008

Evening      *Arrival*

Tuesday 8 January 2008 - Room 2S4

09.00-09.30      Welcome and introduction to the conference and Presentation of the European Science Foundation (ESF) (ESF Standing Committee for Physical and Engineering Sciences) and (ESF Standing Committee for Social Sciences)

*The following discussions will be managed by means of three 10 minute presentations from key colleagues working in the area followed by a discussion of one hour.*

09.30-11.00      Current research on student cognition: defining variables and comparing instruments  
Wim van Dooren, Pessia Tsamir and Lucia Mason

11.00-11.30      *Coffee break*

11.30-13.00      Current research on teacher beliefs: defining variables and comparing instruments  
Joao Pedro da Ponte, Paul Andrews and Zsolt Lavica

13.00-14.00      *Lunch*

14.00-15.30      Current research on students' mathematics-related beliefs; defining variables and comparing instruments  
Markku Hannula, Guenter Toerner and Peter Op't Eynde

15.30-16.00      *Coffee break*

16.00-17.30      Current research on teacher actions: defining variables and comparing instruments  
Paul Conway, Teresa Assude and José Carrillo

17.30-17.45      *Break*

17.45-18.45      Review of the day  
Paul Andrews/Zsolt Lavica  
*This will comprise four brief summaries, one from each of the four key sessions*

19.30-21.00      *Dinner*

Wednesday 9 January 2008- Room 2S8

09.00-09.30      Review of progress - defining the project, Paul Andrews  
*This will be a brief presentation on the previous day's discussions, highlighting key themes, variables and instruments*

- 09.30-11.00 Integrating themes: current research on thematic interactions, other issues for discussion and inclusion  
Paul Andrews Comparative dimension, Pauline Vos Survey methods  
*This will be managed by means of two 15 minute presentations from key colleagues followed by a discussion of one hour*
- 11.00-11.30 *Coffee break*
- 11.30-12.30 Project design – principles and research questions. What exactly will this project set out to achieve?  
*This will be a discussion managed in two groups (Paul Conway, Paul Andrews). It is likely that the discussion will be framed by several key questions, each of which will serve to frame the project design*
- 13.00-14.00 *Lunch*
- 14.00-15.30 Research design – pragmatics. How will the project team achieve its objectives?  
*Having defined potential research questions, colleagues will discuss the manner in which they will be addressed. Two groups (Beno Csapo, Pauline Vos)*
- 15.30-16.30 Project timeline – what is the timescale of the project and what are the issues in respect of personnel?  
*This will be managed through a focused discussion, based on the outcomes of the day's earlier sessions*
- 16.30-17.00 *Coffee break*
- 17.00-18.30 Concluding remarks and summary of follow-up research activities  
Paul Andrews  
*This will comprise a reiteration of decisions made, particularly in respect of roles, responsibilities and deadlines. One major objective of this workshop was to design a project to integrate the four key fields of mathematics education research discussed earlier. During this session, colleagues will, through discussion, define and apportion the various roles and responsibilities necessary for a successful project application.*
- 19.30-21.00 *Dinner*

Thursday 10 January 2008

Morning *departure*



## 5. Statistical information on 24 participants

### Institutional participation by country

|          |   |          |   |                |   |
|----------|---|----------|---|----------------|---|
| Austria  | 2 | Belgium  | 1 | Czech Republic | 1 |
| England  | 5 | Finland  | 1 | France         | 1 |
| Germany  | 2 | Hungary  | 3 | Ireland        | 1 |
| Israel   | 1 | Italy    | 1 | Netherlands    | 1 |
| Portugal | 1 | Slovakia | 2 | Spain          | 1 |

### Participation by gender

Female 10                      Male 14

### Participation by age

|       |    |
|-------|----|
| 20-29 | 1  |
| 30-39 | 4  |
| 40-49 | 7  |
| 50-59 | 11 |
| 60+   | 1  |

## 6. Final alphabetical list of participants

|  |  |
|--|--|
| Paul Andrews<br>University of Cambridge, England         | Teresa Assude<br>University of Provence, France          |
| José Carrillo<br>University of Huelva, Spain             | Sona Ceretkova<br>Constantine University, Slovakia       |
| Paul Conway<br>University College Cork, Ireland          | Benő Csapó<br>University of Szeged, Hungary              |
| Csaba Csíkos,<br>University of Szeged, Hungary           | João Pedro da Ponte<br>University of Lisbon, Portugal    |
| Jose Diego-Mantecón<br>University of Cambridge, England  | Markku Hannula<br>University of Helsinki, Finland        |
| Zsolt Lavica<br>University of Cambridge, England         | Lucia Mason<br>University of Padova, Italy               |
| Gyöngyvér Molnár<br>University of Szeged, Hungary        | Josef Molnár<br>Palacký University, Czech Republic       |
| Peter Op 't Eynde<br>University of Leuven, Belgium       | Birgit Pepin<br>University of Manchester, England        |
| Werner Peschek<br>University of Klagenfurt, Austria      | Bettina Roesken<br>University of Duisburg-Essen, Germany |
| Judy Sayers<br>University of Northampton, England        | Edith Schneider<br>University of Klagenfurt, Austria     |
| Guenter Toerner<br>University of Duisburg-Essen, Germany | Pessia Tsamir<br>Tel-Aviv University, Israel             |
| Wim Van Dooren<br>University of Leuven, Belgium          | Pauline Vos<br>University of Groningen, Netherlands      |