Scientific Report
Discrete structures and related techniques from stochastic analysis

Technion, Israel

4 March 2013 - 08 March 2013

Organizers	Prof. Leonid Mytnik Prof. Dr. Achim Klenke
Co-operating institutions	Technion – Israel Institute of Technology and Johannes Gutenberg-Universität Mainz

1 Summary

The "Discrete structures and related techniques from stochastic analysis" (DSSA) workshop was held from Monday, March 4th to Friday, March 8th at the Faculty of Industrial Engineering and Management, Technion. The workshop was devoted to the analysis of different random discrete structures and their continuous counterparts. In the last two decades these topics have been extensively studied, there are a lot of very interesting results and even more unresolved problems. The aim of the meeting was two-fold. First, we were interested in bringing together world leading experts in population genetic models, continuous tree models, branching processes, random graphs and the KPP equations. The second objective was to give an opportunity to junior researchers (postdocs and PhD students) to learn from leading scientists, present their own research and make new contacts. From both of these perspective the meeting was a great success: we had 4 minicourses and six 50 min lectures given by prominent scientists, and about 50 young participants taking part in the meeting; some of young researchers gave short talks where they presented their results.

2 Description of the scientific content and discussion of the event

Many discrete random structures converge, after suitable rescaling, to very interesting continuous limits. Brownian motion is the classical example of such a continuous object that arises as the scaling limit of random walks. More complicated and more recently studied continuous structures are, for example, continuum random trees, Brownian maps, Schramm-Loewner-Evolutions (SLE), measure-valued diffusions, and in some sense random coagulation, coalescent and fragmentation processes. They arise as scaling limits of fundamental random discrete objects, some of which are reviewed during this school (see topics below). For studying random continuous objects the stochastic analysis plays a crucial role.
Behavior and properties of the continuous objects often shed a light on related questions for corresponding discrete structures, and hence deserve very close attention. There have been

many results in this direction in recent years, but there are still a lot of open questions about relations between discrete and continuous structures.
Some of these areas provided the following topics for the workshop:

- **Population models and related processes** Population models of various kinds, such as population genetic models, math biology models, have been known and studied for a long time. In recent years, they gave rise to beautiful theoretical developments in the fields of fragmentation, coalescents, measure-valued processes, random trees. By now these fields have been established as distinctive areas in probability theory and together with other aspects of population models they were one of the main topics of the school. In particular, they were a theme of two mini-courses given by A. Wakolbinger and A. Sturm.

- **Branching random walks and Gaussian free field** This topic is, in fact, related to the previous one, since branching random walk (BRW) is one of the classical objects in population models. The properties of BRW and its continuous space counterpart, branching Brownian motion (BBM), have attracted a lot of attention in recent years. Branching random walks and their link to Gaussian free field were another topic of the school and the subject of mini-course by O. Zetouni.

- **Random geometric structures** Another theme of very active contemporary research are random graphs. The task of understanding their geometry, properties and limiting behavior has been on the cutting edge of the recent research activity. Many questions originating in physics have been answered, but many are still stay open. The mini-course given by I. Benjamini was devoted to this topic.

The workshop was organized as follows. Most of the morning and afternoon sessions were devoted to minicourses and 50 min talks by leading experts. We freed time on Tuesday and Wednesday from 16:20 until 18:00 for 7 short talks by young participants.

Minicourses

Below is the brief description of the mini-courses given by I. Benjamini, O. Zeitouni, A. Wakolbinger, A. Sturm.

Itai Benjamini gave a mini-course on *Random geometric structures*. He started with a short introduction to coarse geometry, modeled by infinite graphs. Benjamini reviewed basic properties such as volume growth, isoperimetric profile and vertex transitivity, and then he described how the underling geometric structure is reflected in the behaviour of random processes on the graphs, including branching random walk. In the end these random processes were used to construct new classes of natural large graphs.

