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RESEARCH CONFERENCES

ESF Mathematics Conference in Partnership
with EMS and ERCOM

The second European Set Theory Meeting: In honor of Ronald Jensen

Mathematical Research and Conference Center,
Bedlewo • Poland

5-10 July 2009

Chair:

Jouko Väänänen, University of Amsterdam, NL and
University of Helsinki, FI

Organising Committee:

Grzegorz Plebanek, University of Wrocław, PL; **Ralf Schindler**, University of
Münster, DE; **Boban Velickovic**, Université de Paris 7, FR

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(\diamond_{κ}) There is a sequence $\langle S_{\nu < \kappa} \rangle$
s.t. $S_{\alpha} \subseteq \alpha$ ($\alpha < \kappa$) and
for every $X \subset \kappa$, the set
 $\{\alpha \mid X \cap \alpha = S_{\alpha}\}$ is Mahlo in κ .

(\diamond_{κ}^+) There is a sequence $\langle S_{\nu < \kappa} \rangle$
s.t. $S_{\alpha} \subseteq \mathcal{P}(\alpha)$, $\bar{S}_{\alpha} \leq \bar{\alpha}$ ($\alpha < \kappa$)
and for every $X \subset \kappa$, the set
 $\{\alpha \mid X \cap \alpha = S_{\alpha}\}$ is Mahlo in κ .

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Conference Highlights and Executive Summary

Please provide a brief summary of the conference and its highlights in non-specialist terms (especially for highly technical subjects) for communication and publicity purposes. (ca. 400-500 words)

Set Theory has a special place in Mathematical Logic. On one hand it is used as the axiomatic foundation of all mathematics and on the other hand deep technical tools of set theory such as combinatorics, forcing and independence results, descriptive set theoretic techniques, etc. have been applied to a number of different areas of mathematics with spectacular results. In the last couple of decades set theory has reached maturity and impressive progress has been achieved both on the foundational side and on the applied side of the subject.

The central foundational problem is posed by the prevalence of independence results in set theory. We know from Gödel's celebrated Incompleteness Theorem that every explicitly presented, sufficiently rich mathematical theory T is necessarily incomplete, i.e. there exists statements which can neither be proved nor refuted in this theory. While in most areas of mathematics the independence phenomenon is anecdotal, in set theory it plays a central role and such basic questions as Cantor's Continuum Hypothesis (CH) cannot be resolved on this basis of the standard axioms ZFC of set theory due to the work of Gödel and Cohen. One of the main programs of modern set theory is the search for suitable axiomatic extensions of ZFC which would resolve CH and other important problems not resolved by ZFC. The cornerstones of this program was laid out by Gödel himself in the 1940s who called for the search of maximality principles which should be used to supplement the standard axioms. One of the most fruitful directions of this program was initiated by Ronald Jensen in the 1960s and it involves building canonical models for large cardinals axioms – this is the so-called 'Inner Model Program'. This program has achieved great success in the last 20 years or so and 'core models' as they are called were constructed for larger and larger cardinals. However, some fundamental obstacles remains, and one of the central problems is the construction of canonical inner models for supercompact cardinal.

On the applied side, in the last 20 years numerous connections were established between set theory and other mathematical areas. One of the most successful connections involves the use of descriptive set theoretic tools to study general classification problems in various other areas of mathematics. This area was initiated by Kechris, Harrington, Louveau and others in the 1980s and has found deep and fruitful interactions with ergodic theory, operator algebras, group representation theory, etc. Descriptive set theory has provided the framework and basic tools to compare various classification problems in most areas of mathematics and to answer in precise technical terms what it means that a certain class of structures does not admit suitable classification. Another fruitful interaction of set theory has been the use of combinatorics and forcing techniques in the study of Banach spaces and in this direction some fundamental set theoretic results such as for instance the Galvin Prikry theorem have led to the work of Gowers on dichotomy theorems in Banach spaces for which he was awarded the Fields medal in 1998.



I hereby authorize ESF – and the conference partners to use the information contained in the above section on 'Conference Highlights' in their communication on the scheme.

Scientific Report

Scientific Content of the Conference

(1 page min.)

- Summary of the conference sessions focusing on the scientific highlights
- Assessment of the results and their potential impact on future research or applications

A summary of the sessions:

Monday, 6 July Morning session, chair Jean Larson

Menachem Magidor Hebrew University, IL, gave a special lecture on the work of Ronald Jensen. His title was: The core of modern set theory – Ronald Jensen.

Adrian Mathias Université de la Réunion, St.Denis de la Réunion, FR, gave a lecture on provident sets.

Martin Zeman University of California Irvine, US, gave a lecture on combinatorics of small and large cardinals.

John Steel University of California Berkeley, US, gave a talk on the inner model project. His talk was entitled “More mice”.

