



**RESEARCH CONFERENCES**

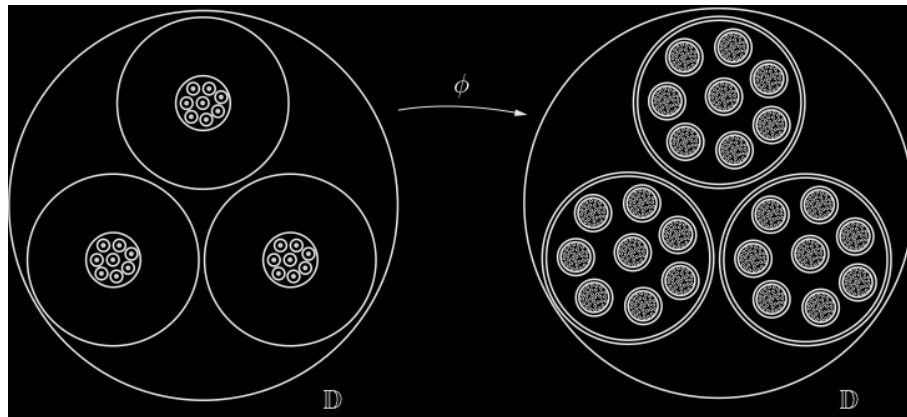
ESF Conference in Partnership with EMS and ERCOM

**Harmonic Analysis,  
Geometric Measure Theory  
and Quasiconformal  
Mappings**

Centre de Recerca Matemàtica, Bellaterra • Spain  
14-20 June 2009

Chair: **Prof Pertti Mattila**, University of Helsinki, Finland

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## Conference Highlights

*Please provide a brief summary of the conference and its highlights in non-specialist terms (especially for highly technical subjects) for communication and publicity purposes. (ca. 400-500 words)*

As the conference title, 'Harmonic analysis, geometric measure theory and quasiconformal mappings', suggests, the main point of the conference was to bring together experts and students from these three areas of mathematical analysis, both such who had mainly worked on only one of these topics or those who had already combined two or three of them in their research. The timing for such a conference was perfect. During the last decade or so surprising amount of deep interplay between these areas has been created. It was a good time to have a clear view on what has happened and where new interesting research directions could be found.

The conference was successful. The scientific level was very high. There was fruitful and up-to-date interaction between the three different main themes. Participants were both senior mathematicians, some of them very distinguished, and young persons at their post doc or Ph.D. student stages. It was very delightful to have many young, around 30 years of age, excellent mathematicians whose career was already in good speed and of whom we will certainly hear much in the future.

There were 21 one hour invited lectures and eight 30 minutes lectures selected by the scientific committee from the applications. They were excellent. They gave a very broad perspective of the area and many exciting new developments were presented. Quite likely this will lead to substantial new research and co-operations. There were only six posters by some young mathematicians. The reason for the small number is presumably that mathematicians prefer to present their ideas in lectures and black-board discussions rather than in posters.

There were 99 registered participants. This was an essentially optimal number. Much more would have been difficult in terms of space. Now the main areas were well represented and the participants could easily communicate and learn to know each other.

CRM provided excellent surroundings for the conference. Both the CRM and ESF (Alessandra Piccolotto) personnel were very helpful and efficient.

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I hereby authorize ESF – and the conference partners to use the information contained in the above section on 'Conference Highlights' in their communication on the scheme.

# Scientific Report

## Executive Summary

(2 pages max)

The conference ‘Harmonic analysis, geometric measure theory and quasiconformal mappings’ was organized in co-operation with ESF and CRM (Centre de Recerca Matemàtica) in the premises of CRM in Bellaterra (Barcelona) June 14-20, 2009. The scientific committee consisted of Pertti Mattila (chair, Helsinki, Finland), Tadeusz Iwaniec (vicechair, Syracuse, NY, USA), Guy David (Paris, France), Carlos Perez (Seville, Spain) and the local organizers Joan Mateu, Joan Orobítg, Xavier Tolsa and Joan Verdera. The director of CRM Joaquim Bruna was in charge from the point of view CRM. Most of the practical matters were effectively and smoothly handled by Alessandra Piccolotto from ESF and Nuria Hernandez from CRM.

The scientific committee selected the invited speakers who were Rodrigo Banuelos, Luca Capogna, John Garnett, Piotr Hajlasz, Alex Iosevich, Tadeusz Iwaniec, Loredana Lanzani, Jani Onninen, Tatiana Toro, Rodolfo Torres and Alexander Volberg from USA, Kari Astala, Eero Saksman and Xiao Zhong from Finland, Pascal Auscher and Guy David from France, Giovanna Citti from Italy, Marianna Csörnyei from England, Daniel Faraco from Spain, Gaven Martin from New Zealand and Laszlo Szekelyhidi from Germany. Consulting the scientific committee the chair selected from the received applications 8 short talks, 6 posters, the recipients of 750 euro full grants (covering accommodation and other local expenses), the recipients of 250 euro partial grants (with no accommodation) for local people and long term CRM visitors, and the participants. All who applied to participate could be accepted, and the final number of registered participants was 99.

