Report:

Spring school in discrete probability, ergodic theory and combinatorics
TU Graz, April 4 – 15, 2011

The program lists of participants and main speakers, as well as abstracts, are attached.

The material is also available at

`www.math.tugraz.at/discrete/index.php?link=events&link2=spring_school2011`

The RGLIS funding was used to pay one of the main speakers (Steif) and young participants from abroad. Two main speakers (Björklund, Karlsson) were funded as visiting professors of TU Graz. The 4th main speaker (Grimmett) was paid by the Doctoral Program "Discrete Mathematics" which is funded by the Austrian Science Fund FWF. Further funding also came from the Austrian Math. Society.

Spring School in Discrete Probability, Ergodic Theory and Combinatorics

Organizers: Ecaterina Sava and Wolfgang Woess

April 4-15: 2011, Graz

Main Speakers

1. Geoffrey Grimmett (Cambridge)
2. Jeffrey Steif (Göteborg)
3. Anders Karlsson (Geneva)
4. Michael Björklund (Zürich)

Local Participants

1. Wolfgang Woess
2. Ecaterina Sava
3. Franz Lehner
4. Elisabetta Candellero
5. Tetiana Boiko
6. Christoph Temmel
7. Lorenz Gilch
8. Markus Hofer
9. Oliver Ebner
10. Daniel Krenn
11. Florian Sobieczky

External participants

1. Reto Spöhel, Germany
2. Johannes Cuno, Germany
3. Jan Czajkowski, Poland
4. Konrad Kolesko, Poland
5. Mashaka Mkandawile, Sweden
6. Benjamin Matschke, Germany
7. Jan Hladky, UK
8. Diana Piguet, UK
9. Justine Simone Louis, Switzerland
10. Rim Essifi, France
11. Olga Glebova, Belarus
12. Tianyi Zheng, USA
13. Antonella Iuliano, Italy
14. Parkpoom Phetpradap, UK
15. Florica Naghiu, Romania
16. Mirjana Rakic, Serbia
17. Simon Aumann, Germany
18. Maria Infusino, Italy
19. Satyanarayana Reddy, India

Detailed program of the Spring School

Organizers: Ecaterina Sava and Wolfgang Woess

April 4-15: 2011, Graz

Main Lectures

- **Geoffrey Grimmett (Cambridge)**
  Title: Uniform spanning trees and other random animals.
  Abstract: Spanning trees, self-avoiding walks, connected clusters, entanglements: these are classes of graphs which, when chosen randomly, possess especially rich probabilistic structure. Their study impacts on such topics as combinatorics, geometry, and interacting systems in probability and physics. Some of their basic theory will be developed in these lectures.

- **Jeffrey Steif (Göteborg)**
  Title: Noise sensitivity and percolation.
  Abstract: In these lectures, I will introduce the notions of noise sensitivity and noise stability for Boolean functions. Many examples will be given. Discrete Fourier analysis plays a central role in this theory. One of the main examples illustrating noise sensitivity is crossing events in percolation theory which will be described in detail. There are three methods to studying sensitivity of percolation; hypercontractivity, randomized algorithms and the geometric study of the spectrum.

- **Anders Karlsson (Geneva)**
  Title: Discrete heat kernels and applications.
  Abstract: I will define heat kernels on graphs and deduce a general expression for the heat kernel on Cayley graphs in terms of Bessel functions. There is also a spectral expression for the heat kernel. Equating these two expressions gives formulas which via certain transformations eventually lead to several applications. These are relevant for some questions in combinatorics, differential geometry, number theory, and statistical physics.

- **Michael Björklund (Zürich)**
  Title: Ergodic theory in additive combinatorics.
  Abstract: The aim is to introduce some useful techniques in ergodic theory for studying problems in additive combinatorics. After a crash course in ergodic theory, we will discuss Furstenberg's approach to Szemeredi's theorem, recent advances on this topic, and how some questions about product sets in groups can be understood via stationary processes.

Titles and Abstract of the Talks

1. **Reto Spöhel, Germany**
   Title: Coloring random graphs online without creating monochromatic subgraphs
   Abstract: Consider the following generalized notion of graph coloring: a coloring of the vertices of a graph G is *valid* w.r.t. some given graph F if there is no copy of F in G whose vertices all receive the same color. We study the problem of computing valid colorings of the binomial random graph $G_{n,p}$ on n vertices with edge probability $p = p(n)$ in the following online setting: the vertices of an initially hidden instance of $G_{n,p}$ are revealed one by one (together with all edges leading to previously revealed vertices) and have to be colored immediately and irrevocably with one of r available colors. It is known that for any fixed graph F and any fixed integer $r \geq 2$ this problem has a threshold $p_0(F, r, n)$ in the following sense: For any function $p(n) = o(p_0)$ there is a strategy that a.a.s. (asymptotically almost surely, i.e., with probability tending to 1 as n tends to infinity) finds an r-coloring of $G_{n,p}$ that is valid w.r.t. F online, and for any function $p(n) = \omega(p_0)$ *any* online strategy will a.a.s. fail to do so.

