Modelling extreme values of processes observed at irregular time step

Extreme Environmental Events

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16th December 2010

Introduction

Objectives

- Spatio-temporal peaks-over-thresholds
- Application: Satellite data, in-situ observations, hindcast data...

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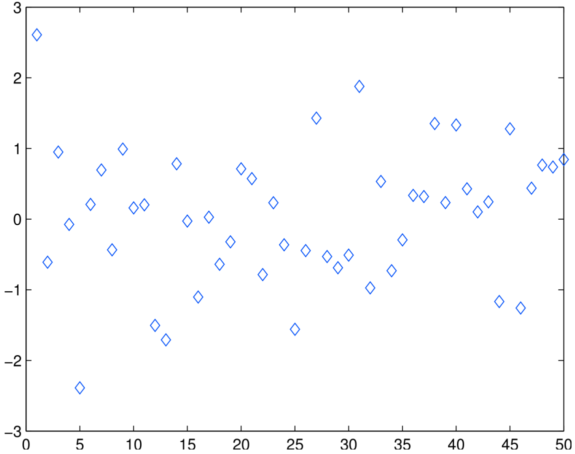

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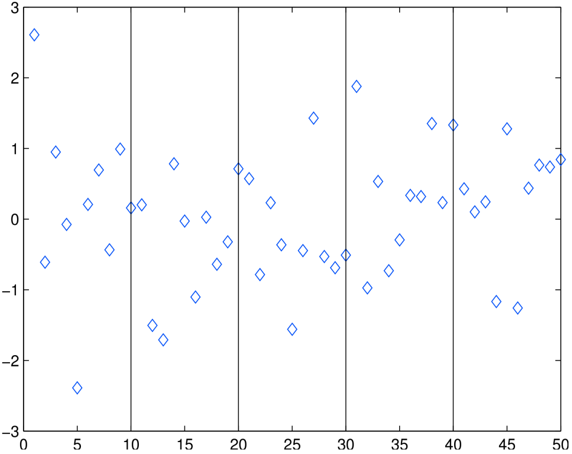

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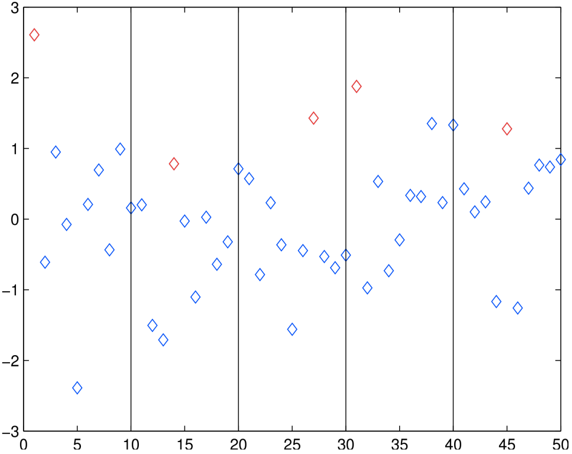

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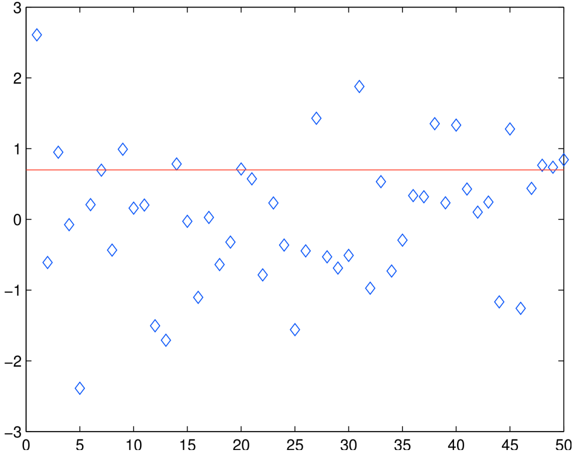

