



Banach Algebras 2009

Stefan Banach International Mathematical Center •
Będlewo, Poland
14-24 July 2009

Chair: **H. Garth Dales**, Leeds University, UK

<http://www.siue.edu/MATH/BA2009/>

Highlights & Scientific Report



Conference Highlights

Please provide a brief summary of the conference and its highlights in non-specialist terms (especially for highly technical subjects) for communication and publicity purposes. (ca. 400-500 words)

This conference, on “Banach algebras” was the 19th in a sequence that goes back to the first meeting in Los Angeles in 1974. The present pattern is for there to be a conference every two years, alternately in Europe and North America. The preceding conference was at the University Laval in Quebec, Canada, in July, 2007.

The conference was attended by 94 participants and about 12 associated people. Essentially all world leaders in research in Banach algebras and their applications attended the meeting. It seems that participants represented about 27 different countries. Numbers include the following: Australia (1), Canada (12), India (2), Iran (5), Poland (10), Russia (6), US (9), UK (15). About 50 participants came from the EU. The conference extended for 8 days of lectures, with a break at the middle weekend. All lectures and discussion were in English.

There were 10 plenary lectures, each of 1 hour, expounding significant areas of research. For example, Professor Ruan (Illinois) opened the conference with a masterly survey of the exciting topic of ‘Locally compact quantum groups’; this is a rather new area of mathematics that blends classical harmonic analysis and the newer theory of operator spaces to allow many traditional topics to be seen within a new synthesis. Professor Vaes (Leuven) gave a brilliant exposition of the tremendous recent advances of himself and others in understanding the significant class of algebras of operators called ‘von Neumann algebras’. The final lecture of the conference, by Professor Runde (Edmonton) announced a very significant new result. For about 30 years mathematicians have sought to understand the algebraic structure of the algebra of all operators on a Banach space (these algebras generalize algebras of matrices). A challenge has been to prove that, for certain standard sequence spaces l^p , the corresponding algebra is not ‘amenable’; this was known for p equal to 1 and 2. Now Runde has a formidable proof that answers this question positively for all possible values of p .

There were about 62 ‘main lectures’ of 40 minutes each, with two at the same time expounding more special topics. These were arranged in groups reflecting main topics within our subject; they included several lectures on harmonic analysis, Banach function algebras, operators on Banach spaces, and the cohomology of Banach algebras.

An additional evening lecture by our Honorary President, Professor Zelazko, was devoted to a ‘History of Polish mathematics’.

A considerable amount of informal discussion took place between lectures; new mathematics was created and links and contacts were made that are likely to lead to future collaborations, some new. For example, there will be a ‘working semester’ in Leeds, UK, in May/June 2010, on the topic ‘Banach algebra and operator space techniques in topological group theory’; this is closely related to the topic of the Bedlewo Conference.

Mathematical papers based on lectures at the conference will be published in two volumes of the ‘Proceedings of the Banach Center’. These volumes are expected to appear in October 2010.

The conference was supported by the ESF under the ESF-EMS-ERCOM partnership, by the Polish Academy of Sciences, and by Adam Mickiewicz University, Poznan. We are grateful for this support.

For further information on the conference, see <http://www.siue.edu/MATH/BA2009/>

I hereby authorize ESF – and the conference partners to use the information contained in the above section on ‘Conference Highlights’ in their communication on the scheme.

Scientific Report

Executive Summary

(2 pages max)

At our conference, there were 10 plenary lectures, each of 1 hour, expounding significant areas of research, and about 62 main lectures of 40 minutes each, with two at the same time, expounding more special topics. There were also about 10 valuable poster presentations.

All lectures were, broadly, devoted to Banach algebras and their application, but within this area, there were several themes, which we summarize; of course, some talks were related to more than one theme. The abstracts of all talks, and the slides of many lectures, are available on our web site <http://www.siue.edu/MATH/BA2009/>

Scientific Content of the Conference

(1 page min.)

- Summary of the conference sessions focusing on the scientific highlights
- Assessment of the results and their potential impact on future research or applications

- 1) Operator spaces and harmonic analysis. In recent years the classical subject of harmonic analysis, which refers to Banach algebras such as the group and measure algebras of a locally compact group and their 'dual' objects, including the Fourier and Fourier—Stieltjes algebra of a group has been melded with the rather new subject of operator spaces to make a new synthesis, based on the notion of a locally compact quantum group, a notion introduced by Vaes, one of our plenary speakers, and others.

