

Scientific Report

ESF Exploratory Workshop on

Singularities in Mechanics: Description and Formation

Paris, France, 21-25 January 2008

Convened by:

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1. Executive Summary :

Partial differential equations (PDEs) play a key role in describing real world phenomena as they appear in most physics and engineering problems. It is therefore an important part of mathematical research into PDEs to study well-posedness and qualitative behaviour of their solutions, and in particular to investigate the possibility of blow-up. However, the wealth of singularities that actually exist in the standard equations describing the mechanics of fluids and solids has not been appreciated until recently. By “singularities” we mean all phenomena involving strong focusing or rapid oscillations, leading to non-smooth behaviour in some continuum description. In particular, it is crucial to note that the singularity itself often lies at the heart of many important phenomena, such as crack propagation, drop pinch-off, drop spreading, or nonlinear wave propagation. This means there is a growing interest into the structure of an individual singularity, and mechanisms for their formation.

Activities in Europe are very diverse and wide-ranging, which makes it a unique centre of innovation in the field of continuum mechanics, and in the study of singularities. For this workshop, we invited leading researchers from Europe together with a few selected experts from the US and Israel. Another distinguishing feature of the field of singularities is that it cuts across disciplines. In this workshop, we therefore invited scientists of very diverse backgrounds, from experimental physics and engineering, pure and applied mathematics, and theoretical physics. Although connected directly to the study of singularities, many of our invitees would ordinarily not participate at the same workshop, and many met for the first time during the workshop.

To educate participants about the approach to singularities in different fields, we chose 4 world-leading experts to deliver a series of talks each, with a total length of 3 hours. This was Miguel Escobedo and Jeffrey Rauch in Mathematics, Tom Witten in theoretical physics, and Jay Fineberg in experimental physics. This program was complemented by a series of talks on recent

results of applied and theoretical aspects of singularities. The main results were :

- 1 We produced an overview of singular phenomena observed in physics and engineering; this will provide a pool of problems for mathematicians and theorists to work on.
- 2 We have assured an exchange of the most powerful tools to be used in mathematics for the study of singularities.
- 3 We have produced an overview of the self-similar structure of singularities. It has emerged recently that this structure is more complex than anticipated, and contains very slow corrections to self-similar behaviour.

2. Scientific content :

The program was divided into 4 major overview talks of 3 hour each. They were delivered by 2 mathematicians, 1 theoretical physicist, and 1 experimentalist. In view of the diversity of the problem of singularities, this provided the participants with an opportunity to learn about the principal methods employed in the study of singularities.

- 1 Jay Fineberg, University of Jerusalem, talked about experimental methods, using the example of crack propagation in brittle materials. This provided an excellent opportunity to learn about the extraction of experimental data and comparison between theory and experiment. The main problem consists in the matching between different regions: an outer region far from the crack, a self-similar region close to it, and a small region where microscopic effects become important. This situation is very characteristic for singularities in general.
- 2 Tom Witten, University of Chicago, talked about singularities in thin sheets from the point of view of a theoretical physicist. A number of different singularities occur, such as points and ridges. The main open problems concern the interaction between these singularities. This is closely related to the matching of the self-similar region and the outer solution on one hand, and the inner region on the other, where stretching of the sheet plays a role.
- 3 Miguel Escobedo, University del Pas Vasco talked about a set of problems of recent interest, where one attempts to treat the dynamical equation for a distribution of certain quantities, rather than the quantity itself. In view of the large solution space, this presents significant problems.
- 4 Jeffrey Rauch, University of Michigan, talked about multiscale problems, using the example of wave propagation. The problem of rapid variation close to the singularity, and how this behaviour is related to the outside, is of course central to the understanding of singularities.

The rest of the program discussed a wide range of singular problems of current interest, for example, dynamical free surface problems, polymeric thinning, sliding drops, granular gases, charged drops, high-Reynolds number flow around bodies, vortices, waves, and Navier-Stokes turbulence. This was done from a variety of perspectives ranged from experimental observation to the proof of theorems.

To mention a few highlights, Marco Fontelos (Madrid) reported on the dynamical formation of singularities on drops in electric fields. Classically, this is regarded as a stationary problem, analysing Taylor's classical cone solution. Now it has been discovered that there is a dynamical analogue to this solution, which maybe even more relevant, for example for the break up of droplets as they occur in rain clouds.

Sylvia Serfaty (Paris and New York) reported on vortex singularities in Ginzburg-Landau models with magnetic field, important to understand superconductors. The main point was that the concept of singularities led to a reduced description of a complex problem, in terms of an effective “particle dynamics”.

