**A report on the visit at Technische Universität Graz**
**for the period Sept.24-Oct.4, 2012**

The main goal of my visit was to work with Prof. Mihyun Kang at the Technische Universität Graz on our joint project related to the *phase transition* phenomena in random graphs models.

During this visit we focused on *Marcus-Lushnikov processes*, in which two components of sizes $i$ and $j$ are merged at a given rate $K(i,j)$. Then, the evolution of the expected number of components of size $i$ at time $t$, denoted by $c_i(t)$, is described by *Smoluchowski's coagulation equation*: for $t \geq 0$ and for $i \in \mathbb{N}$,

$$c_i'(t) = \frac{1}{2}\sum_{j=1}^{i-1} K(i-j,j)c_{i-j}(t)c_j(t) - \sum_{j=1}^{\infty} K(i,j)c_i(t)c_j(t).$$

Thus, for instance, if $K(i,j) = ij$, then the Marcus-Lushnikov process corresponds to the classical Erdős-Rényi random graph process [2], which starts with $n$ isolated vertices and edges are added randomly one at a time. The *distribution of component sizes* for different functions $K(i,j)$ have been studied by various authors, both mathematicians and physicists (see, for instance, [1, 3, 4, 5, 6]). However, although a substantial number of results on this model can be found in this literature, many of them are based on some non-rigorous heuristic; in our opinion, in this area there is a large space for further studies.

During this short visit we have worked on two types of problems. Firstly, we have tried to show rigorously that some random variables (such as the number of components of a given size) are sharply concentrated around their expectation during the whole process. Secondly, we have looked at the very end of the process, when there are just a few components which are about to merge into one.

Although our preliminary investigations have not led to publishable results yet, we plan to pursue our studies of Marcus-Lushnikov processes. Needless to say, we will acknowledge the ESF on any publications which emerges from this project.

# References


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