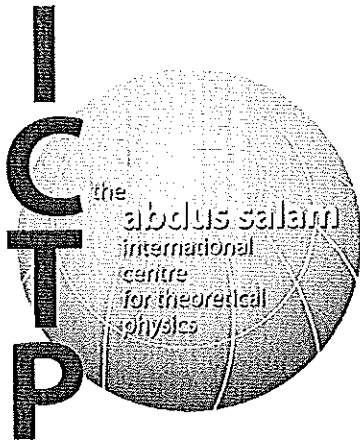


EUROPEAN  
SCIENCE  
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SETTING SCIENCE AGENDAS FOR EUROPE



ESF- ICTP Conference

## Geometric Analysis

ICTP, Miramare - Trieste, Italy  
24-29 June 2012

Chair: **Claudio Arezzo**, *ICTP, IT*  
Co-Chair: **F. Pacard**, *Ecole Polytechnique, FR*;  
**R. Schoen**, *Stanford, US*; **G. Tian**, *Princeton, US & Beijing, CN*

[www.esf.org/conferences/12400](http://www.esf.org/conferences/12400)

Highlights & Scientific Report

## Conference Highlights

*Please provide a brief summary of the conference and its highlights in non-specialist terms (especially for highly technical subjects) for communication and publicity purposes. (ca. 400-500 words)*

The conference has taken place the week after a two weeks school during which seven courses on various aspects of Geometric Analysis had been given by some of the major scientists in the field (including the four chairmen of the event).

Courses touched the following themes:

- 1) Positive Energy and Positive Mass Theorems in General Relativity and Differential Geometry;
- 2) Einstein Equations on complex manifolds;
- 3) Ricci flow in real and complex geometry;
- 4) Geometry and Analysis on surfaces in Euclidean spaces.

The Conference followed the above scheme, with long sessions with frontline research presented and open problems discussed, and one hour seminars.

Fifteen speakers presented their work and oriented discussions among participants. Highlights of these are:

- 1) Variational approach to the Einstein equations on complex manifolds;
- 2) Solution of the Willmore Conjecture;
- 3) Interactions of Mathematics and Physics in the study of the Geometry of 4-manifolds.
- 4) Algebraic aspects of the Kahler-Einstein equations;
- 5) Analytic aspects of the Einstein constraint Equations.

Each of the above themes has recently developed into a highly specialistic field and this conference was a rare occasion to gather together some of the major figures in these fields and stimulate interactions on these topics.

Seven of the fifteen speakers come from European Institutions, five from North American, two from Asia and one from South America.

110 participants took part to the event, about forty of which working in Europe representing ten different european nationalities.

As a single highlight of the scientific content of the Conference, we point out the four hours lectures by A. Neves (Imperial College, London) and F. Coda Marques (IMPA, Rio) on their recent (still unpublished) solution of the Willmore Conjecture.

The Willmore Conjecture predicts the only equilibrium state of a "doughnut shaped" (i.e. a surface with one hole) subject to forces which generalize the classical case of soap bubbles where the only forces considered are the surface tension and the amount of air contained in the soap film. The equilibrium state of closed soap bubbles have been mathematically described only in 1984. The Willmore Conjecture has deep connections to fundamental questions in general relativity, cell biology and lens design.

It is of crucial importance, since the proof has yet to be published, that experts and young scientists have occasions like this conference to discuss the new ideas introduced and explore the new horizons they open.

I hereby authorize ESF – and the conference partners to use the information contained in the above section on 'Conference Highlights' in their communication on the scheme.

# Scientific Report

## Executive Summary

(2 pages ma)

The conference has taken place the week after a two weeks school during which seven courses on various aspects of Geometric Analysis had been given by some of the major scientists in the field (including the four chairmen of the event).

Courses touched the following themes:

- 1) Positive Energy and Mass Theorems in General Relativity and Differential Geometry (R. Schoen);
- 2) Einstein Equations on complex manifolds (Tian, Arezzo);
- 3) Ricci flow in real and complex geometry (Brendle, Topping);
- 4) Geometry and Analysis on surfaces in Euclidean spaces (Pacard, Riviere).

Seven of the fifteen Conference speakers come from European Institutions, five from North American, two from Asia and one from South America.

110 participants took part to the event, about forty of which working in Europe representing ten different european nationalities. Scientists from thirtyeight countries have taken part to the School and Conference.