The purpose of the conference was to provide researchers working in harmonic analysis, quasiconformal mappings or geometric measure theory with a scientific event designed to promote a deep interaction between the three subjects. The main goal was to bring together the leading worldwide experts in each of the subjects listed above and young researchers, including post docs and advanced doctoral students working in related topics. This goal was achieved very well and the conference was scientifically successful.

## Scientific Content of the Conference

(1 page min.)

- Summary of the conference sessions focusing on the scientific highlights
- Assessment of the results and their potential impact on future research or applications

The purpose of the conference was to bring together experts and students from harmonic analysis, geometric measure theory and quasiconformal mappings. This goal was achieved very well, all these areas were broadly presented and their fruitful interplay was clear throughout the conference. In fact, interplay of different topics of mathematics was substantial in a wider sense. In addition to the above three topics, for example stochastics, functional analysis, complex analysis and potential theory came up in many interesting ways in several lectures. The overall impression one could get during the conference was that mathematical analysis is living an exciting period with many surprising and often unexpected interconnections.

The role of stochastics in analysis has increased in many ways during recent years. This was also apparent in the conference. Astala spoke in his opening talk about conformal welding. This is an

exciting project concerning problems of mathematical physics and involving interplay of the famous stochastic Loewner equations of Schramm and others with quasiconformal and conformal mappings. Another aspect where stochastics was constantly present, in particular in the talks of Banuelos, Hytönen and Saksman, was estimates for various singular integral operators, most notably the Ahlfors-Beurling transform.

Partial differential equations is one of the most central parts of mathematics with its many applications to physics and elsewhere. They were also strongly present in the conference. Faraco, based on his joint work Astala and others, talked about some recent spectacular progress of applications of quasiconformal mappings to the so-called Calderon's inverse problem which has applications to tomography. Iwaniec, Martin and Onninen explained in their lectures parts of their extensive ongoing work in various aspects of partial differential equations and their connections to quasiconformal and harmonic mappings.

Analysis in subriemannian spaces, such as the Heisenberg groups, often with strong connections to geometric measure theory has become very active during recent years. This was also evident in the conference in the talks of Capogna, Citti, Hajlasz and Zhong.

But not all good progress has depended in the interplay and connections, as is quite natural to expect. There have also been striking results inside each of the main areas as such. For example, in harmonic analysis Torres discussed many sharp new results for fractional integral operators in weighted spaces. In geometric measure theory Csörnyei presented nearly optimal results on classical Sard's theorem.

## Forward Look

(1 page min.)

- *Assessment of the results*
- *Contribution to the future direction of the field – identification of issues in the 5-10 years & timeframe*
- *Identification of emerging topics*

There has been important interplay between harmonic analysis, geometric measure theory and quasiconformal mappings, and other fields, for many decades, but it has usually meant applying one result from one area to the other. Only during the last 10-20 years this has become more systematic. Such interplay is now actively sought by large groups of researchers. This has not only lead to interesting and surprising results describing the relations between different areas but also to new methods with applications to unexpected problems. We are now at the stage where such research can be expected to intensify and diversify with many new exciting results. What will really happen in the future, even in a very near future, is impossible to say, but some possibilities can be speculated.

It is very probable that stochastics will play an increasingly central role in analysis. There are many open important problems related to mathematical physics and stochastic Loewner equations where further use of quasiconformal mappings and perhaps more effective use of geometric measure theory could play a decisive role. Recent developments on stochastic methods for singular integrals have been so successful that much more progress could be expected. Perhaps the long-standing open problem on the norm of the Ahlfors-Beurling transform could finally be solved with applications to quasiconformal mappings.

During about 1994-2004 an amazing progress took place, developed by many mathematicians and completed by Tolsa, on classical problems of removable singularities of bounded analytic

functions, behaviour of the Cauchy singular integral operator on subsets of the plane and geometric rectifiability properties of such sets. There are similar problems in higher dimensions with interesting partial results, some of them discussed by Volberg and Mas in their talks. But these partial results are still very far from the essentially complete results in the plane. Getting progress on this will be one of the big challenges in near future. Another topic where geometric measure and harmonic analysis will certainly meet many times in coming years are questions related to behaviour of Fourier transforms of singular measures and Hausdorff dimension. Some recent progress was discussed by Iosevich but several fundamental problems are open, such as the Kakeya conjecture with its connections to restriction conjectures of Fourier analysis.

On Heisenberg groups and more general subriemannian spaces there have been important developments of harmonic analysis (singular integrals), geometric measure theory (rectifiability and structure of surfaces) and quasiconformal mappings, but there has been very little interplay with these areas. Will there be soon and where would it lead?

- Is there a need for a foresight-type initiative?
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## Atmosphere and Infrastructure

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- *The reaction of the participants to the location and the organization, including networking, and any other relevant comments*

As far as I can tell, the participants reacted very positively. They seemed to like the location and found the atmosphere very pleasant and stimulating.