Report on the ESF research conference on
Algebraic Methods in Dynamical Systems (2010-320)
Bedlewo, Poland, 16-22 May 2010

Partnership: ESF, EMS, ERCOM.
Conference Title: Algebraic Methods in Dynamical Systems.
Dates: 16-22 May 2010.
Chair: Zbigniew Hajto (Jagiellonian University Poland) and Teresa Crespo (University of Barcelona, Spain)
Rapporteur: Marius van der Put (University of Groningen, The Netherlands).

(1) *General Comments*

The theme "algebraic methods in dynamical systems" of this conference is a broad subject with surprisingly many research directions.
Some of these specializations are:
differential Galois theory, parametrized differential Galois groups and differential groups, Picard-Vessiot theory, Painlevé equations, isomonodromic deformations, hierarchy of partial differential equations, non linear differential/difference Galois theory, q-difference equations, Hamiltonian systems, Morales-Ramis-Simó theory, differential algebra, the dynamics of billiard problems, stability of mechanical systems, Hilbert's 16th problem and polynomial foliations, singularities of integrable systems and power geometry *and more*.
The titles of the talks and the corresponding abstracts demonstrate the richness of the theme "algebraic methods in dynamical systems". Moreover,

the lecturers and the audience came from many countries and often belong to different "schools of mathematics".

There are very few mathematicians (if any) knowing all these topics in detail. Still the subjects are related in many ways. Good expositions (and there were many in this conference) of a topic are of great importance for researchers working on other topics. Especially, explaining relations and even finding new relations between the themes was an important part of this conference. Maybe the best thing of this conference is that the participants obtained an overview of the richness of " algebraic methods in dynamical systems" of which their own specialization is a part.

(2) *Quality of Scientific Programme, Presentation and Discussion*

The lectures of the conference were, for a part, given by experts in one of the areas of research mentioned in (1). This was complemented by lectures given by junior researchers presenting their recent work. The quality of the lectures was quite high. Despite the variety of topics, most of the talks were understandable by the audience. The level of difficulty of the lectures was variable (as always) and some lectures assumed a considerable mathematical background.

(3) *Informal Networking and Exchange; Atmosphere*

The lectures were scheduled as follows:
Three 1-hour lectures in the morning, two 1-hour lectures in the afternoon followed by four or five lectures of 30 minutes. In between the lectures and during the lectures there was ample time for discussions, questions, proposals and remarks. A poster session was added to enable young mathematicians to present the problems of their research.

As you may observe, the programme fully occupied the days of the conference (with the exception of Wednesday afternoon). From more than one participant came the suggestion to restrict the number of lectures per day somewhat and create some free time in the middle of the day (after lunch). This system is adopted by other mathematical centers of conferences such as those of Oberwolfach and CIRM Luminy. This rapporteur has the same opinion.

(4) *Balance of Participants*

There was a fair amount of young participants, working on their Ph.D. or shortly after their Ph.D. work. Many nationalities were present. We especially mention a group of Polish, a group of Ukrainians, a group of Russians and a group of researchers from Israel. These groups have probably less opportunities to participate international conferences in Western Europe. From outside Europe we mention the presence of American mathematicians and two Japanese ones. One of the American mathematicians is Michael F. Singer, a well-known expert in the algebraic theory of differential/difference equations. This conference was in fact dedicated to him on the occasion of his 60th birthday.

(5) *Outlook and Future Developments*

During the meeting new projects for collaboration were started. More future collaboration is expected between the groups for Ukraine, Russian Federation, Israel, Japan and the European participants.

At present the majority of the talks of the conference week can be found on the website of the Banach Center at Bedlewo. One intends to publish the conference talks and related items in a volume of the Banach Center Proceedings.

In a special session of the meeting, on Friday evening, the participants were invited to give proposals for future work and to discuss the future scientific problems in "algebraic methods in dynamical systems". Here is short list of the suggestions.

For differential algebra and differential Galois theories, more on: algorithms, connections with model theory (i.e., a topic in logic), differential/difference schemes, equations from mathematical physics, applications.

For ODE, PDE's and foliations, more on: Oscillatory properties of surfaces in space, search for new functions for measuring zeros of, say, solutions of connections which are quasi-unipotent.

For Power Geometry, more on: developing methods together with differential algebra to analyze higher variational equations and integrability.

For differential/difference Galois theory, more on:

relations between various groups, applications to PDE's other than Hamil-toninan systems, stability, Darboux curves, integrals of motion.

For Morales-Ramis-Simó theory, more on:
Not only proving non integrability of certain systems, but especially developing methods to find new integrable systems.

For ODE's, more on:
Global extensions of geometric objects related to ODE's, such as the Poincaré map. More on geometry of complex limit cycles, saddles and connections. Simultaneous uniformization of hyperbolic leaves.

Integrability, more on:
comparing the various notions, Lie methods.

(6) *Follow-up*
This meeting is the second one with the title "Algebraic Methods in Dynamical systems". The first one was held in Barcelona in 2008. The next meeting with this title will be organized by Prof. José Cano, to be held at the university of Valladolid in 2012.

(7) *Organization and Infrastructure*

The participants were very satisfied with the facilities (computer, beamer, wifi) and praised the assistance of the on-site administration of ESF.

(8) *Summary & Overall Assessment*
Was the conference succesful; were its aims achieved?

This is a rather difficult question to answer, since it is not easy to measure success and moreover the aims of the conference were not sharply defined.

What can be said is that this conference inspired some of the lecturers to present their new work into a presentable form and even to find new results. The long term effect of this conference is that the participants have now good ideas of each others work and will more easily contact each other in the future for questions. Also a good deal of actual collaboration is expected.

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