

## Science Meeting – Scientific Report

**Proposal Title:** *'Inhomogeneous Random Systems 2014'*

**Application Reference N°:** 5271

### 1) Summary

The conference was organized by François Dunlop (LPTM, Université de Cergy-Pontoise), Thierry Gobron (LPTM, CNRS - Université de Cergy-Pontoise) and Ellen Saada (MAP5, CNRS - Université Paris Descartes)

It took place on Tuesday, January 28 and Wednesday, January 29, 2014 at the Institut Henri Poincaré, Paris, and brought together more than 80 participants, mathematicians and physicists, working on disordered or random systems.

The scientific aspects of the conference are provided below. Additional information is available on the conference website:

<http://irs.math.cnrs.fr>

Ellen Saada, Thierry Gobron, Francois Dunlop.

### 2) Description of the scientific content of and discussions at the event

The aim of this annual workshop is to bring together mathematicians and physicists working on disordered or random systems, and to discuss recent developments on themes of common interest. Each of the two days is devoted to a specific topic and organized together with a moderator, specialist of the field. This year, the themes were “Synchronization”, with Giambattista Giacomin as moderator, and “Time delays in stochastic systems”, with Jacek Miękisz.

**Synchronization.** Synchronization is a crucial mechanism which appears in a variety of real world phenomena, when a number of units (particles, cells, oscillators, individuals, circuits...) show a synchronous, or approximately synchronous, dynamical activity. Synchronicity may have different origins: it is often the result of interactions between units, or may be due to an external pacemaker. Synchronicity may be identified in systems made of just a few or very many units, like in multicellular organisms or in colonies of organisms. Moreover the onset of synchronicity may lead to enhancing the characteristics of the interacting units, while in other instances synchronicity may induce

substantial changes of the behavior of the single units. The purpose of this day is to give snapshots of ideas, of research directions and of unifying concepts in this field, that uses tools of dynamical systems (random and deterministic, finite and infinite dimensional) and that often demands novel probabilistic ideas, coming from random graph theory and non-equilibrium statistical mechanics.

**Time delays in stochastic systems.** It is known that time delays may cause oscillations in solutions of ordinary differential equations. In biological systems it takes time for biochemical reactions to be completed and for signals to be transmitted between various parts of the system. We will discuss combined effects of time delays and stochasticity on the behavior of gene regulatory and neural networks and other systems.

### 3) **Assessment of the results and impact of the event on the future directions of the field.**

This meeting brings together participants with different backgrounds, mostly mathematician and physicist, but also people from theoretical biology. Synchronization phenomena and time delays in stochastic systems can be encountered in a variety of fields, in physical and chemical systems as well as in biology, and at various level of organization. An overview of these various aspects by specialists of the field was clearly appreciated by the participants.

### 4) **Programme of the meeting and full list of speakers and participants**

---

Tuesday January 28:      SYNCHRONIZATION

---

Moderator: Giambattista Giacomini (Paris).

9.00- 9.10      Opening

9.10-10.00      Giambattista Giacomini (Paris): Synchronization phenomena and statistical mechanics.

10.00-10.50      Nils Berglund (Orleans): Noise-induced phase slips, log-periodic oscillations and the Gumbel distribution.

10.50-11.10      Coffee Break

11.10-12.00      Arkady Pikovsky (Potsdam): Synchronization in ensembles of oscillators: theory of collective dynamics.

12.00-12.50      Bastien Fernandez (Marseille): Properties of synchronization graphs in discontinuous forced systems.

12.50-14.20      Lunch

14.20-15.10      Khashayar Pakdaman (Paris): On some synchronization phenomena

in neuronal assemblies.

15.10-16.00 Christof Kueelske (Bochum): Discrete and continuous rotators.

16.00-16.20 Coffee Break

16.20-17.10 Paolo Dai Pra (Padova): Collective periodic behavior in dissipative ferromagnetic systems with mean-field interaction.

---

Wednesday 29 January: TIME DELAYS IN STOCHASTIC SYSTEMS

---

Moderator: Jacek Miekisz (Warszawa)

9.10-10.00 Jacek Miekisz (Warszawa): Introduction.

10.00-10.50 Andre S. Ribeiro (Tampere): Delays as regulators of the dynamics of genetic circuits.

10.50-11.10 Coffee Break

11.10-12.00 Janek Wehr (Tucson): Stochastic delay equations with colored noise -- theory and experiment.

12.00-12.50 Fatihcan Atay (Leipzig): Consensus and synchronization problems in networks with time delays.