   We establish a general correspondence between this probabilistic problem and a deterministic two-player game in which the random process is replaced by an adversary that is subject to certain restrictions inherited from the random setting. This characterization allows us to compute, for any F and r, a value $\gamma = \gamma(F, r)$ such that the threshold of the probabilistic problem is given by $p_0(F, r, n) = n^{-\gamma}$. Our approach yields polynomial-time coloring algorithms that a.a.s. find valid colorings of $G_{n,p}$ online in the entire regime below the respective thresholds, i.e., for any $p(n) = o(n^{-\gamma})$.

2. **Mashaka Mkandawile, Sweden**
   Title: Coloring from random lists
   Abstract: Let $B(n, m)$ be a bipartite graph with n vertices in each side and m edges. For each vertex in $B(n, m)$ we draw uniformly at random a list of size k from some base set of size n. The research intends to design a heuristic algorithm for existence of perfect matching in $B(n, m)$.

3. **Jan Hladky, UK**
   Title: Hamilton cycles in dense vertex-transitive graphs
   Abstract: Lovasz asked whether every connected vertex-transitive graph contains a Hamilton path. We prove that if the graph is sufficiently dense, then it contains even a Hamilton cycle. The regularity lemma is the main tool in the proof.

4. **Satyanarayana Reddy, India**
   Title: Pattern polynomial graphs
   Abstract: Let A be the adjacency matrix of a graph X. The set of all polynomials in A with coefficients from the field of complex numbers C forms an algebra called the adjacency algebra of X, denoted by $A(X)$. A graph X is said to be a pattern polynomial graph if $A(X)$ is a coherent

algebra. In this study we explore some necessary conditions for a graph to be a pattern polynomial graph.

5. **Jan Czajkowski, Poland**
   Title: Clusters in middle-phase percolation on hyperbolic plane
   Abstract: I consider p-Bernoulli bond percolation on graphs of vertex-transitive tilings of the hyperbolic plane with finite sided faces (or, equivalently, on transitive, nonamenable, planar graphs with one end) and on their duals. It is known (Benjamini and Schramm) that in such a graph G we have three essential phases of percolation, i. e.

   $$0 < p_c(G) < p_u(G) < 1,$$

   where p_c is the critical probability and p_u – the unification probability. I prove that in the middle phase a. s. all the ends of all the infinite clusters have one-point boundary in ∂H^2. This result is similar to some results of Lalley.

6. **Christoph Temmel, TU Graz**
   Title: K-independent percolation on trees
   Abstract: Consider the class of k-independent bond, respectively site, percolations with parameter p on an infinite tree T. We derive bounds on p in terms of k and the branching number br(T) of T for both a.s. percolation and a.s. non-percolation. The bounds are tight for the whole class, coincide for bond and site percolation and are continuous functions of T. This extends previous results by Lyons for the independent (k=0) case and by Bollobas and Balister for the 1-independent bond percolation case. Central to our argumentation are moment method and capacity estimates a la Lyons supplemented by explicit percolation models. An indispensable tool is the minimality and explicit construction of Shearer's measure on the k-fuzz of the integers.

7. **Olga Glebova, Belarus**
   Title: Krausz dimension and its generalizations in special graph classes
   Abstract: A krausz (k, m)-partition of a graph G is the partition of G into cliques, s.t. any vertex belongs to at most k cliques and any two cliques have at most m vertices in common. The m-krausz dimension $kdimm(G)$ of the graph G is the minimum number k such that G has a krausz (k, m)-partition. We prove that the problem $kdim(G) \leq 3$ is polynomially solvable for chordal graphs. We show that the problem of finding m-krausz dimension is NP- hard for every $m \geq 1$, even if restricted to $(1, 2)$-colorable graphs, but the problem $kdimm(G) \leq k$ is polynomially solvable for $(, 1)$-polar graphs for every fixed $k, m \geq 1$.

8. **Mirjana Rakic, Serbia**
   Title: Doubly biased Maker-Breaker Connectivity game
   Abstract: We study $(a : b)$ Maker-Breaker Connectivity game, played on the edge-set of K_n , the complete graph on n vertices, where the winning

sets are all spanning trees of K_n . Maker and Breaker take turns in claiming previously unclaimed edges of K_n , with Breaker going first. Breaker claims b edges per turn, while Maker claims a edges per turn. Maker wins the game, if by the end of the game, he manages to claim all edges of one spanning tree. Breaker wins otherwise. We will determine the winner of the game for almost all values of a and b. This is joint work with Dan Hefetz and Milos Stojakovic.