Anja Sturm gave a mini-course on *Coalescent processes with multiple mergers and spatial structure*. She started with a short introduction to classical coalescent processes, such as

Kingman's coalescent, that describe particles that merge ("coalesce") over time. Coalescent processes with multiple mergers that have attracted much attention over the last decade were shown to arise in a variety of population settings, in particular when the offspring distributions have heavy tails. In the end Sturm considered the genealogies of models in which the large population and large space limits are taken jointly as well as touched on some other variants of spatial population models in continuous space and with non constant population sizes.

Anton Wakolbinger gave a mini-course on *Random genealogies under selection*. Modelling and analyzing the interplay of (random) reproduction, mutation, and selection is a major topic of mathematical population genetics. In the scaling limit of large population size and many generations per time unit, the evolution of type frequencies becomes an object of stochastic analysis, whereas the genealogies consisting of the individuals' ancestries still keep a discrete flavour. In this mini-course, Wakolbinger reviewed central concepts in this topic: the ancestral selection graph, the look-down representation with selection, and the "common ancestor type process" along an immortal line.

Ofer Zeitouni gave a mini-course on *Branching Random Walk and the Gaussian Free Field*. The model of branching random walks, and its close relative branching Brownian motion, describes the evolution of particles that undergo both random motion (diffusion) and branching. Zeitouni described recent progress concerning fine properties of the leading particles, and the link with a class of Gaussian fields called the (discrete) Gaussian free field, in dimension 2.

50 min Talks

Now we are going to provide a short description of 50 minutes talks by senior participants.

Nina Gantert gave a talk *The maximum of a branching random walk in random environment*. It is based on a joint work with Thomas Höfelsauer. She considered real-valued branching random walks in an inhomogeneous random environment given by an i.i.d sequence of branching mechanisms. Gantert investigated the asymptotics of the rightmost position of the random walk, which is in sharp contrast to the case of a deterministic environment.

Anita Winter gave a talk *Evolving genealogies of interacting branching diffusions: The mean field finite system scheme*. The talk was based on a joint work with Andreas Greven and Lea Popovic. She constructed and studied the evolution of genealogies for spatially branching diffusions. Such models arise as limits of aparticle models with rapid branching and as the McKean Vlasov limits. Winter characterized the limit dynamics by a well-posed martingale problem in the space of marked ultra-metric (finite) measure spaces. Finally she established the mean field finite system scheme on the level of genealogies.

Thomas Duquesne gave a talk *Hereditary Tree Growth and Lévy forests*. He introduced the notion of a hereditary property for rooted real trees and also considered reduction of trees by a given hereditary property. He considered the metric structure of trees, and the framework is the space $\mathbb{T}$ of pointed isometry classes of locally compact rooted real trees equipped with the Gromov-Hausdorff distance. Some of the main results of his joint work with Mathias Winkel are a general tightness criterion in $\mathbb{T}$ and limit theorems for growing families of trees. In the talk Duquesne showed how to apply these results to Galton-Watson trees with exponentially distributed edge lengths. He also obtained a characterization of the laws of Lévy forests in terms of leaf-length erasure and the invariance principles for discrete Galton-Watson trees, including the super-critical cases.

Robert Adler gave a talk on *On Random Algebraic Topology*. Over the past few years there has been considerable activity in exploiting the power of algebraic topology to investigate areas outside of mathematics. Even more recently the intrinsically random nature of the world is beginning to bring statistical and probabilistic tools to bear on these problems, leading to the birth of a new area of 'random algebraic topology'. In this talk Adler gave some examples of the (few) results in this area, primarily limit theorems for the Betti numbers of random complexes built over random point processes.

Noam Berger in his talk based on joint work with G. Amir and T. Orenshtein showed a zero one law for directional transience for a one-dimensional cookie-driven random walk under the conditions of ellipticity and ergodicity.