12.50-14.20 Lunch

14.20-15.10 Tobias Galla (Manchester): Stochastic processes with delays and their application to gene regulation and epidemics.

15.10-16.00 Raul Toral (Palma de Mallorca): Stochastic description of systems with delay: applications to models of protein dynamics.

16.00-16.20 Coffee Break

16.20-17.10 Axel Hutt (Villers-les-Nancy): Additive noise tunes the stability in nonlinear systems.

17.10-18.00 Martin-Luc Rosinberg (Paris): Entropy production and fluctuation theorems in stochastic systems with time delay.

---

Abstracts:

Nils Berglund (Orléans): Noise-induced phase slips, log-periodic oscillations and the Gumbel distribution.

We will start by recalling classical results on synchronization of two coupled oscillators, and their description by an effective phase dynamics. Noise acting on such a system may induce a temporary loss of synchronicity, called a phase slip. The mathematical description of the distribution of phase slips requires methods going beyond large-deviation theory, and reveals some striking properties. In particular, Martin Day has discovered that exit locations through unstable periodic orbits depend periodically on the logarithm of noise intensity, a phenomenon

called cycling. We will explain how the cycling profile is related to the Gumbel distribution, known from extreme-value theory. Based on joint work with Barbara Gentz (Bielefeld).

Paolo Dai Pra (Padova): Collective periodic behavior in dissipative ferromagnetic systems with mean-field interaction.

Various models, mostly inspired by life sciences, consist of many particles driven by random noise and interacting through a mean-field potential; this potential is subject to dissipation and diffusion, so it is random and varying in time. We analyze some particular models, where we show that in the thermodynamic limit the system may exhibit time periodic behavior. We expect this behavior to be somewhat "universal" within ferromagnetic systems.

This research is developing in collaboration with F. Collet (Padova), M. Fischer (Padova), G. Giacomin (Paris), D. Regoli (Pisa).

Bastien Fernandez (Marseille): Properties of synchronization graphs in discontinuous forced systems.

When a contractive map is forced by a chaotic discontinuous system, the asymptotic response function that defines the attracting invariant set can be highly irregular, with a dense set of discontinuities.

In this talk, I'll describe the properties of such function in a basic example of linear real contractions forced by (generalized) Baker's maps.

In this setting, it is also natural to ask whether the invariant distributions of the base and factor systems share the same characteristics and in particular, whether the factor distribution of an absolutely continuous SRB measure in the base can be absolutely continuous. I will show that absolute continuity holds for almost every value of the factor contraction rate.

Giambattista Giacomin (Paris): Synchronization phenomena and statistical mechanics.

The word "synchronization" gathers a variety of phenomena and models coming from very different scientific domains. The first part of the talk will be an overview of synchronization phenomena and models from a mathematical perspective. Notably the emphasis will be on phase reduction and on the arising models (Kuramoto models). The second part of the talk will focus on synchronization for Kuramoto models in the limit of infinitely many interacting components and on the natural connection to nonequilibrium statistical mechanics.

Christof Külske (Bochum): Discrete and Continuous rotators.

We describe connections between systems of discrete and continuous rotators, and show how these connections can be used in the study of some systems with synchronization, both on the lattice and in mean field.

- 1) B. Jahnel, C. Külske, A class of nonergodic interacting particle systems with unique invariant measure, arXiv:1208.5433v2, to appear in Annals of Applied Probability
- 2) B. Jahnel, C. Külske, Synchronization for discrete mean-field rotators, arXiv:1308.1260

Khashayar Pakdaman (Paris): On some synchronization phenomena in neuronal assemblies.

One of the features of neuronal assemblies in nervous systems is to engage in collective regular or irregular activity, broadly referred to as synchronization. Motivated by experimental observations, this presentation will go over our approaches in modelling and analyzing the conditions for the occurrence of such dynamics in specific systems.

Arkady Pikovsky (Potsdam): Synchronization in Ensembles of Oscillators: Theory of Collective Dynamics.

In the talk theoretical approaches to the problem of synchronization of populations of coupled oscillators are discussed. Theories of Watanabe and Strogatz and of Ott and Antonsen are presented together with their generalizations. Different effects in nonlinearly coupled and strongly heterogeneous populations are discussed.

Jacek Miękiś (Warszawa): Introduction.

Time delays usually lead to oscillations in dynamical systems. We will introduce a simple time-delayed random walk which illustrates the mechanism of such a behavior. However, the

occurrence of oscillations may depend upon origins of time delays as it can be seen in some evolutionary games and gene expression models. We will briefly review models where time delays shift stationary states and those where stochastic stability of stationary states depends on time delays.