Afternoon session, chair Ralf Schindler

Ronald Jensen Humboldt-Universität zu Berlin, DE: Ronald Jensen reported on his recent development of a theory of subproper and subcomplete forcings. These forcings generalize proper forcings, subproper forcings and may add reals, but subcomplete forcings do not. Namba forcings or what Jensen calls L-forcings and many other similar forcings are shown to be subproper or even subcomplete. The theory is expected to have many applications.

Itay Neeman University of California Los Angeles, US Lecture: Itay Neeman talked about his recent result according to which the failure of the singular cardinal hypothesis at κ is compatible with the tree property holding at the successor of κ . This solves a long-standing open problem and constructs models in which the singular cardinals hypothesis fails in a surprisingly new way.

Grigor Sargsyan University of California Berkeley, US, The mouse set conjecture: Sargsyan presented the results of his Ph.D. thesis which contains breakthroughs in inner model theory. He gives an analysis of HOD of models of AD^+ up through $AD_R + \text{``}\Theta \text{ is regular''}$ and shows that HOD is a “hod mouse.” He applies this to get optimal lower bounds for the existence of divergent models of AD^+ and of the existence of an ω_1 -dense ideal on ω_1 in the presence of CH.

Poster session

The general opinion was that the poster session was a great success. It was arranged on two consecutive days with the same posters, and both times there was a lot of audience and also a lot of interaction between the presenters and the audience. The presenters were:

- 1 Borodulin-Nadzieja, Piotr: Some cardinal invariants of density ideal.
- 2 Brech, Christina: New amalgamation techniques to construct thin-very tall compact scattered spaces by forcing.
- 3 Farkas, Barnabas: Combinatorics of analytic P-ideals and related forcing problems
- 4 Flašková Jana On certain I-ultrafilters and ip rich sets
- 5 Ikegami Daisuke Forcing absoluteness and regularity properties.
- 6 Keskinen, Lauri: Infinitary Second Order Languages
- 7 Kraszewski Jan On two coverings.
- 8 Kulikov, Vadim: Weak Ehrenfeucht-Fraïssé Games.
- 9 Kysiak, Marcin: How to construct Bernstein sets with algebraic properties without transfinite induction?
- 10 Ralowski, Żebrowski, Robert, Szymon: Generalized (I,J) - Luzin sets.
- 11 Sakai, Hiroshi: Partial versions of stationary reflection and square
- 12 Schlicht, Philipp: A dichotomy for projective equivalence relations.
- 13 Spadaro, Santi: Noetherian type: an ordered sibling of weight.
- 14 Szymon, Glab: The Laczkovich-Komjath property of sigma-ideals.

Tuesday, 7 July Morning session, chair Joan Bagaria

Menachem Kojman Ben Gurion University of the Negev, IL, Singular cardinals in the 20-th century. Kojman gave an overview of the state of the art of singular cardinals and made the point that singular cardinals, despite their somewhat dismissive name, are actually “natural” and interesting cardinals.

Tomek Bartoszynski National Science Foundation, US: Bartoszynski’s talk on covering the real line by a few translations of a compact set was based on joint work with Saharon Shelah. The main result is a surprising ZFC-result about approximate covering.

James Cummings Carnegie Mellon University, Pittsburgh, PA, US. Combinatorial principles. Cummings answered a question asked by Viale: If W is an extension of V with the same cardinals, and κ is inaccessible in V while $\text{cf}(\kappa) = \omega_1$ in W , must $\text{square-}\kappa\text{-}\omega_1$ hold in W ? The new result of Cummings says that it is consistent from large cardinals that there can be such V and W such that $\text{square-}\kappa\text{-}\omega_1$ fails in W .

Hugh Woodin University of California Berkeley, US. The search for ultimate L . In his surprising and interesting talk Woodin argued as followed: Mitchell and Steel defined the fine-structural version of extender models up to the level of superstrong cardinals. Assuming the so called Iteration Hypothesis and the existence of superstrong cardinals, Neeman and Steel proved existence up to the level of superstrong cardinals. If the Mitchell-Steel theory can be extended to the level of suitable extender sequences then modulo iteration hypotheses one would have an ultimate version of L , denoted L^Ω . The axiom $V = L^\Omega$ would be compatible with essentially all known large cardinal hypotheses. The successful construction of L^Ω would complete the Jensen Program for finding the ultimate core model.

Afternoon session, chair Boban Velickovic

Inessa Epstein University of California Los Angeles, US, gave a talk on recent results on the descriptive set theory of orbit equivalence.

Taras Banach Ivan Franko Lviv National University, UA, gave a talk on descriptive set theory and dimension.

Todor Tsankov University of Paris VI, FR, gave a talk on equivalence relations generated by profinite actions.

Wednesday, 8 July Morning session, chair: Istvan Juhasz.

Philip Welch University of Bristol, UK, very appropriately gave a short history of the Jensen Indiscernibles Lemma.

Dilip Raghavan University of Toronto, CA, gave a talk on cofinal types of ultrafilters.

Janusz Pawlikowski Wroclaw University, PL, gave a talk on universally Baire operations.

