Scientific report - Geometry, spectrum and random walks for graphs

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Time and place of the visit
Technical University Graz, July 2nd - 7th 2012

Purpose of the visit
The purpose of the visit was to seek the collaboration with Prof. Wolfgang Woess and his group towards questions concerning analysis and spectral theory on graphs.

Description of the work carried out during the visit and main results obtained.
During the visit we discussed the connection between various notions of boundaries. Usually the Royden compactification of a graph is too complicated to analyze. However, in the case where the functions of finite energy are included in the space of bounded functions, the boundary value problem with respect to the can be uniquely solved. This situation is equivalent to the fact that the graph has finite diameter with respect to a certain metric which appears in non-commutative geometry. Yet, another metric was studied earlier by Carlson. The boundaries that arise in these different ways were subject to investigation during my visit. We could identify sufficient conditions when the boundaries in question coincide. On the other hand we found counter examples when the boundaries differ. Next steps towards a criterion when the Royden boundary and the Martin boundary coincide are still be worked out.
There were also most inspiring discussions about applications of Mourre theory to various models. Implications of Puiseux expansions to the spectral theory of trees of finite cone type were explored.

Future Collaborations and Project Publications.
The project on boundaries will result in a publication where the support by the ESF will be acknowledged.