

Research Project RGLIS 4915:
Harmonic Analysis of products of Markov chains and graphs
Scientific Report

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The support received from the European Science Foundation within the activity entitled “Random Geometry of Large Interacting Systems and Statistical Physics” made me able to visit Professor Wolfgang Woess at the Institut für Mathematische Strukturtheorie of the Technische Universität of Graz, Austria, in the period from the 7th to the 19th of May 2012. It really was for me a very fruitful and stimulating experience from the scientific point of view.

Purpose of the visit

The purpose of my visit was to collaborate with Professor Wolfgang Woess in a project work aiming at investigating interesting relations between probability, combinatorics and harmonic analysis.

Description of the work carried out during the visit

Starting from the study of the construction called zig-zag product of graphs, that I described in a talk that I gave on May 8th at the Institut für Mathematische Strukturtheorie of the Technische Universität of Graz, I had many interesting and stimulating discussions with Professor W. Woess and members of his research group, who suggested me to consider iterations of this construction, also related to the problem of constructing expander graphs, in order to establish when a limit graph exists and which are its properties in terms of spectrum, random walks, number of ends. In particular, it is expected that the limit graphs have certain self-similarity properties, which make them particularly interesting in the framework of random processes evolving on them. It is also known that these graphs have deep connections with Group theory, via the notions of semidirect product and Cayley graph. Hence, it is natural to ask what is the group associated with the limit graph of this construction. In this direction, I started a collaboration with Dr. Ecaterina Sava, a post-doc of the host department, who was a Ph.D. student of Professor W. Woess. Another idea that we intend to develop is to define in some natural way a zig-zag product of finite reversible Markov chains, inspired by the definition for graphs, and perform a spectral analysis allowing to describe the k -step probability transition of the associated random walk.

On the other hand, I also started a collaboration with Dr. Elisabetta Candellero, a Ph.D. student of Professor W. Woess, with the aim of investigating connections between the zig-zag construction and the Lamplighter Markov chain.

I also had the opportunity to discuss with other people from the host department. In particular, I discussed with Professor Franz Lehner about generalized wreath products of graphs and groups, but also around some new ideas and conjectures involving spectral properties of Markov operators on zig-zag products of graphs and their automorphism group.

Description of the main results obtained

Together with Dr. E. Sava, I constructed an explicit example of zig-zag product of an infinite regular tree of degree 3 with a triangle. The graph that we get seems to be highly connected, but it still has infinitely many ends. It is natural to expect that iterations of this construction provide a 1-ended graph. We also have constructed an explicit example where the iteration map is a contraction, so that the Banach fixed-point Theorem can be applied and the uniqueness of the limit graph can be deduced. We plan to generalize this construction and to establish which conditions have to be satisfied by the factor graphs in order to get uniqueness. Furthermore, we have produced an explicit example of a sequence of zig-zag product of graphs, related to the self-similar group theory, showing that in this case the limit does not depend on the particular root chosen for the Schreier graph of the self-similar group, and that the zig-zag construction is very regularizing, making the automorphism group of the resulting graphs larger and larger.

During my collaboration with Dr. E. Candellero, we established an interesting connection between two different versions of the Lamplighter random walk, on one hand, and the random walk on the zig-zag product of a suitable Hamming cube with a second regular graph, on the other hand. We plan to develop a spectral analysis of these models, in connection with both the expansion properties and the probabilistic interpretation of this construction.

Future collaboration with host institution

I intend to continue collaborating with Dr. E. Sava and with Dr. E. Candellero. In particular, we plan that Dr. E. Sava will come to Rome, probably in September 2012, to intensively develop our work plan and also work to draft a scientific paper containing our results.

Projected publications/articles resulting or to result from the grant

My cooperation with Dr. E. Sava and Dr. E. Candellero is still a “work in progress” at the moment, but we hope that this collaboration will produce at least two scientific articles, where, of course, an acknowledgement to the support received by the European Science Foundation will appear.