Report for short visit RGLIS/4076

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1 Purpose

The purpose of the visit has been an exchange of ideas regarding cluster expansion of hard-core lattice gases / polymer systems [SS05, FP07] and to see if certain improvements proposed by the visitor are feasible. The core question we have focused on is how to get closer to the following Dobrushin-style condition [SS05],

$$\rho \leq \max \frac{\mu}{(1+\mu)^D}\,, \tag{1}$$

for a polymer system with maximal degree (i.e. the number of incompatible polymers for any give polymer, not counting itself) D and activity ρ. It is derived by a purely inductive approach à la Dobrushin [Dob96], improves the Dobrushin condition

$$\rho \leq \max \frac{\mu}{(1+\mu)^{D+1}}\,, \tag{2}$$

by replacing $D+1$ by D in the power of the denominator of (1). Furthermore it is known to be exact on D-regular trees [She85] and no derivation of (1) via cluster-expansion techniques is known.

2 Results

Our discussions quickly centered on lifting the idea of excluding always one neighbour, thereby reducing the number of incompatible polymers by 1, from the level of the polymer system to the cluster expansion. Using a spanning tree identity by Penrose [Pen67] this should at least include a partition scheme of a cluster $G(\gamma_0, \vec{\gamma})$ with the following property: the spanning trees invariant under this scheme never have children of all the polymer types incompatible with the polymer type of the parent node.

Our current status is: We believe to have

- a partition scheme with the above outlined property.

- an improvement of the Fernandez-Proccacio [FP07] condition

$$\rho \leq \max \frac{\mu}{\mu + (1+\mu)^D} \tag{3}$$

(here for the D-regular tree with activity ρ). The improvement subtracts some terms to $\mu + (1+\mu)^D$ but does not reach $(1+\mu)^D$.

3 Outlook and comments

The plan for the near future is to first prove rigorously the above claims and second two some calculations (also on derived level-2 criteria) for several classes of interesting graphs. In the medium term we plan to follow-up with another meeting, discussing the above claims and discussing related more speculative ideas.

Given the early stage no publications are planned yet.

Finally I wand to add that it has been an intellectually stimulating visit, weeding out a lot of wrong perceptions of mine, stimulating several ideas and putting us onto a common discussion base.

References

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[She85] J. B. Shearer. On a problem of Spencer. *Combinatorica*, 5(3):241–245, 1985.

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