SCIENTIFIC REPORT ABOUT THE SHORT VISIT SUPPORTED BY THE ESF PROGRAM "RANDOM GEOMETRY OF LARGE INTERACTING SYSTEMS AND STATISTICAL PHYSICS"

Grantee: **Prof. Bálint Tóth**
Host: **Prof. Dmitri Ioffe**
Host institution: **Technion – Israel Institute of Technology**
Dates: **15-29 January 2012**

1. Purpose of the visit:

Continuation of ongoing joint research project on investigation through stochastic representations of quantum spin systems and interacting Bose lattice gases. In particular: we investigate mean field variants of these models.

We started this collaboration in 2010, after realizing that our earlier separate work on quantum spin systems have many common features and joining our expertise could lead to new approaches. During an earlier visit of Ioffe at Budapest we outlined and started a very concrete program. We gave precise stochastic formulation for the quantum systems, using Feynman-Kac-type and related representations.

The specific purpose of the present visit was to finalize the work started in summer 2011 in Budapest. More precisely, to carry out the Freidlin-Wentzell-type large deviation analysis for the models under investigation.

2. Description of the work carried out during the visit:

On the way we encountered some unexpected surprises which slowed down our progress. It turned out that our planned work is closely connected with the so-called Mott transition of Bose-Hubbard lattice gas system. That is: insulator-superconductor phase transition controlled by the density of particles. Since this was an unexpected turn, we had to read and understand a number of papers on this problem. The upshot is that we could relate our work and approach to earlier work of R. Fernandez, J. Frohlich, D. Ueltschi, respectively, J-B. Bru and T.C. Dorlas. We did understand these connections and applied successfully our stochastic geometric approach to the model in question.

3. Description of the main results obtained:

We could describe the phase diagram of the mean-field (Curie-Weiss type) Bose-Hubbard system in the ground state (zero temperature), with good hope of extending these results to positive temperature. We found an alternative approach to that of Bru and Dorlas, which seems to give more information about the superfluid-insulator (Mott) transition and certainly it is mathematically more transparent. Unfortunately, we were not yet able to produce a paper about these results, as it was originally planned.

4. Future collaboration with host institution:

The collaboration between the two parties and the two institutions will continue. A longer visit of Ioffe and a shorter visit of Nicholas Crawford to Budapest are planned for spring 2013. Also, shorter encounters between Ioffe and Toth at various other locations (Bonn, Bristol) are planned in the nearer future.

5. Projected publications / articles resulting or to result from the grant:

On the basis of preliminary results already obtained (during this visit and also earlier collaboration) we expect that the research efforts will lead soon to a number of joint publications. Specifically, we hope two types of results in two different (groups of) papers: (a) ground state and finite temperature phase diagram for Curie-Weiss (infinite range hopping) models of quantum spin systems and Bose gas; (b) [resulting from earlier joint work] to derive a closed form stochastic description of phase diagrams for quantum Heisenberg and transverse field Ising models on homogeneous trees (so-called Cayley graphs).

6. Other comments:

Besides the research and collaboration described above we also continue joint thinking about random walks with various long memory mechanisms and other stochastic models arising in the context of statistical physics. In particular, a model of interface separating two phases in a 2-dimensional model, investigated by Ioffe and Shlosman may be (actually: seems to be) related in some way to the models of self-repelling walks and processes (the *true self-repelling motion*) investigated by Toth and Werner in the late nineties.

Also: Toth and Nicholas Crawford (Technion) consulted repeatedly about random walks with long memory induced by self-interaction through local time (e.g. reinforced rw).

Report completed: 20 February 2012.

Bálint Tóth