Fatihcan Atay (Leipzig): Consensus and Synchronization Problems in Networks with Time Delays.

This talk will consider the network consensus problem as a special case of synchronization. Although the problem is a deterministic one, I will indicate some close relations to Markov processes, and specifically to random walks on graphs. I will then introduce time delays into the models and study their effects on network dynamics. I will consider both discrete and distributed delays, while distinguishing between signal propagation and signal processing delays.

Tobias Galla (Manchester): Stochastic processes with delays and their application to gene regulation and epidemics.

Many of the systems modelled in biology have memory, examples are translational or transcriptional delays in gene regulation, or recovery periods in the context of infectious diseases. At the same time such systems are often composed of a finite number of particles, subject to discrete reaction events. We here focus on the mathematical description of such non-Markovian systems with intrinsic stochasticity. Master equations cannot be formulated straightforwardly and it is not clear how to derive systematic Gaussian approximations. We demonstrate that progress can be made using a path-focused view, based on generating functionals. These do not describe the time-evolution of one-time probability distributions, instead they capture the probabilities of entire paths. We derive analytical expressions for Gaussian approximations for a wide class of delay systems, and apply these to two biological problems in which delay is relevant. One is the susceptible-infective-recovered model in epidemiology and the other a model of delayed autoinhibition in gene regulation. This allows us to characterise the phenomena arising from the combination of intrinsic noise and delayed dynamics.

T. Brett, T. Galla, Phys. Rev. Lett. 110, 250601 (2013) ; T. Galla, Phys. Rev. E 80, 021909 (2009)

Axel Hutt (Villers-lès-Nancy): Additive noise tunes the stability in nonlinear systems.

The talk motivates the study of additive noise by a numerical result on the stochastic Swift-Hohenberg equation revealing a shift of stability by additive noise. The corresponding analytical study based on stochastic center manifolds elucidates the underlying mechanism. A subsequent center manifold study of delayed scalar stochastic equations introduces the similar analysis in delayed systems revealing the same stabilizing and underlying mechanism.

Andre S. Ribeiro (Tampere): Delays as regulators of the dynamics of genetic circuits.

Time delays are a key component of the dynamics of genetic circuits. Relevantly, many of these delays have a kinetics which is, to a great extent, sequence dependent and, thus, evolvable. Here, we review recent findings on the role that time delays play in cellular processes. First, we address the effects of delays that take place in transcription initiation, along with the most recent measurements of the mean and variance of the duration of these events in live, individual cells. We further address the ability that regulatory molecules of transcription have to affect both mean and fluctuations of these delays. Next, we address the effects of sequence-dependent delays and how they may be used to regulate RNA and protein production kinetics. Also addressed is the delay in the intake of regulatory molecules and its effects on the cell to cell diversity in gene activity. Finally, we consider recent findings on the potential effects of the aforementioned delays on the dynamics of genetic circuits and, consequently, on cellular phenotypes and phenotypic diversity.

Martin-Luc Rosinberg (Paris): Entropy production and fluctuation theorems in stochastic systems with time delay.

Stochastic thermodynamics has emerged over the last decade as a general theoretical framework that extends the central concepts of thermodynamics (work, heat, entropy) to the level of single stochastic trajectories [1]. It is aimed at studying the nonequilibrium behavior of small systems (colloidal particles, macromolecules, nanomechanical oscillators, etc...) subjected to large and measurable fluctuations. The so-called fluctuation theorems play a key role in this context, and the second law of thermodynamics for Markov systems is now understood as

resulting from a universal identity that relates the entropy production to the probability of observing a trajectory and its time reversal. Recent developments, at the crossroad between statistical physics and information theory, have generalized this framework to stochastic systems operating under feedback control [2,3]. In this talk, I will present ongoing work [4] that focuses on Langevin systems submitted to a continuous, non-Markovian feedback control. The non-Markovian character results from a time lag between the input and output signals, which is a common feature in many biological and artificial systems. As a consequence, the feedback-controlled systems settle into a nonequilibrium steady state where entropy is permanently produced (and cooling or heating is achieved depending on the delay). We show that non-Markovianity gives rise to a specific contribution to the entropy production due to an apparent breaking of causality when time is reversed. This so-called "Jacobian effect" is illustrated by detailed analytical and numerical calculations for linear delay Langevin equations with additive Gaussian white noise. In particular, we show that a comprehensive path integral description of the steady-state behavior is available in the overdamped limit of the stochastic motion.