Our conference opened with a powerful survey by Zhong—Jin Ruan, who showed how both group/measure algebras on one side and their 'dual objects' on the other side can be viewed in a unified way. This was followed by talks of Zhigou Hu, who cast the same ideas in the context of modules, and by Ebrahim Samei, who concentrated on the second duals of our algebras.

We recall that every Banach algebra A has a second dual A'' that is a Banach algebra with respect to two different 'Arens products', and that A is Arens regular if these two products coincide; the topological centre measures how close A is to being Arens regular: at the opposite extreme are the strongly Arens irregular (SAI) algebras, whose topological centre is minimal. For example, all group algebras of a locally compact group are SAI. Matthias Neufang surveyed this area in an inspiring plenary lecture, and, in particular, explained his (with Pahl and Steprans) recent exciting theorem that the measure algebra of many compact groups is SAI; this result resolves a long standing conjecture. A related concept of a strong topological centre was explained by Denis Poulin, a student of Neufang.

- 2) A related topic is that of the cohomology of Banach algebras, a subject that arises from the work of B. E. Johnson and Helemskii in the 1970's. The 'best' (in some sense) Banach algebras are those Banach algebras A which are amenable, which means that certain cohomology groups $H^1(A, E')$ vanish for all Banach A -bimodules E ; this means that every derivation from A into E' has a simple, inner, form. The algebra A is weakly amenable if $H^1(A, A')$ vanishes.

This theme was connected to the first theme in a powerful plenary lecture of Viktor Losert, who gave exciting new results about derivations and crossed homomorphisms arising in the context of locally compact groups and their group algebras. Further connections were developed by Nico Spronk, who discussed the weak amenability of Fourier algebras on compact groups, by Mahya Ghandehari, who discussed the amenability of Rajchman algebras, which are related to measure algebras; Robert Archbold discussed the cohomology theory of C^* -algebras.

- 3) The calculation of specific higher cohomology groups $H^n(A, E)$ and related cohomology groups, including cyclic cohomology groups, for a Banach algebra A is of great interest as an end in itself as a way of illuminating the general theory. Such calculations were exhibited by Yemon Choi, by Frederic Gourdeau, and by Michael White; specific forms for some derivations was demonstrated by Matthew Heath, and Zinaida Lykova looked at higher dimensional versions of amenability for various tensor products.

The subject of the cohomology of Banach algebras is vast.

- 4) A fourth theme is that of algebras of linear operators. Let E be a Banach space. Then $B(E)$ denotes the famous Banach algebra of all bounded linear operators on E . Closed ideals in $B(E)$ include the compact operators $K(E)$ and the approximable operators $A(E)$.

In general, it is difficult to calculate the family of all closed ideals in $B(E)$ for a specific space E , and this has only been achieved in a few cases. This was explained in a lecture of Andras Zsak; calculations involving the explicit Banach spaces of James and Schrier type were given by Niels Laustsen and his student, Alistair Bird, and involving explicit Banach spaces were given by Phillip Brooker. Since Zsak, Laustsen and Bird come from the UK and Brooker is from Australia, the conference was a fine opportunity for these people to meet 'from other ends of the world', and to compare their results.

A fine survey talk of David Blecher related the theme of operator algebras to that of operator spaces, described earlier.

Forward Look

(1 page min.)

- *Assessment of the results*
- *Contribution to the future direction of the field – identification of issues in the 5-10 years & timeframe*
- *Identification of emerging topics*

Our assessment is that many strong results were announced and discussed at our conference; some are mentioned in our survey of the scientific highlights of the conferences. There were many strong results announced at our conference, and this speaks of the vitality of the subject.

We consider the future direction of the field and the identification of emerging fields.

The topic of Banach algebras and its applications has been active for several decades, and different aspects of the subject have ebbed and flowed over the years. It will be seen from a glance at the list of participants at the conference that our subject is a very international one. However, the countries in which the leading experts work have similarly changed somewhat with time.

Historically the strongest country in our subject was the USA; however several key experts such as Bade (Berkeley) and Curtis (UCLA), who were the leaders of strong schools of graduate students, have retired, and so there are no longer such schools in the US. On the other hand it is clear that Canada has several very active groups with strong leaders; this is reflected in the fact that two recent conferences have been at Edmonton and Quebec, and a future one will be at Waterloo.

