ESF Exploratory Workshop on

Multivariate Interpolation - Its Relation To Algebraic Statistics, Classical Algebraic Geometry and Computational Complexity Theory

Scientific Report

Sestri Levante, Italy, 16 - 19 October 2007

Convened by:

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Executive report:

Planning for this workshop began over two years ago. The organizers were aware that certain topics in (seemingly) unconnected parts of mathematics had a common central problem -- namely how does one efficiently do multivariate interpolation -- roughly speaking, how does one fit relatively nice functions to data which includes controlled "bad" behaviour.

We contacted people who do research in classical algebraic geometry and commutative algebra, algebraic statistics, phylogenetics and computational complexity and explained what we had in mind. We were greeted, on all sides, by enthusiasm for the idea of the workshop and a willingness to both participate in the workshop and to chart future developments. We were greatly aided by Professor Maria Virginia Catalisano (Univeristy of Genova) and Dr. Enrico Carlini (Politecnico di Torino) who were on site and did most of the "leg work" negotiating with the hotel, printing programmes and notices, procuring small supplies, installing blackboard and overhead projectors in the conference room and acquiring computers that could be used for internet access during the conference itself..

Our idea was to bring together established experts in the areas mentioned above and also bright young people who might become interested in the problems which would arise from this mixing of ideas. We wanted the older, more experienced people, to explain their area of expertise to the others, with emphasis on this common central theme, and we wanted the younger people to explain their research which was connected to these central problems. The hope was that each group would interest the others in their work and

expose new ways to approach the problems that each group brought to the table.

We choose, as a location for the conference, an out-of-season resort hotel, relatively isolated and quiet, on the Ligurian Coast – Sestri Levante. Others had recommended it for its quiet surroundings and park, which would permit participants to walk and discuss things in a beautiful environment. We had also heard that the management of the hotel "I castelli" was very reasonable and would give us a good price for the lodgings and food. Indeed the peacefulness of the location was a wonderful backdrop to the activity and energy of the workshop itself.

According to the guidelines of the European Science Foundation, we planned a workshop of 3 full days (beginning on Tuesday afternoon and finishing with lunch on Friday – see the attached programme). The programme was intense, but not so packed with talks that it would prohibit people from discussing the talks with each other. We had, after every morning and afternoon session, a period called "The Second Chance" in which the workshop participants could requestion the speakers of that session in the light of the other talks.

The atmosphere during the entire workshop was excellent and friendly with an easy mixing between the more senior and more junior participants. We saw people in continuous dialogue about the problems which arose at the sessions and it was clear that people were interested in the new areas being exposed to them. Many new contacts were established and the bright young researchers we brought to the workshop were introduced to both the best experts in each area as well as exciting new problems which were available for them to consider.

The participants were unanimous in asking us (Ranestad and Geramita) when we would follow up this workshop to treat some of the specific issues raised by the speakers. Clearly it was a group that interacted well and we believe that this means they will make significant advances in most of the areas touched by the subject of the workshop.

Financial Support

In addition to the generous support of the ESF we were aided by a grant of 3,500 Euro by GNSAGA of the Italian Research Council. We were also (indirectly) supported by the Research Councils of all the countries of the participants. Those funds paid for the travel of these participants to the workshop. The funds from the ESF paid for the room and board of 25 of the participants and also for the incidental expenses connected with renting space for the talks.

Scientific Report:

Main objectives of the workshop

30 mathematicians were invited from various areas to discuss important problems in multivariate interpolation. The aim was to present the distinct approaches and recent progress in statistics, geometry, complexity theory and computational algebra in an attempt both to find common ground and to instigate future research in the fundamental questions of the area.

Setting

Of the 30, two cancelled at the last minute. The setting proved very conducive for the purpose, with a lecture room at the hotel with all the proper equipment. The local organizers Maria Virginia Catalisano and Enrico Carlini had done an excellent job preparing a participant package for each upon registration with all necessary information. We could therefore start on time with a welcoming address from an ESF –representative, followed by two introductory lectures.
A combination of breaks and "second chances" allowed for plenty of discussion following the lectures. Having successfully assembled central figures in three well-established areas, many key problems showed both the distinct nature of the three fields and a common ground in terms of subject matter.

Scientific content

Of the many new and exciting ideas that were presented, some are worth particular attention.

Secant varieties and tropical geometry. Degenerations may be used to compute the dimension and the degree of secant varieties. Tropical geometry is a particular case of such a degeneration, and we have only seen the start of its use in attacking classical problems. Ciliberto, Chiantini and Draisma explained new and exciting results.

The equations of secant varieties are of particular interest in computations, but have only recently come into focus again in research. Some intriguing problems presented by Sturmfels remain open, while a new approach in the special important case of toric varieties was presented by Sullivant.

This progress is partially inspired by application of algebraic statistics in phylogeny. In another, but closely related direction Casanellas reported on phylogenetic inference using algebraic geometry. Together these developments show how algebraic statistics is in a key position to push new ideas in the interface between well established fields.

Gröbner basis in computer algebra is a dominating concept underlying many basic algorithms. It is itself related to a degeneration which in many cases is too coarse. Border basis, as an alternative, was introduced by Robbiano and Abbot. It relates to a finer degeneration, and therefore opens up for more applications.

For further details we refer to the program listing below and to the website
http://calvino.polito.it/~carlini/webESF/webESF.html
where notes from most of the talks are available.

Outcome

A number of new and open problems were identified as key research topics in the area.

1. (Geramita) Let X be the Segre embedding of a multiple product of projective spaces. Find the degree and the geometry of syzygies of the secant varieties of X. In particular, develop the results of Landsberg and Manivel to produce equations and syzygies of resolutions. (Initial cases explored by Sidman and Cox)

2. (Sturmfels) Let X be P^1x P^1x P^1 in P^{15} . Study the hypersurface $(X + X) (X + X)$ i.e., multiply the secantvariety by itself pointwise. What is the degree of this variety? Defining equations? It is known to be a hypersurface.