Among the speakers V. Guedj (Toulouse) and C. Le Brun (Stony Brook) gave a three hours presentation (each) on "Variational approach to the Kahler-Einstein equation" and "Hermitian Metrics, Einstein Manifolds, and Conformal Geometry". These lectures included open problems discussions and were accompanied by beautiful and comprehensive set of notes distributed to all participants (and available online).

A. Neves (Imperial College) and F. Coda Marques (IMPA) gave a joint four hours presentation of their recent solution to the Willmore Conjecture.

Other one-hour talks were:

X. Zhu (Peking)

Perelman's entropy and Kähler-Ricci flow on a Fano manifold

J. Li (Peking)

On the symplectic mean curvature flows

J. Chen (University of British Columbia)

Compactness of  $W^{2,2}$  branched conformal immersions of surfaces and applications

A. Malchiodi (SISSA)

Variational analysis of singular Liouville equations

J. Corvino (LaFayette College)

Deformation and gluing constructions for scalar curvature

R. Gicquaud (Tours)

Solutions of the constraint equations with non-constant mean curvature

J. Fine / Université Libre de Bruxelles, Belgium

Quantization and Mabuchi energy

G. La Nave / University of Illinois at Urbana Champaign, USA

V-solitons and Kähler-Ricci flow on symplectic quotients

R. Berman / Chalmers University, Gothenburg, Sweden

Real-Monge-Ampere equations and Kähler-Ricci solitons on toric log Fano varieties

S. Sun / Imperial College London, UK

Structure of Gromov-Hausdorff limit of Kähler-Einstein manifolds

A. Della Vedova / Princeton University, USA  
 On the uniqueness of extremal Kähler metrics

All the scientific and social activities took place inside the ICTP facilities.

## Scientific Content of the Conference

(1 page min)

- Summary of the conference sessions focusing on the scientific highlights
- Assessment of the results and their potential impact on future research or applications

(1 page min)

As mentioned in the previous sections, highlights of these were:

- 1) Variational approach to the Einstein equations on complex manifolds;
- 2) Algebraic aspects of the Kahler-Einstein equations;
- 3) Solution of the Willmore Conjecture;
- 4) Interactions of Mathematics and Physics in the study of the Geometry of 4-manifolds.
- 5) Analytic aspects of the Einstein constraint Equations.

On point one and two the main lectures were given by V. Guedj (Toulouse), 3 hours, who also made available to all participants an amazing set of notes (roughly 60 pages) describing this topic from some of its classical results up to most recent advancements.

On points 1 and 4 the one hour lectures were given by R. Berman (Chalmers), X. Zhu (Beijing), J. Fine (Brussels), S. Sun (Imperial), G. La Nave (Urbana) and A. Della Vedova (Princeton). Of particular importance were the lectures by Berman and Sun who spoke about very recent unpublished, even in preprint form at the time of the conference, work in collaboration with Berndsson and Donaldson respectively.

There is no doubt these works will shape research in this area for the next few years.

Four hours have been devoted to the lectures by Marques and Neves about the solution of the Willmore Conjecture. Specifically they talked about

On the Willmore Conjecture - "Canonical families"  
 "Min-max and Width"  
 "Proofs of Main Theorems"

The Willmore Conjecture has appeared in mathematical, physical and biological literature (under different names) more than 100 years ago and has resisted the attack of many of the most prominent mathematicians of the XIX Century. Marques-Neves have solved this problem introducing some new ideas that will certainly resist the assault of time. It seems difficult to overestimate the impact these lectures might have had on the more than a hundred participants, many of them at the beginning of their career. The other topics in the list of highlights were touched in the following lectures:

C. LeBrun  
 Hermitian Metrics, Einstein Manifolds, and Conformal Geometry  
 X. Zhu  
 Perelman's entropy and Kähler-Ricci flow on a Fano manifold  
 J. Li  
 On the symplectic mean curvature flows  
 J. Chen

Compactness of  $W^{2,2}$  branched conformal immersions of surfaces and applications

A. Malchiodi

Variational analysis of singular Liouville equations

J. Corvino

Deformation and gluing constructions for scalar curvature

R. Gicquaud

Solutions of the constraint equations with non-constant mean curvature

LeBrun's talk and open problem discussion is recorded in a set of notes which has become available to all participants through ICTP website.