[1] See e.g. U. Seifert, Rep. Prog. Phys. 75, 126001 (2012) and references therein. [2] T. Sagawa and M. Ueda, Phys. Rev. E 85, 021104 (2012). [3] T. Munakata and M. L. Rosinberg, J. Stat. Mech. P05010 (2012); ibid P06014 (2013). [4] T. Munakata, M.L. Rosinberg, and G. Tarjus (in preparation).

**Raúl Toral (Palma de Mallorca): Stochastic Description of Systems with Delay: Applications to Models of Protein Dynamics.**

The combined effects of delay and stochasticity are not completely understood, despite the fact that both effects appear simultaneously in a large variety of processes of relevance in many areas of science, such as physics, ecology or chemistry. From the mathematical point of view, stochastic processes including delay are difficult to analyze due to their non-Markovian character. Most of the previous approaches have focused on stochastic differential equations or random walks in discrete time. However the consideration of discrete variables and continuous time are the natural description of many systems such as gene regulations, chemical reactions, population dynamics or epidemic spreading, in which discreteness can be a major source of fluctuations. In this talk, I will introduce some simple, yet general, stochastic birth and death processes including delay and will discuss some of the inherent difficulties for their study. Most often, the delay time is taken to be a constant with zero fluctuations, a non very realistic assumption for the applications, since it is unusual to have a deterministic delay when the rest of the dynamics is stochastic. I will take this consideration into account by allowing the delay times to be random variables with arbitrary probability density functions. I will then present different approaches that have been developed for their analytical treatment, allowing us to derive effective Markovian models that incorporate most of the features of the delay. We apply the methodology to a protein-dynamics model that explicitly includes transcription and translation delays. In the case of delay in the degradation we rigorously derive the master equation for the processes and solve it exactly. We show that the equations for the mean values obtained differ from others intuitively proposed and that oscillatory behavior is not possible in this system. We discuss the calculation of correlation functions in stochastic systems with delay, stressing the differences with Markovian processes. The exact results allow us to clarify the interplay between stochasticity and delay.

Work in collaboration with Luis F. Lafuerza: -Role of delay in the stochastic creation process, Physical Review E84, 021128 (2011). - Exact solution of a stochastic protein dynamics model with delayed degradation, Physical Review E84, 051121 (2011). -Stochastic description of delayed systems, Philosophical Transactions A371, 20120458 (2013).

**Janek Wehr (Tucson): Stochastic delay equations with colored noise --- theory and experiment.**

In a recent experiment it was shown that a noisy electrical circuit exhibits an Ito-to-Stratonovich transition as some parameters are varied. I will explain this effect by studying a associated system of time-delayed stochastic differential equations. The results were obtained jointly with experimental physicists G. Pesce and G. Volpe, and with mathematics graduate students S. Hottovy and A. McDaniel. They were published in a recent issue (November 12) of Nature Communications.

