SCIENTIFIC REPORT FOR ESF

ALEXANDER FRIBERGH

I visited Munich for a week between January 6th and January 14th.

1. Purpose of the visit

The main goal was to work with Nina Gantert and Serguei Popov on biased random walks on the interlacement set of $\mathbb{Z}^d$ for $d \geq 3$.

This model is reminiscent of the biased random walk on supercritical clusters in the sense that we are performing a biased random walk on a random graph which is very irregular. In the percolation case, it is known that, in any dimension $d \geq 2$, the limiting speed of the random features a sharp phase transition from positive to zero speed as the bias increases [2]. We wanted to see if this kind of property was conserved in the interlacement set.

Studying model is interesting since random walks in random environments with long range correlations (in the environment) are poorly understood and hardly any method exists to study these model. One of the key properties used to study directionally transient i.i.d. random walks in random environments is the existence of regeneration structure and the fact that these regeneration times are i.i.d. When introducing long range correlations, this key point does not hold anymore and essentially the proofs for all significant results break down, in particular nothing general is know about the asymptotic speed of the random walk.

2. Description of the work carried out and the results obtained during the visit

We were able to show that an interesting phenomenon occurs for the interlacement set in dimension $d = 3$. Indeed, in that case, the speed is always 0 and that actually $||X_n|| \leq n^{\varepsilon}$ for any $\varepsilon > 0$, which contrasts strongly with the situation in the percolation case.

In dimensions $d \geq 4$, we expect the speed to exhibit a similar phase transition but we are not yet able to prove that. It seems that our current techniques should be enough to prove that the speed is zero for high biases but they will not extend to prove the positivity of the speed for small biases, let alone the phase transition from positive to zero speed.

In the last days of my visit, we found a sketch of proof for trying to prove that, in some weak sense, the speed is positive for small biases. Nevertheless, certain steps in this sketch of proof are a bit shaky and we did not have time to follow through and check whether or not these argument can be made rigorous.

3. Possible future collaborations

As an added benefit of visiting Munich I was led to talk to Benedikt Rehle, a master student of Nina Gantert, and discuss with him some of my previous works. He is currently working on a project to extend some results I obtained with Gérard Ben Arous, Nina Gantert and Alan Hammond [1], and I think that some of my insights could him make some progress on this project. I discussed with Nina Gantert some potential project that he could work on for his pHD and it may be possible that this turns into a new collaboration at some point.

I also got the opportunity to talk with Noam Berger, we discussed some potential projects related to a special version of the DLA and some fracturing models. Those projects are still rather vague but we plan to discuss them again in the future.

4. Projected papers

It appears very likely that some publication will come out of this visit. The result on the sub-polynomial speed of the biased random walk on the interlacement set is in the process of being written and should be finished within a few months. This result on its own would be good enough for a publication, but we may postpone the publication to see if we can obtain further results in higher dimensions which would make the article a bit more complete.

Conclusion

To sum it up, this visit was successful since Nina Gantert, Serguei Popov and I managed to prove a major difference between biased random walks on the percolation set and the interlacement set. Also, I think at some point another project will come out of this visit, indeed the subjects I discussed with Noam Berger seemed to be very exciting.

References

[1] Ben Arous, G., Fribergh, A., Gantert, N. and Hammond, A. (2010). Biased random walks on a Galton-Watson tree with leaves. *Ann. of Probab.*.

[2] Fribergh, A. and Hammond, A. (2012). The phase transition of the speed of biased random walk on percolation clusters. *Comm. Pure Appl. Math.*.