9. **Antonella Iuliano (Italy)**
   Title: On a bilateral birth-death process with alternating rates
   Abstract: We study a bilateral birth-death process characterized by two different transition rates from even states and from odd states. We evaluate the generating functions of the probability of even and odd states. These allows to determine the transition probabilities, the mean and the variance of the process for arbitrary initial state. Certain symmetry properties of the transition probabilities are pinpointed. Some features of the birth-death process confined to the non-negative integers by a reflecting boundary in the zero-state are then analyzed. In particular, making use of a Laplace transform approach we obtain a series form of the transition probability from state 1 to the zero-state.

10. **Lorenz Gilch, TU Graz**
    Title: Branching Random Walks on Free Products
    Abstract: In this talk I will give an introduction to branching random walks on free products. A branching random walk is a discrete-time process, which can be described in the following way. An initial particle starts at some vertex of the Cayley graph of the free product. At each instant of time, each particle produces in a first stage some offspring according to an offsping distribution and in a second stage each of the offspring particles moves independently to a neighbour element in the free product. That is, each particle performs its own independent single random walk from its place of birth. I will summarize facts and results about the behaviour of survival of the process and also about the behaviour, how the particle cloud moves towards the boundary concerning speed and direction.

11. **Maria Infusino, Italy**
    Title: Discrepancy of Kakutani's sequences
    Abstract: In this talk we intend to present the concept of uniformly distributed (u.d.) sequences of partitions, firstly introduced by Kakutani in 1976. He proposed a construction which generates a whole class of u.d. sequences of partitions of [0, 1], called Kakutani's splitting procedure. This result received a considerable attention in the late seventies, but there have not been any quantitative results about the behaviour of these sequences for thirty years. Only recently the interest for this subject has been revived and discrepancy bounds have been given for a class of Kakutani's sequences and for some of their generalizations. In particular, we show a special example of Kakutani's sequence for which it is possible to give the explicit value of its discrepancy: the Kakutani-Fibonacci sequence of

partitions. This sequence has a discrepancy of order $1/N$, which is the best possible. The Kakutani-Fibonacci sequence belongs to a larger class of u.d. sequences of partitions called LSsequences. For all these sequences there exist explicit estimates of the discrepancy and a countable subclass of LSsequences has low-discrepancy. LSsequences are in turn a subclass of u.d. sequences of partitions produced through successive $\rho-$refinements of [0, 1]. This technique is a generalization of Kakutani's splitting procedure and it allows to obtain new families of u.d. sequences of partitions. Discrepancy bounds have been recently introduced for a class of $\rho-$refinements, including a countable number of Kakutani's sequences.

12. **Parkpoom Phetpradap, UK**
Title: Large deviation for the range of a simple random walk
Abstract: For a simple random walk on the integer lattice $\mathbb{Z}^d$ in dimension $d \geq 3$, we consider the number R_n of sites visited up to time n. From a law of large numbers of Dvoretzky and Erdös (1950) we know that $\frac{1}{n}R_n$ converges almost surely to κ, the probability that the random walk never returns to the origin. We show that, for $0 < b < \kappa$, we have
$$\lim_{n\to\infty} \frac{1}{n^{\frac{d-2}{d}}} \log \mathbb{P}(R_n \leq bn) = -\frac{1}{d} I^\kappa(b),$$
where I^κ is an explicitly given rate function. This complements an analogous result for the volume of the Wiener sausage, obtained by van den Berg, Bolthausen and den Hollander (2001).

13. **Benjamin Matschke, Germany**
Title: A few examples on how to use equivariant topology in combinatorics and geometry
Abstract: Many problems in geometry and combinatorics contain a natural symmetry such that one can try to apply equivariant algebraic topology methods to handle them. This talk will be a very quick survey on a few methods and examples to illustrate that machinery.

14. **Konrad Kolesko, Poland**
Title: Tails of solutions of inhomogeneous smoothing transform
Abstract: We study fixed points of an inhomogeneous smoothing transform, i.e. solutions of the equation $X \stackrel{d}{=} C+\sum_{i=1}^N T_i X_i$, where $(C, T_1, \ldots, T_N)$ is a given sequence of non-negative random variables and $X_1, \ldots, X_N$ independent copies of X. Assuming that there exists a positive α such that $\sum_i T_i^\alpha = 1$, and $\sum_i T_i^\alpha \log T_i = 0$ (the critical case) we describe the tail of X.