3. (Winiarski) Find a criterion to look for:
Interpolation of algebraic sets. Let X be an algebraic set in C^n. Suppose we have a sequence of generic points on the set. This defines a decreasing sequence of ideals I_m (the intersection of the ideals of the first m points).
It follows that this intersection ideal is the ideal of this algebraic set. Calculate the Gröbner basis of the I_m. There exists some m_0 such that the full Gröbner basis becomes stable. Find a stopping criterion, i.e. when do we know we have attained the stable Gröbner basis?
A variation (Schreyer, Peters): Suppose V defined over Q is unirational. Pick algebraically independent numbers for your points. Now find equations with integer coefficients .

4. (Robbiano) Is B_O connected? The similar problem for Hilbert schemes of zero-dimensional schemes is solved. For the definitions, see M. Kreuzer and L. Robbiano, Deformations of border bases, arXiv:0710.2641, To appear in Collectanea Mathematica.

5. (Bocci) Consider X = { f = l_1 l_2 l_3... l_d } a product of linear forms, and its embedding X = {f^a } in the space of polynomials of degree ad. Conjecture: Sec^{m-1} (X) is defective only if a=2.

6. (Sturmfels) Compute a particular Gröbner basis, of the ideal

$$\langle x_1^{a(1)}, x_2^{a(2)}, \ldots, x_n^{a(n)} (x_1 + \ldots + x_n)^{a(n+1)} \rangle$$

where the exponents $a(1), \ldots, a(n+1)$ are large, on the order 10^6. Describe the initial ideal?
Roberts and Reichman have given information about the Hilbert series when n is one more than the dimension.

7. (Geramita) Let H be the Hilbert function of a set of points in P^2. We know examples of point sets X which have Hilbert function H and which achieve ALL possible resolutions compatible with H. In fact, there is a special set of points (the k-configurations) with maximal resolution for H and every other possible resolution arises from it by "consequtive cancellation" in that resolution. The generic resolution for that Hilbert function occurs at the bottom of the tree of all possible resolutions.
What is the situation for interpolating double point schemes whose support has Hilbert function H? What is the maximum resolution (it is known to exist, but it is not known what it is, in general)? Is there a generic resolution as in the case of simple point schemes?

On the organizational side the workshop inspired further cooperation. Two concrete measures should be mentioned:

8. The intention to submit a proposal for an RNP within ESF at the next call for applications.

9. The submission of a proposal for a program covering the topics of the workshop at the Institut Mittag Leffler in the spring of 2011.

Program

Tuesday 16. Oct. Arrival
1400– 1600 Registration

1600–1700: Welcome and first lecture (introductory/survey)

Ciro Ciliberto, Rome: "Degenerations and interpolation"

1730–1830: Lecture (introductory/survey)

Bernd Sturmfels, Berkeley: "Open Problems in Algebraic Statistics"

1930: Dinner

Wednesday 17. Oct.

0900–1000 Lecture (introductory/survey)

Lorenzo Robbiano, Genova: "Computer Algebra and Multivariate Interpolation (Part I)"

1030–1130 Lecture (introductory/survey)

Hans Michael Moeller, Dortmund: "Multivariate interpolation from the numerical point of view"

1200–1300 Lecture (introductory/survey)

Markus Bläser, Saarbrucken: "Complexity of graph polynomials and interpolation"

1300 Lunch

1500–1545 Lecture (research topic)

John Abbott, Genova: "Computer Algebra and Multivariate Interpolation (Part II)"

1615–1700 Lecture (research topic)

Luca Chiantini, Siena: "On the dimension of secant varieties"

1730–1815 Lecture (research topic)

Jan Draisma, Eindhoven: "Tropical geometry, secant varieties, and multivariate interpolation. "

1830–1900 Second chances (Open questions on todays lectures)

1930 Dinner

Thursday 18. Oct.

0900–0945 Lecture (research topic)

Seth Sullivant, Harvard: "Secant Varieties in Algebraic Statistics"

1015–1100 Lecture (research topic)

Henry P. Wynn, London: "Smooth interpolators via algebraic methods "

1130–1215 Lecture (research topic)

Eva Riccomagno, Warwick: "Repeated measurements and algebraic statistics"

1230–1300 Second chances (Open questions on the morning lectures)

1300 Lunch

1500–1545 Lecture (research topic)

Marta Casanellas, Barcelona: "The geometry of evolutionary models"

1615–1700 Lecture (research topic)

Joaquim Roé, Barcelona: "Recent progress on the Nagata and Harbourne–Hirschowitz conjectures"

1730–1815 Lecture (research topic)

Peter Buergisser, Paderborn: "Average volume, curvatures, and Euler characteristic of random real algebraic varieties"

1830–1900 Second chances (Open questions on the afternoon lectures)

1930 Dinner

Friday 19. Oct.

0900–0945 Lecture (research topic)

Wolfram Decker, Saarbrucken: "The Boij Söderberg conjecture "

1015–1100 Lecture (research topic)

Frank–Olaf Schreyer, Saarbrucken: "Over constrained mechanisms and curves of genus 7"

1130–1200 Second chances (Open questions on the morning lectures)
1215–1300 Open problems (Common discussion)

1300 Lunch

1500 Departure

List of participants

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Statistics on participants:

Male: 24
Female: 4

Age:
20–39: 13
40–59: 12
60+:   3

Country of origin:
England 2
France   1
Germany   5
Holland   1
Italy 10
Norway 2
Poland 1
Spain   2
Sweden 1
United States 3