## **Full list of speakers and participants**

ATAY Fatihcan	Max Planck Institute for Mathematics	Leipzig, Germany
BERGLUND Nils	MAPMO - Université d'Orléans	Orléans, France
BIANCHI Alessandra	Università di Padova	Padova, Italy
BLONDEL Oriane	Univ. Paris Diderot	Paris , France
BODINEAU Thierry	CMAP - École Polytechnique	Palaiseau, France
BURIONI Raffaella	Università di Parma	Parma, Italy
CABANA Tanguy	CIRB - Collège de France	Paris, France
CASARTELLI Mario	Università di Parma	Parma, Italy
COLLET Francesca	University of Bologna	Bologna , Italy
COLLET Pierre	CPHT - Ecole Polytechnique	Palaiseau, France
COMETS Francis	Universite Paris Diderot	Paris, France
CONTE Robert	ENS Cachan	Cachan, France
CONZE Jean-Pierre	IRMAR, Université de Rennes 1	Rennes, France
CORTINES PEIXOTO NETO	Aser	
	Universite Paris Diderot	Paris, France
COURBAGE Maurice	Universite Paris Diderot	Paris, France
DAI PRA Paolo	Università degli Studi di Padova	Padova, Italy
DE CONINCK Joël	Université de Mons	Mons, Belgium
DERRIDA Bernard	LPS - ENS	Paris, France
DUGUET Yohann	LIMSI-CNRS	Orsay, France
DUNLOP François	LPTM - Université de Cergy-Pontoise	Pontoise, France
DUTERCQ Sébastien	MAPMO - Université d'Orléans	Orléans, France
ERIGNOUX Clément	CMAP - École Polytechnique	Palaiseau, France
EVANS Martin	University of Edinburgh	Edinburgh, Scotland
FERNANDEZ Bastien	CPT - Université d' Aix-Marseille	Marseille, France
FORMENTIN Marco	Università degli Studi di Padova	Padova , Italy
FOURT Guy		Clermont-Ferrand, France
FRICKER Christine	INRIA	Le Chesnay, France
FROUVELLE Amic	CEREMADE - Université Paris-Dauphine	Paris, France
GALLA Tobias	The University of Manchester	Manchester, England
GANDOLFO Daniel	CPT	Marseille, France
GENOVESE Giuseppe	LPMA - Université Denis Diderot	Paris, France
GENTZ Barbara	University of Bielefeld	Bielefeld , Germany
GIACOMIN Giambattista	Université Paris Diderot	Paris, France
GIANFELICE Michele	Università della Calabria	Arcavacata di Rende, Italy
GOBRON Thierry	LPTM - Université de Cergy-Pontoise	Pontoise, France
GUPTA Shamik	LPTMS - Université Paris-Sud	Orsay, France
HILHORST Danielle	Université Paris-Sud	Orsay, France
HUILLET Thierry	LPTM - Université de Cergy-Pontoise	Pontoise, France
HUTT Axel	INRIA	Villers-les-Nancy, France
HUVENEERS François	CEREMADE - Université Paris-Dauphine	Paris, France
INGLIS James	INRIA	Sophia-Antipolis, France
KERGOSIEN Yannick	Université Paris 13	Bobigny , France
KOTECKY Roman	Center for Theoretical Study	Praha, Czech Republic
KUELSKE Christof	Ruhr-Universität	Bochum, Germany
KUOCH Kevin	Université Paris-Descartes	Paris, France
LANNOO Bruno	K.U.Leuven	Leuven, Belgium
LE NY Arnaud	Université Paris-Est	Créteil, France
LECOMTE Vivien	LPMA - Université Denis Diderot	Paris, France
LEMAIRE Sophie	Université Paris-Sud	Orsay, France
LEOS ZAMORATEGUI Arturo		
	LPMA - Université Denis Diderot	Paris, France
LIMIC Vlada	Univ Paris Sud Orsay, France	

LÖCHERBACH Eva	AGM - Université de Cergy-Pontoise	Pontoise, France
LOUIS Pierre-Yves	Université de Poitiers	Chasseneuil, France
LUCON Eric	MAP5 - Université Paris Descartes	Paris, France
MacLAURIN James	INRIA	Sophia-Antipolis, France
MAES Christian	KU Leuven	Leuven , Belgium
MERLET Glenn	IML	Marseille, France
MIEKISZ Jacek	University of Warsaw	Warsaw , Poland
MIRACLE-SOLE Salvador	CPT - Université d'Aix-Marseille	Marseille, France
NARDINI Cesare	Ens de Lyon	Lyon, France
NGUYEN Gia Bao	Université de Nantes	Nantes, France
NGUYEN Vu-Lan	LPMA - Université Denis Diderot	Paris , France
NOVAGA Matteo	Università di Pisa	Pisa, Italy
OGRYZKO Vasily	Institut Gustave Roussy	Villejuif, France
OGUEY Christophe	LPTM - Université de Cergy-Pontoise	Pontoise, France
PAKDAMAN Khashayar	Université Paris Diderot	Paris, France
PAUTRAT Yan	Université Paris-Sud	Orsay, France
PIKOVSKY Arkady	University of Potsdam	Potsdam, Germany
POLESZCZUK Jan	University of Warsaw	Warsaw, Poland
POQUET Christophe	Université Paris Dauphine	Paris, France
RIBEIRO Andre S.	Tampere University of Technology	Tampere, Finland
RIVIER Nicolas	Institut de Physique	Strasbourg, France
ROSINBERG Martin-Luc	LPTMC	Paris, France
RUIZ Jean	CPT	Marseille, France
SAADA Ellen	MAP5 - Université Paris Descartes	Paris, France
SADOC Jean-François	Université Paris-Sud	Orsay, France
SARI Tewfik	IRSTEA	Montpellier, France
SHLOSMAN Senya	CPT	Marseille, France
SLOWMAN Alexander	University of Edinburgh	Edinburgh, Scotland
SUPERNANT Jacques		Paris, France
TANRE Etienne	TOSCA - INRIA	Sophia-Antipolis, France
TARJUS Gilles	LPTMC	Paris, France
TORAL Raul	Departament de Física	Palma de Mallorca, Spain
TOUBOUL Jonathan	Collège de France	Paris, France
WEHR Janek	The University of Arizona	Tucson, USA
WIESEL Christian	Universität Bielefeld	Bielefeld, Germany
WON Yong Sul	Imperial College	London, United-Kingdom