Hatim Zaag (Paris) presented very recent work on the intricate structure of the semi-linear wave equation, which has the interesting property that different types of singularities may exist depending on initial conditions. One kind exhibits standard, self-similar scaling, the other contains logarithmic corrections to scaling. All results were rigorously established.

3. Assessment of the results:

According to the participants of the workshop, it has been a great success. A significant part of this success lies in the fact that it familiarised researchers with work going on in other fields, as there is usually very little direct communication between experimentalists and pure mathematicians. Therefore, many invitees met for the first time during the workshop, as otherwise there is little chance to meet or to learn about another community’s work. Thus theorists left with a significant store of important physical problems, and experimentalists learned what kind of data might allow for significant tests of mathematical predictions.

To summarise the outcome has been :

- 1 We produced an overview of singular phenomena observed in physics and engineering; this will provide a pool of problems for mathematicians and theorists to work on.
- 2 We have assured an exchange of the most powerful tools to be used in mathematics for the study of singularities.
- 3 We have produced an overview of the self-similar structure of singularities. It has emerged recently that this structure is more complex than anticipated, and contains very slow corrections to self-similar behaviour.

After the program, there has been a very significant amount of follow-up activity, not the least owing to the program on “Singularities in mechanics” at the Institut Henri Poincaré, which was running until the 4th of April. During the ESF conference and for the period following it, Marco Fontelos and Jens Eggers have kept a record of mathematical activities, entitled “The role of singularities in singularities of PDEs”, which is also a review paper commissioned by Nonlinearity. Owing to the activity during the workshop, and the activity that resulted from it, we were able to assemble a catalogue of self-similar behaviours, which is far more varied than anticipated. Essentially, simple self-similar behaviours is often broken, and there are corrections taking place on a logarithmic timescale. We found that in all examples known in the literature, this slow dynamics can be described by a low-dimensional dynamical system. This had not been anticipated, and hopefully will lead to significant future developments in the field.

4. Statistical information

This workshop attracted an extremely diverse group of researchers from all over the world, and Europe: for example from France, Germany, Spain, Italy, Canada, USA, Russia, Israel, Brasil, and Chile. The degree of diversity in the background of the participants was also highly unusual. They came from the fields of pure and applied mathematics, experimental and theoretical physics, and from engineering.

Total number of participants: 28 (ESF Representative not included)

Geographical repartition (by country of work):

| | | | |
|----|----|----|---|
| BR | 1 | IL | 1 |
| CA | 1 | IT | 1 |
| CL | 1 | RU | 1 |
| ES | 4 | UK | 1 |
| FR | 10 | US | 7 |

Gender repartition:

| | | | |
|------|----|--------|---|
| Male | 24 | Female | 4 |
|------|----|--------|---|

Statistics on age structure: no data available.

5. Final Programme

« Formation and structure of singularities »

January Monday 21st – Friday 25th, 2008

Amphitheater Hermite

Program

The four long lectures are shown in blue.

Monday January 21st

1.30pm – 2.00pm Welcome -Coffee
2.00pm – 2.15pm Presentation of the ESF.
John Bush (MIT, Boston) «Singularities galore : On viscous sheet retraction and the impact of hydrophobic bodies».
2.15pm – 3.00pm Hamid Kellay (CPMOH, Bordeaux), «Some experiments displaying singular behaviour : thinning, break-up, coalescence, and line undulations».
3.00pm – 3.45pm Coffee Break.
3.45pm – 4.15pm Laurent Limat (MSC, Paris) «Sliding drops».
4.15pm – 5.00pm

Tuesday January 22nd

9.00am – 10.30am Jay Fineberg (Hebrew University, Jerusalem), «1) The dynamics of rapid fracture».
10.30am – 11.00am Coffee Break.
Miguel Escobedo (Universidad del Pas Vasco, Spain), «1) Singular solutions for the Uehling Uhlenbeck equation : The linearised problem».
11.00am – 12.30pm Buffet
12.30pm – 2.15pm Evariste Sanchez-Palencia (IJLRA, Paris) «Examples of propagating and non propagating singularities in thin shell theory».
2.15pm – 3.00pm Clément Mouhot (CEREMADE, Paris) «Cooling process and self-similarity in the mathematical kinetic theory of granular gases».
3.00pm – 3.30pm Coffee Break.
3.30pm – 4.00pm Opening discussion
4.00pm – 5.00pm