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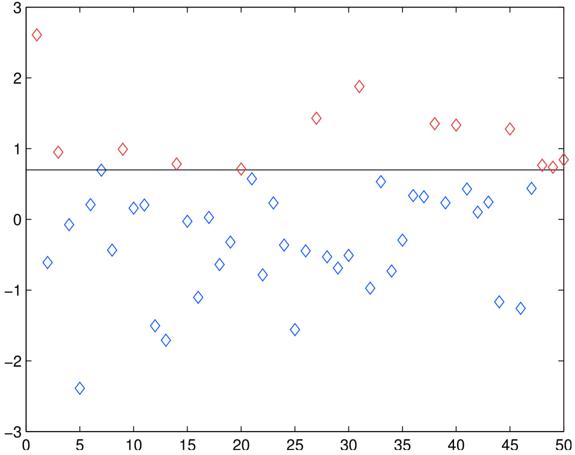

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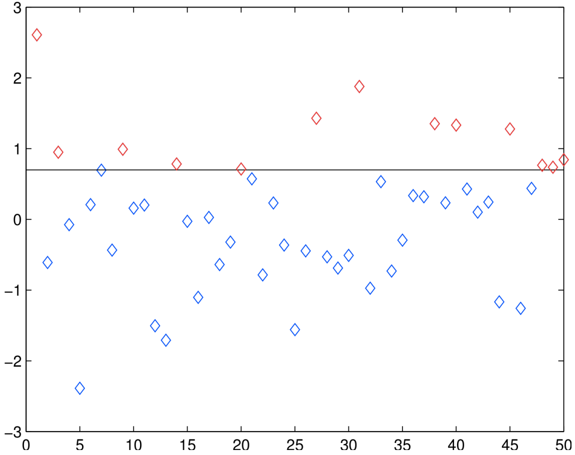

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Solutions to time-dependence in POT

2 approaches:

1. **Declustering** (Coles, 2001)
   - Identify Clusters
   - Fit the marginal distribution to maximum excess within each cluster
   - Estimate the extremal index to **correct** the return period
2. **Modelling all exceedances** of the threshold (Smith,1997; Ribatet 2009)
   - Advantages: no need to define a cluster; all excess are kept
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Smith's max-stable process

Extension of **max-stable concept for processes** (de Haan, 1986: Smith, 1990), with unit Fréchet marginal distributions.

$$\forall t \in \mathbb{R},\ Z(t) = \max_{i=1,\dots} \{\lambda_i f(t, Y_i)\}$$

with: $f(s,t) = \varphi(s-t)$, φ p.d.f of $\mathcal{N}(0,1)$, (λ_i, Y_i) points of a Poisson process with intensity $\lambda^{-2} d\lambda / ds$

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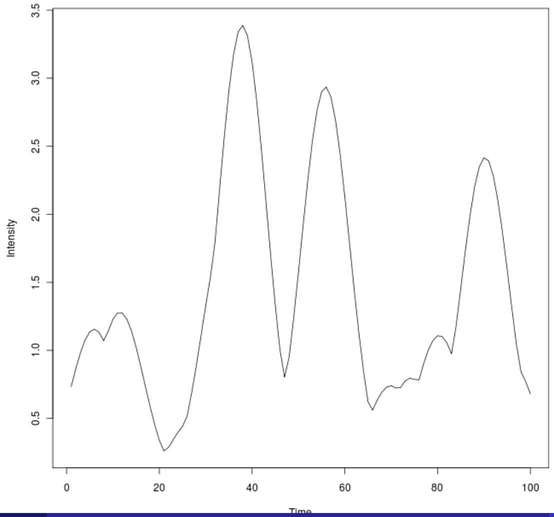

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$\Rightarrow$ use of a **composite likelihood method** (Varin, 2008):

$$CL(z_{t_1}, \ldots, z_{t_n}; \theta) = \prod_{i=1}^{n-K} \prod_{j=i+1}^{i+K} p(z_{t_i}, z_{t_j}; \theta)$$