There were rather a large number of participants from the UK. However it is the case that quite a number of the UK experts have recently or will soon retire, and current financial exigencies in the UK, especially with a dramatic drop in the funding that EPSRC devote to mathematics, will mean that research in our subject will be supported in the future at a much lower level. To a lesser extent this is true for much of western Europe.

There are many strong experts from the FSU; almost all were trained in the excellent Moscow State University. It is not clear whether or not this will continue after the retirement of Helemskii. There is also a very strong school in Poland, the historic home of Banach and the founders of functional analysis.

Our subject has been the abstract underpinning of many aspects of mathematics and its applications – for example, it has significant applications in Fourier analysis, signal analysis, control theory, spectral theory, numerical analysis, etc. However it is not easy to justify the subject in terms of its short-term economic impact on industry, and so it will struggle to receive funding, in view of the present policies of several key governments and the EU.

The subjects that are most active at the moment seem to be the following.

- 1) Fundamental structures of so called II-1 factors in von Neumann algebra theory.
- 2) Locally compact quantum groups and operator-space theory.
- 3) The understanding of rather 'exotic' Banach spaces, and of the linear mappings that operate on them.

- 4) The connection between the amenability of locally compact groups and related algebras, and especially studies of various classes of non-amenable groups.

It seems that these are the topics that will be the objects of particular study in the next 5 years, and it is likely that conferences will concentrate on these topics.

For example, this will be the case for our own next conference in Waterloo in 2011. The working semester at Leeds in May/June 2010 will concentrate on locally compact quantum groups and operator-space theory, and there will be a meeting on operator spaces at Banff in June 2010. It is likely that there will be a Thematic Programme on 'Banach algebras and harmonic analysis' at the Fields Institute, Canada, from July to December, 2014.

- Is there a need for a foresight-type initiative?

All initiatives that advance our subject are to be welcomed; there is substantial mathematical scope for such activities.

Business Meeting Outcomes

- *Election of the Organising Committee of the next conference*
- *Identified Topics*
- *Next Steps*

There is an 'International Steering Committee' that oversees the progress of our meetings on Banach algebras. The Chairman of this Committee at the meeting was Professor H. G. Dales. However Dales announced at the meeting that he was stepping down (after many years). The new Chairman of this Committee will be Professor V. Runde of Edmonton, Canada. The other members of the Committee will be Jean Esterle (Bordeaux, France), Niels Gronbaek (Copenhagen, Denmark), Krzysztof Jarosz (Illinois, US), Niels Laustsen (Lancaster, UK), Zina Lykova (Newcastle, UK), Alexei Pirkovskii (Moscow, Russia), Sandra Pott (Paderborn, Germany), and Thomas Ransford (Laval, Canada).

It was agreed that the next conference in our sequence, the 20 th Conference, will take place at Waterloo University, Canada, in July, 2011. The organizers will be Ken Davidson, Brian Forrst, and Nico Spronk of the Waterloo Department. Funding is expected from NSERC (Canada), from the Department at Waterloo, and other sources.

The topic of the conference will again be Banach algebras and its applications, but it is expected that there will be some special emphasis on Banach algebras of operators, which is a strength of the group at Waterloo.

We expect that the 21 st Conference in our sequence will take place in Europe in 2013. Informal discussions on the location are taking place.

Atmosphere and Infrastructure

- *The reaction of the participants to the location and the organization, including networking, and any other relevant comments*

It seems that all the participants were content with the smooth organization of the conference, the happy atmosphere engendered, and the time for discussions outside the formal lectures. The organization at Bedlewo worked very well, and the facilities are fine; this is a very convenient place to meet, being accessible to mathematicians from both western countries and to those coming from Russia and neighbouring countries.

The costs of rooms, food, and conference facilities at Bedlewo are considerably lower than they would be at a university in the UK, for example. This makes the location very attractive.

For some time we believed that essentially our only financial support would be that coming from the Polish Academy of Sciences. This provides some free rooms at Bedlewo, but no travel expenses. This arrangement would have prevented some people that we wished to hear from attending. It made a very substantial difference to our conference that we were able to receive, at a later stage financial support from the ESF; without this the conference would have significantly less valuable.