Wednesday January 23rd

9.00am – 10.30am Jay Fineberg (Hebrew University, Jerusalem), «2) Frictional motion».
10.30am – 11.00am Coffee Break.
Tom Witten (James Franck Institute, Chicago), «Crumpling singularities».
11.00am – 12.30pm : Buffet
12.30pm – 2.15pm Elisabeth Guazzelli (IUSTI, Marseille) «Falling clouds of particles».
2.15pm – 3.00pm

3.00pm – 3.45pm Marco Fontelos (Universidad Autonoma, Madrid), «Singularities in the evolution of charged drops».
 3.45pm – 4.15pm Coffee Break
 4.15pm – 4.45pm Christophe Eloy (IRPHE, Marseille) «Flag Flutter : Potential flow around a rectangular plate».
 4.45pm – 5.15pm Eytan Katzav (LPS-ENS, Paris) «Stability and roughness of crack paths in 2D heterogeneous brittle materials».
 5.15pm – 6.00pm Alfonso Gañan Calvo (U. Sevilla) «Focusing of liquids by convergent gas streams».
 6.00pm Cocktail
 7.30pm Dinner

Thursday January 24th

9.00am – 10.30am Jeffrey Rauch (U. Michigan), «1) Asymptotic Analysis of internal layers».
 10.30am – 11.00am Coffee Break
 11.00am – 12.30pm Tom Witten (James Franck Institute, Chicago), «Crumpling singularities».
 12.30pm – 2.00pm Buffet
 Miguel Escobedo (Universidad del Pas Vasco, Spain), «2) Singular solutions for the Uehling Uhlenbeck equation : The non linear problem».
 2.15pm – 3.45pm Coffee Break
 3.45pm – 4.15pm Sylvia Serfaty (Paris) «Vortex patterns in the 2D Ginzburg-Landau Model with Magnetic Field».
 4.15pm – 5.00pm

Friday January 25th

9.00am – 10.30am Jeffrey Rauch (U. Michigan), «2) Propagation of linear wave packets through periodic media».
 10.30am – 11.00am Coffee Break
 11.00am – 11.45am Hatem Zaag (Paris) «Existence of characteristic points for blow-up solutions of a semilinear wave equation».
 11.45am – 12.30pm Walter Craig (Mac Master U.) «Bounds on Kolmogorov spectra for the Navier Stokes equations».
 12.30pm – 1.30pm Summary of discussions on follow-up and future activities

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5. List of Participants

| | |
|--|-------------|
| 1. Professor Claude Bardos, Paris, FR | Participant |
| 2. Professor Stefanella Boatto, Rio de Janeiro, BR | Participant |
| 3. Professor John Bush, Cambridge, US | Participant |
| 4. Professor Walter Craig, Hamilton, Ontario, CA | Participant |
| 5. Dr. Francisco De La Hoz Mendez, Bilbao, ES | Participant |
| 6. Professor Jens Eggers Bristol, UK | Convenor |
| 7. Dr. Christophe Eloy Marseille, FR | Participant |
| 8. Professor Miguel Escobedo Lejona, ES | Participant |
| 9. Mrs. Gioia Failla Messina, IT Participant Tempe, US | Participant |
| 10. Professor Jay Fineberg, Jerusalem, IL | Participant |
| 11. Dr. Marco Fontelos, Madrid, ES | Participant |
| 12. Professor Alfonso Gañan Calvo, Sla Cartuja, ES | Participant |
| 13. Professor Elisabeth Guazzelli, Marseille, FR | Rapporteur |
| 14. Dr. Slim Ibrahim, Arizona State University, US | Participant |
| 15. Dr. Christophe Josserand Paris, FR | Co-Convenor |
| 16. Dr. Eytan Katzav Paris, FR | Participant |
| 17. Professor Hamid Kellay Talence, FR | Participant |
| 18. Dr. Laurent Limat Paris, FR | Participant |
| 19. Dr. Clément Mouhot Paris, FR | Participant |
| 20. Professor Yves Pomeau, Tucson, US | Participant |
| 21. Professor Jeffrey Rauch, Ann Arbor, US | Participant |
| 22. Professor Sergio Rica, Santiago, CL | Participant |
| 23. Professor Laure Saint-Raymond, Paris, FR | Co-Convenor |
| 24. Dr. Evariste Sanchez-Palencia Paris, FR | Participant |
| 25. Professor Sylvia Serfaty New York, US | Participant |
| 26. Dr. Pavel Shushpannikov Lubercy, RU | Participant |
| 27. Professor Howard Stone, Cambridge, US | Participant |
| 28. Professor Tom Witten, Chicago, US | Participant |
| 29. Dr. Hatem Zaag, Villetaneuse, FR | Participant |