Sy Friedman University of Vienna, AT, described the outer model program and related quasi-lower bounds on consistency strength.

Thursday, 9 July Morning session, chair Grzegorz Plebanek

Mirna Dzamonja University of East Anglia, UK, The isomorphism problem: The talk, presenting a joint work with J. Väänänen, focused on combinatorics of trees with singular height and size. The results were used to give a characterisation of isomorphism of chain models using an appropriate version of some classical infinite games.

Istvan Juhasz, Alfréd Rényi Institute of Mathematics, HU, The convergence and character spectra of compact spaces: The author defined two kinds of cardinal spectra for the class of compact topological spaces; in each case a spectrum of a space is a set of cardinal numbers that arise by performing some natural topological constructions. The concept of such spectra gives new insight into some important problems in set-theoretic topology.

Jindrich Zapletal University of Florida Gainesville, US, Ramsey theorems and creature forcing: The author presented several new Ramsey-type results for Borel partitions of infinite products of finite sets. This subject originated in some natural questions of purely combinatorial nature but is also connected with forcing - one of the applications of the presented theorems was a generalization of a result due to Laver on products of Sacks forcing.

Stevo Todorcevic University of Toronto, CA and University of Paris VII, FR, Classifying structures belonging to $H(\omega_2)$ - recent contributions: The talk surveyed old and new results on the structure of trees, focusing on several quasi orders one can define in an attempt to get some classification theory. In particular, well quasi orderability and Lipschitz comparability of trees were discussed.

Afternoon session, chair Menachem Magidor

Slawomir Solecki University of Illinois, Urbana-Champaign, US, gave a talk on point realizations of Boolean actions of Polish groups.

Justin Moore Cornell University, US, gave a talk on some open problems related to proper forcing and CH.
Moti Gitik Tel Aviv University, IL, Lecture gave the final lecture. His topic was pcf theory.

a. An assessment of the results

Concerning the concrete new themes which were discussed and which are likely to lead to new breakthroughs in the next 5-10 years we mention the following:

- a. Very recent work on the 'suitable extender sequences' by Woodin, raising the possibility of building canonical models for supercompact cardinals and completing the Jensen program for finding the ultimate core model.
- b. Classification by countable structures, as pursued in particular by Gregg Hjorth, has provided a precise mathematical language for proving non-classifiability results in various areas of mathematics and has allowed to answer some long standing open problems in operator algebras.
- c. The study of automorphisms of the Calkin algebra, as recently investigated by Ilijas Farah, using set theoretic principles, has opened the possibility of numerous new applications of combinatorial set theory to C^* -algebras. There is a whole new area of combinatorial set theory which needs to be developed in order to answer some fundamental open problems in this area such as, for instance, the Naimark problem.

Forward Look

(1 page min.)

- Assessment of the results
- Contribution to the future direction of the field – identification of issues in the 5-10 years & timeframe
- Identification of emerging topics

State of the art:

In the conference most of the central areas of modern set theory were presented and at the end we had a forward looking plenary discussion. The panel consisted of some leading set theorists (M. Magidor, J. Steel, J. Väänänen) as well as three prominent young set theorists (I. Epstein, J. Moore and I. Neeman) representing three different areas of modern set theory. The discussion was very fruitful and it revolved around the identification of major emerging topics, discussion of key open problems and issues such as how to help young people in the subject and improve the awareness of the general scientific community of the tremendous recent achievements in set theory. This was the first panel discussion of this sort in a very long time and is likely to have a lasting impact on the subject for years to come. It was particularly enlightening for young researchers who are trying to enter the field and are often perplexed by the sheer technical complexity of the subject.

Emerging topics:

- Woodin's work on the 'suitable extender sequences' which raises the possibility of building canonical models for supercompact cardinals and completing the Jensen program for finding the ultimate core model.
- Hjorth's notion of classification by countable structures has provided a precise mathematical language for proving non-classifiability results in various areas of mathematics and has allowed to answer some long standing open problems in operator algebras.
- The recent work of Farah who used set theoretic principles to study the automorphisms of the Calkin algebra opens the possibility of numerous new applications of combinatorial set theory to C^* -algebras. There is a whole new area of combinatorial set theory which needs to be developed in order to answer some fundamental open problems in this area such as, for instance, the Naimark problem.

Visions for the future:

Numerous other open problems and areas of new research were discussed. In the audience there were a number of young researchers who were inspired by the discussion and since we plan to hold similar panels in the future it will be interesting to see if some of these young researchers succeed in solving the challenges posed to them by the older generation.

- Is there a need for a foresight-type initiative?

Since this conference was part of the long term project of developing set theory in Europe and bringing it closer to the center of mathematics it would be very useful to continue with such panel discussions in the future. Given the richness of possible topics it may be advisable to make more focused discussions with specific topics in mind.

Atmosphere and Infrastructure

- *The reaction of the participants to the location and the organization, including networking, and any other relevant comments*

There is a separate questionnaire about this