Application to peaks over a high threshold
Model

Model for excess

- Observations $\{X_{t_i}\}_{i=1,\dots,n}$
- If $X_{t_i} > u$, $X_{t_i} = \widetilde{X}_{t_i}$ where $\widetilde{X}_t$ is a max-stable process:
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    $\Rightarrow Z_t = -1/\log G(X_t;\mu,\sigma,\xi) \sim$ Fréchet(1)
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- Under study: consistency of the uncensored one-lagged CL estimator (i.e. with $K = 1$)
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  - Obtaining asymptotic normality
  - Choice of K?
  - Results with censoring
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  - Convergence
  - Influence of parameters values
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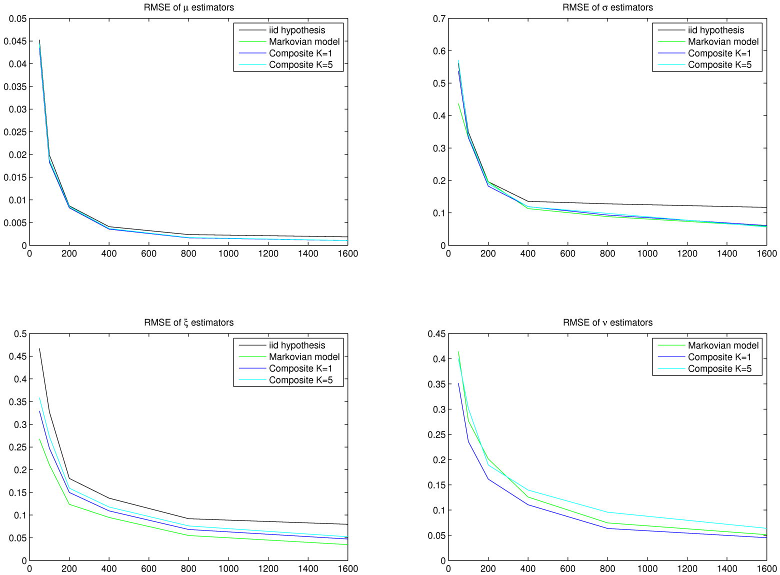

Figure: RMSE of the different estimators

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Question: Is this model able to catch extremal properties of usual time series models?

- 10 years return level: level which is exceeded on average once every 10 years
- Frequency and length of extremal events
- Sum of the values above the threshold

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Method:

- Simulation of $X_t = \varphi X_{t-1} + \mathcal{N}\left(0, \sqrt{1-\varphi^2}\right)$ of length 100 years (with 1 obs./day)
- Fitting the model
- 100 times:
  - Simulation of the fitted model over 10 years, and computation of the statistics
  - Simulation of the $AR(1)$ over 10 years, and computation of the statistics

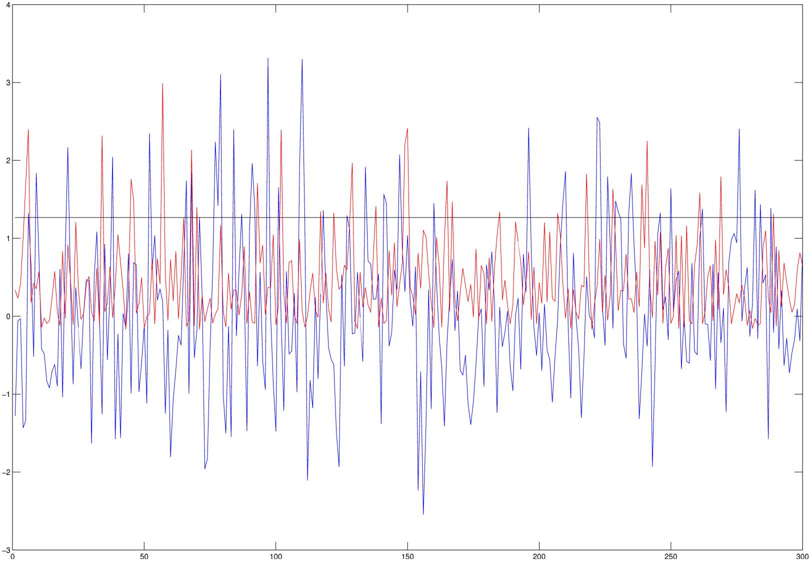

Figure: Example of simulated data (Blue) vs. fitted model (Red) and selected threshold (Black)