Dima Ioffe gave a talk *Asymptotic ground states and stochastic representation for mean field models in transverse field*. The talk was based on a joint work with Anna Levit and dealt with the limiting behavior of ground states. The particular case of the spin-$\frac{1}{2}$ Ising model was worked out in detail. It was shown that for a large class of interactions, asymptotic ground states undergo a sequence of first order transitions as the strength of the transverse field varies. Also in the case of multiple wells, the limiting ground states necessarily develop stationary shocks between successive local minima.

Short Talks Sessions

It was really a great pleasure to have two short talks sessions. The talks during these sessions were given by young participants and covered a range of topics including random graphs, superdiffusions, random trees, and discrete Gaussian free field. These talks were very successful. The list of speakers and titles of these two sessions follows.

1. Elisabetta Candellero (University of Birmingham): *Clustering in Random Hyperbolic Graphs;*

2. Martin Slowik (Technische Universität Berlin): *Invariance principle for the Random Conductance Model in a degenerate ergodic environment;*

3. Karl-Patric Glöde (Universitaet Erlangen-Nuernberg): *Dynamics of Genealogical Trees for Autocatalytic Branching Processes;*

4. Maren Eckhoff (University of Bath): *The strong law of large numbers for superdiffusions;*

5. Thomas Rippl (Goettingen): *Compact Support Property of the Stochastic Heat Equation with Colored Noise;*

6. Gugan Thoppe (Tata Institute of Fundamental Research, Mumbai): *An online network tomography algorithm;*

7. Oren Louidor (UCLA): *The extremal process of the Discrete Gaussian Free Field.*

3 Assessment of the results

This workshop gathered outstanding scientists working on population genetic models, continuous tree models, branching processes, random triangulations, random graphs, and the KPP equation. Bringing together leading representatives and junior scientists from these groups led to a fruitful exchange of ideas circulating in these fields dealing with different discrete structures and their continuous counterparts. Junior researchers, and especially PhD students and postdocs, had a great opportunity to learn from leading scientists and make new contacts. The meeting has been an extraordinary stimulus for young scientists working on random discrete and continuous structures — one of the main topics of the contemporary research in probability theory.

4 Final programme of the meeting

Here is the final program of the meeting supported by ESF, GIF and Moshe Yanai Fund. The abstracts of the talks can be seen at the workshop website:
http://www.mathematik.uni-mainz.de/∼klenke/dssa2013/index.htm

March 4th to 8th, 2013

Time	Monday	Tuesday	Wednesday	Thursday	Friday
08:50	Opening				
09:00	Lecture 1/1 Benjamini	Lecture 1/2 Benjamini	Lecture 1/3 Benjamini	Talk 5 Ioffe	Lecture 3/4 Wakolbinger
09:50	Short Break				
10:00	Lecture 2/1 Sturm	Lecture 2/2 Sturm	Lecture 2/3 Sturm	Lecture 2/4 Sturm	Lecture 4/4 Zeitouni
10:50	Coffee Break		Coffee Break	Coffee Break	
11:10			Talk 4 Adler		
11:20	Talk 1 Berger	Talk 2 Duquesne		Talk 6 Winter	Transfer to the train
12:10	Lunch Break		12:00-12:35 Lunch Break (at the Faculty)	Lunch Break	
14:00	Lecture 3/1 Wakolbinger	Lecture 3/2 Wakolbinger	12:45-18:30 Akko (Acre) tour	Lecture 3/3 Wakolbinger	
14:50				Short Break	
15:00	Lecture 4/1 Zeitouni	Lecture 4/2 Zeitouni		Lecture 4/3 Zeitouni	
15:50	Coffee Break			Coffee Break	
16:20	Talk 3 Gantert	Short talks: 1. Candellero 2. Slowik 3. Glode		Short Talks: 1.Eckhoff 2.Rippl 3.Thoppe 4.Louidor	
17:10					
18:00-20:30	Coler visitors center Reception-Dinner			Dinner El Gaucho restaurant	

Lectures will take place at the Faculty of Industrial Eng. & Management, Cooper Building, room 112.