Scientific Report:

Clustering effects in spatial preferential attachment networks

Visit of Dr E Jacob (Lyon) hosted by Prof P Mörters (Bath)

Purpose of the visit. The main aim of this visit was a scientific collaboration on the study of a preferential attachment network in which all randomness stems from randomly chosen spatial positions of the vertices. We wanted to prove that this model enjoys particular features, which are desirable for the modeling of real networks. Our main focus was on empirical degree distributions and empirical clustering coefficients.

Description of the work carried out during the visit. An initial idea in our proofs was to replace the concentration of measure arguments, which are often used to derive limit theorems for empirical quantities in power law networks, by a geometric law of large numbers as found in the work of Yukich and Penrose for geometric random models. This approach was successful, and we were able to modify the approach of Yukich and Penrose in a way suitable for our class of models. Key step is to establish an infinite idealised graph, which describes the local neighbourhood of a typical vertex in the network. More precisely, considering a functional depending on this local neighborhood, the law of large numbers states that, under certain integrability conditions, the average of this functional taken over all the vertices in the network, converges in probability to the expectation of this functional in the infinite idealised graph. Thanks to this theorem, the understanding of the network relies largely on the understanding of the infinite picture. Using this approach, we could get the expected results for the empirical degree distributions and average clustering coefficients. We also identified global clustering coefficients, and the rescaled edge lengths distribution, which both show an interesting phase-transition.

Description of the main results obtained.

1. The empirical indegree distribution converges to a limiting distribution, which is known explicitly and, by varying the model parameters, can be a power-law with any parameter $\tau \in (2, \infty)$.

2. The empirical outdegree distribution converges to a limiting distribution. It is not Poisson, as in some other preferential attachment models. However, it has light tails.

3. The empirical network average clustering coefficient converges to a positive constant.

4. The empirical global clustering coefficient converges to a positive constant if $\tau > 3$, and to zero if $\tau \leq 3$.

5. The empirical rescaled edge lengths distribution converges to a limiting distribution. Its tail is polynomial with exponent $2 - \tau$ if $2 < \tau \leq 3$, with exponent -1 if $\tau > 3$.

We are currently writing a paper gathering these results.

Future collaboration

The collaboration has been successful, and will continue. The most important directions for future research are

(a) Generalisation of the current result to a larger model class. This may be possible at very little cost, and might be done at the writing-up stage.

(b) Study of the connectivity properties of the network, like existence, size and diameter of a giant component. The local limit we have constructed will certainly help in the study of these properties. However, we would, at least, need a much better understanding of this infinite graph.

Further visits, either of Dr E Jacob to Bath, or of Prof P Mörters to ENS Lyon, are planned for the second half of the year. We are very grateful to ESF and the RGLIS committee for the support, which allowed this visit to happen.