## Forward Look

(1 page min)

- *Assessment of the results*
- *Contribution to the future direction of the field – identification of issues in the 5-10 years & timeframe*
- *Identification of emerging topics*

There is no doubt the following themes, central in Conference and which have been already at the center of research for the last decades, will continue to be the main topics in Geometry and Analysis:

- 1) Existence of Einstein metrics
- 2) Algebraic obstructions to the above problem
- 3) variational problems of surfaces in euclidean spaces
- 4) geometric flows in real and complex analysis.

The Conference has been a rare occasion to mix these themes and to present them in a unified way to an audience largely composed of scientists at early stages of their career. The fact that many lecturers have provided notes for their courses/talks will certainly help leaving a lasting impression on all participants. The interaction of leading experts among them and with the younger participants has been deep and proficous.

Scientists from thirtyeight countries have taken part to the School and Conference.

Via the formal discussions during the morning lectures, where topics and open problems were introduced, an informal ones during breaks of the programme, it is clear that the Conference has had an important role in making accessible to a large audience classical problems, new techniques and emerging topics. In particular the following topics have been longly discussed and we expect participants to the Conference to significantly contribute to the future developments of these fields:

- 1) Tian-Yau-Donaldson Conjecture relating stability and existence of Kahler-Einstein metrics on algebraic manifolds of positive curvature (Berman, Sun);
- 2) Variational problems of surfaces in three manifolds, in particular the new technique, introduced by Marques-Neves in solving the Willmore Conjecture, has been discussed at lenght as well as its possible application to other important contexts, e.g. non flat ambient spaces.
- 3) Ricci and Calabi flows on real and complex manifolds: after Perelman's solution of the Poincare Conjecture this has become a huge field of research. During the Conference various aspects have been carefully studied in particular: existence and convergence of the Ricci flow on complex manifolds of positive curvature (Zhu, Fine and Della Vedova). An important variant of this approach has recently emerged thanks to the work of La Nave and Tian. This has been a topic of discussion and has attracted the attention of many senior and young participants. There is no doubt that this will be a lasting topic of research.

- Is there a need for a foresight-type initiative?

NO

### Business Meeting Outcomes

- Election of the Organising Committee of the next conference
- Identified Topics
- Next Steps

Due to change in agreement, this was the last conference of the scheme so there was no need to take any further decision.

### Atmosphere and Infrastructure

- The reaction of the participants to the location and the organization, including networking, and any other relevant comments

The following Questionnaire has been distributed to all participants to the Conference. We can provide a scanned version of the replies upon request.

The overall response was extremely positive both on the scientific and the organizational sides.

### ICTP-ESF School and Conference on Geometric Analysis

11 - 29 June 2012

To help us gauge the merits of this activity, and to enhance the planning of next year's activities, we should be grateful if you would kindly complete this questionnaire, and return it, at the end of the activity, to the "Questionnaire Box" in the Main Lecture Hall.

1(a) Were you paid by ICTP

No  Yes  subsistence  travel

1(b) Were you supported by ESF ?

No  Yes

2. Which continent are you from: .....

3. Did you find the scientific level of the activity

too low  satisfactory  too high  ?

4. Was the duration of the activity

too short  satisfactory  too long  ?

5. Was the number of lectures per day

too few  satisfactory  too many  ?

6. Were most of the lectures clear?

not very clear  clear  very clear

7. Which lectures did you enjoy listening to most ?

.....

8. From which lectures did you learn most ?

.....

9. Was the overlap of the topics covered with your research interests

high  low

10. How could the lectures be improved ?

.....

11. Please make any comments and suggestions you may have for future Schools

.....

12. Miscellaneous comments regarding your stay at the ICTP

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**Sensitive and Confidential Information**

This report will be submitted to the relevant ESF Standing Committees for review.

In order to promote transparency, it is ESF policy to also publish the Scientific Reports on its website. Any confidential information (i.e. detailed descriptions of unpublished research, confidential discussions, private information) should therefore not be included in this report. Confidential issues can be addressed in the next page, which will not be published.



I hereby authorize ESF to publish the information contained in the above Scientific Report on the ESF Research Conferences Webpages. No sensitive or confidential information (see above) has been included in this report.

## Confidential Issues

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▪ Any other issues, not to be included in the published report.

Date & Author:

Trieste 9<sup>th</sup> August 2012

Claudio Arezzo