Scientific Report:

Connectivity of spatial preferential attachment networks

Visit of Dr E Jacob (Lyon) hosted by Prof P Mörters (Bath)

Purpose of the visit. The main purpose of this short visit was to determine the regime of existence of a robust giant component (or robustness regime) in a spatial preferential attachment network model, which we studied in a previous work supported by an RGLIS grant[1]. We have shown there that this model is scale-free, just like the usual preferential attachment models (PA models), but has a very different local geometry characterized by a positive average clustering coefficient. The study of the robustness regime is the next natural property of this model to consider.

Thanks to previous work, we know this issue is closely linked to the robustness of percolation in an infinite random geometric graph, which has some features of Poisson Boolean models and Poisson random-connection models.

Description of the work carried out during the visit. In usual PA models, the power law exponent $\tau \in (2, +\infty)$ determines the robustness regime. There is a robust giant component if and only if $\tau \leq 3$, and then this component is unique and has doubly logarithmic diameter (if $\tau < 3$). In our model, an intricate first moment calculation led us to the conjecture that the phase transition should occur at the critical value $\tau = 2 + 1/\delta$. This new parameter $\delta \in (1, \infty]$ is the asymptotic decay rate (in x) of the probability of being connected to a typical vertex located at distance x.

It has proven not difficult to see that $\tau > 3$ implies non-robustness, and we quickly obtained a partial robustness result if $\tau < 2 + 1/\delta$. Therefore we focused on proving non-robustness for some $\tau \leq 3$. First, for $\delta = \infty$, our model shows similarities with a Poisson boolean model. In Meester and Roy, non-robustness of percolation for such a model is shown by a coupling with a multitype branching process, and a "trick", the fact that balls which are contained in another ball cannot help constructing an infinite component. Inspired by these ideas, we searched for a way to explore a connected component by counting only the connections that could effectively help us to discover new domains in space.

We eventually found quite an intricate construction, inducing remarkably easy computations. A simple operator analysis then shows non-robustness, for any τ in the case $\delta = \infty$. To our surprise, the proof could be adapted to show non-robustness for finite $\delta > 1$ if $\tau < 2 + 1/(\delta - 1)$.

[1] arXiv:1210.3830

Description of the main results obtained. Here are the results for which we should have all the ingredients of the proof.

1. There is *never* a robust giant component when $\tau > 3$. This is in line with the idea that spatial correlations do not increase connectivity.

2. There is *at least one* robust giant component when $\tau < 2 + 1/\delta$. This giant component contains a subgraph of the same order with doubly logarithmic diameter. This is far from a complete result. We believe there is actually a unique giant component, with doubly logarithmic diameter.

3. There is *no* robust giant component when $\tau > 2 + 1/(\delta - 1)$.

The case $2 + 1/\delta \leq \tau \leq (2 + 1/(\delta - 1)) \wedge 3$ remains unsolved, even if we believe this is a case of non-robustness. Our results already show that the phase transition of the robustness regime in our model is richer and more complex than in the non-spatial PA models. The parameter δ, which appeared as unessential in most of our previous work, becomes decisive. Whatever the value of γ, that is whatever the strength of preferential attachment, if δ is large enough, that is if the spatial correlations are strong enough, they can destroy the robust giant component.

Future collaboration

The results are encouraging after a 10-days work, but they are still at an early stage. Two challenges await us straight away. First, carry the proof on a much finer analysis to show non-robustness for $\tau > 2 + 1/\delta$. Second, generalize the results to higher dimensions. The first two points in our results use soft first moment calculations that can be adapted, but our intricate construction cannot give any interesting bound in dimension 2 and higher. We already thought about another construction that could potentially give better bounds in higher dimensions. It is not clear whether this other construction can give any result for finite δ.

Further research will focus on the description of the robustness regime. We believe there is a unique giant component. Its diameter should be doubly logarithmic with a constant factor depending on δ and greater than the constant 4 observed in non-spatial PA models. New ideas will be needed, different from those of non-spatial models.

Further visits, either of Dr E Jacob to Bath, or of Prof P Mörters to ENS Lyon, are planned for the year to come. We are very grateful to ESF and the RGLIS committee for the support, which allowed this visit to happen.