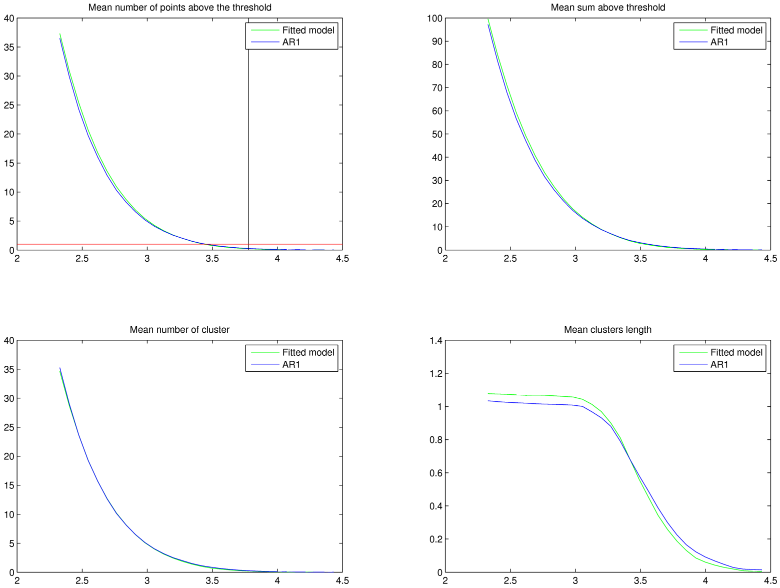

Figure: Statistics computed for an AR(1) process with $\varphi = 0.2$

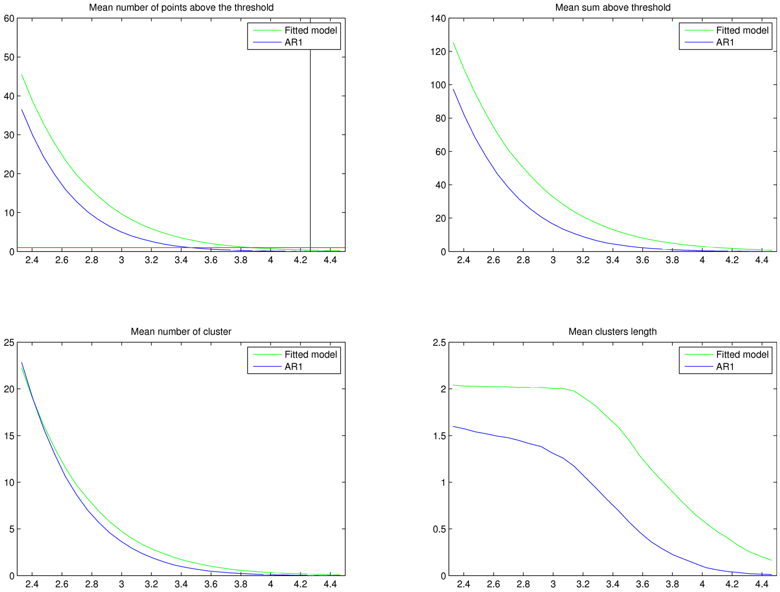

Figure: Statistics computed for an AR(1) process with $\varphi = 0.8$

Contents

Conclusion

… and prospects

- We obtained a flexible model that can account for irregular data spacing and missing values
- Ongoing: application to satellite and in-situ data
- Perspectives:
    - Theoretical results :asymptotic normality, choice of K, influence of censoring…
    - Extensions: space-time modelling…

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