SCIENTIFIC REPORT FOR ESF

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I visited Switzerland for two weeks: one week spent in Zürich and the other in Geneva.

1. Purpose of the visit

Between November 11th and 18th, I stayed in Zürich to work with Vladas Sidoravicius. Our goal was to study once reinforced random walks in $\mathbb{Z}^d$ for $d \geq 3$. Seen differently this is a model of random walks that are more likely to move on their past trace.

It has been conjectured that depending on the strength of the reinforcement two regimes will occur:

(1) if the strength of the reinforcement is weak then the walk should scale as a Brownian motion and hence should be transient,
(2) if the strength of the reinforcement is strong then the walk will be recurrent.

Our main goal was to make progress towards proving the first part of the conjecture.

In the second week from the 18th to the 25th, I stayed in Geneva to work with Alan Hammond. We studied the scaling limits of biased random walks on the percolation cluster of $\mathbb{Z}^d$. This is the natural continuation of two of our previous papers. The first one is about scaling limits for biased random walks on Galton-Watson trees with leaves [1], in which we showed that scaling limits exist only along exponential subsequences. The second paper [2], proves the phase transition from positive to zero-speed regime for the speed of biased random walks on the percolation cluster of $\mathbb{Z}^d$.

2. Description of the work carried out and the results obtained during the visit

Firstly, let me address the work made during my stay in Zürich. We had previously observed that a walk that is more likely to erase the last steps of its past trajectory than to move to new sites verifies an invariance principle (provided the reinforcement to retrace our steps is not too strong). This result already existed but we had found an alternative and simple proof. The issue with this result is that walk is not really interacting with the geometry of its past trace, this makes a significant difference with the original model of once reinforced random walk that we are trying to solve.

We managed to extend this result to see that a walk interacting with the last k distinct edges that were added to its trace verifies an invariance principle (provided the reinforcement towards those k edges in not too strong). Although this result is not our ultimate goal, it already appears to extend a result by Holmes [3] and Holmes and Sakai [4] on walks interacting with the last visited edge (which is our result for $k = 1$). This model of interaction with the last k vertices captures some complex interactions with its past trajectory. For example, if the trace forms a loop then the interacting walk is allowed to erase it entirely in one step. This important phenomenon is totally absent from the model in which we are more likely to retrace the last steps of the walk.

The methods used to prove this result are very robust (they can be extended for a random k under mild conditions) and for this reason we have hope that they may be a possibility to push these methods to eventually obtain a result on the behavior of once-reinforced random walks.

Concerning the work that was done with Alan Hammond in Geneva our goal was to draw a parallel between biased random walks on a percolation cluster and on Galton-Watson trees [1]. This meant that we expected to prove that scaling limits exist for biases with an irrational direction and exist only along exponential subsequences when the direction is rational. Our main focus for now was the case where $d \geq 3$ (which differs from the 2-dimensional case).

This project is a substantially long one so it is by no means possible to finish it over a week. Nevertheless, we managed to give a credible plan to tackle the problem and discussed in depth the more critical points of the proof: the existence of an environment seen from the particle (in a weak sense) and some control on its Radon-Nikodym derivative with respect to an unconditioned environment. It seems very likely that we will eventually be able to prove the result that we were after. There is a long writing process ahead but I am pretty confident that the main difficulties have been addressed.

We also discussed the case $d = 2$, and it is probable that, after having finished the case $d \geq 3$, we would also be able to deal with this case too.

3. Possible future collaborations

During my stay in Geneva I also got the opportunity to meet with Hugo Duminil-Copin and, along with Alan, we discussed the Alexander-Orbach conjecture in dimension 2, or, in other words, we tried to understand the behavior of the heat kernel of a simple random walk on the critical cluster in dimension 2. We did not make any significant progress at the time, but nevertheless we have not given up on the project and this could lead to a future collaboration.

As I mentioned before the work with Alan Hammond on biased random walk on percolation clusters presents significant differences between dimension $d = 2$ and $d \geq 3$. After finishing the case $d \geq 3$, it would be natural for us to start a future project to handle the case of $\mathbb{Z}^2$.

4. Projected papers

It appears very likely that we will publish an article called "Scaling limits for the biased random walks on the supercritical percolation cluster of $\mathbb{Z}^d$ for $d \geq 3$" with Alan Hammond. The timeframe for this article to be finished should be around 1 to 2 years.

Although further work is probably needed it is likely that another article with Alan Hammond called "Scaling limits for the biased random walks on the supercritical percolation cluster of $\mathbb{Z}^d$ for $d = 2$" would appear at some point within 2-3 years.

The project with Vladas Sidoravicius still needs new ideas for us to reach our main goal. Even if the project were to stay at the current the point, it would still be possible to publish a paper on our intermediate result since it extends previously existing results in a simple manner. Nevertheless, at the moment, it is unclear if we are going to publish an article or simply write a note on this result.

References

[1] Ben Arous, G., Fribergh, A., Gantert, N. and Hammond, A. (2010). Biased random walks on a Galton-Watson tree with leaves. *Ann. of Probab.*.

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[3] Holmes, M. (2009). The scaling limit of senile reinforced random walk. *Electron. Commun. Probab.* **14**, 104–115

[4] Holmes, M. and Sakai, A. (2007) Senile reinforced random walks. *Stochastic Process. Appl.*, **117 (10)**, 1519–1539.