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visit from IRE NASU, Kharkiv, Ukraine to GGIEMR, the University of Nottingham, UK

Topic: "ANALYSIS OF 3-D AXIALLY SYMMETRIC TWO-REFLECTOR QUASIOPTICAL ANTENNAS"

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The purpose of the visit was a collaborative effort around the research into three-dimensional (3-D) modeling of quasioptical two-reflector antennas with rotational symmetry using a fast and accurate numerical method based on the singular integral equations and interpolation-type discretization.

Two-reflector antennas are used in various communication and radar system across the whole millimeter-wave range. Well-known examples are Cassegrain antenna, which has paraboloidal and hyperboloidal reflectors, Parabola-Cone (PACO), axis-displaced Cassegrain, and axis-displaced ellipse. Accurate modeling of such antennas with convergent and economic numerical methods allows analyzing their performance, studying physical effects in detail, and obtaining optimized configurations.

Transportable atmospheric Radar (TARA), which is a combination of parabolic reflector and conical shield, is a real system used at the Delft Technical University. This is also a two-reflector antenna however the reflectors are welded together along the rim of paraboloid. The aim of such design is to reduce the signals received from all directions in the plane of paraboloid rim because TARA looks into zenith and only the signals reflected from that direction are relevant.

I was going to derive basic equations for an arbitrary two-reflector co-axial configuration. The numerical efforts were planned to be concentrated on the analysis and optimization of TARA as a computationally simpler case. This is because studying the reflectors of quasi-optical size (for instance, some 20 to 50 lambda) within realistic computer time on a moderate PC needs developing not simply a convergent algorithm but a truly economic one.

The electromagnetic field scattered by an axially symmetric PEC zero-thickness screen has to satisfy Maxwell equations, edge condition, radiation condition, and PEC-type boundary condition. When working towards my Ph.D. thesis, I had developed a numerical method that reduced the corresponding wave-scattering problem, for each of the azimuthal harmonics of the current density components, to two coupled one-dimensional integral equations (IE) with varying coefficients. One of these IEs is hypersingular and another is singular. Their discretization is based on the Nystrom-type interpolation scheme and specifically tailored quadrature formulas of interpolation type. This method has guaranteed (mathematically proven) convergence in full range from quasi-statics to quasi-optics.

However for quasi-optical antennas this numerical method needed some improvements. These improvements relate to calculation of the so-called modal Green's function (MGF) $S_M = \int_0^{2\pi} \exp(-ikL) L^{-1} \cos(M\psi) d\psi$ (L is the distance between two points on the rotation surface) for large values of L, and analytical calculation of the smooth kernels limit when the integration point tends to the observation point.

Description of the work carried out during the visit.

According to the proposal, I have been working on the optimization of my algorithm. In quasi-optical range the unknown current densities have many oscillations on the reflector surface. This calls for high order discretizations. To reduce the calculation time it is needed to develop faster numerical methods to calculate the matrix elements. The calculation of MGF and its first and second derivatives takes almost all calculation time. Therefore, the first step in the method improvement is to obtain optimal quadrature formulas for MGF in the quasi-optics range. To calculate the MGF I have used quadrature formula of interpolation type. In this quadrature formula the integrand is approximated by the trigonometric polynomial and the integrals of these trigonometric polynomials are expressed analytically. The quadrature formula has exponential convergence. If one increases the degree of trigonometric polynomial, which approximates the integrand, then it is seen that the relative error of computations starts quickly tending to zero after crossing certain value of that degree. The problem is to evaluate this degree a priori. For this I have majorized the trigonometric function coefficients and thus evaluated the quadrature formula order n needed to obtain a relative error smaller than the fixed value for any MGF parameter.

Another problem consists in efficient calculation of smooth kernels of integral equation in the case of integration and observation points' coincidence. Smooth kernels contain difference between MGF and its derivatives and their asymptotics. MGF has singularity in the case of integration and observation points' coincidence. Therefore, we have uncertainty of the form $\infty - \infty$ in the smooth kernels and have to calculate the smooth kernel limits when the ntegration point tends to the observation point. Numerical calculation of this limit is simple but very rough in the quasi-optics range. Therefore, the second step is analytical proceeding to the limits for the smooth kernels in the case of the integration and the observation points' coincidence. The MGF can be expressed through some C^2-smooth functions, the first and the second kind elliptic integrals (in the case of parameterization of rotation contour using C^2 functions). The finding of the mentioned above smooth kernel limits is based on the well-known asymptotics of the first and second kind elliptic integrals.

After that I have considered the real-life TARA-like shielded paraboloidal reflector antenna (Fig. 1).

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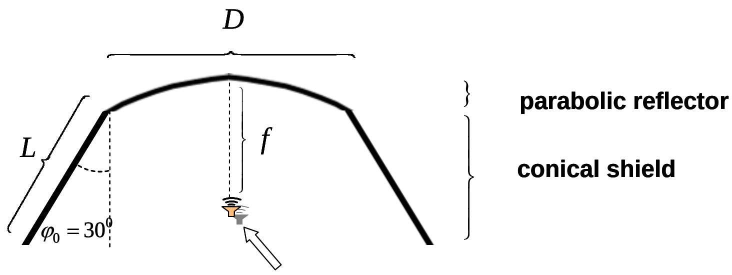

Fig. 1. The cross-sectional geometry of TARA

My goal was determining the optimal TARA shield length and shield inclination angle for the best focusing (in the reception mode) and for the largest directivity and the lowest sidelobes in the direction that is orthogonal to the rotational axis (in the transmission mode).

The TARA is used for studying atmospheric phenomena such as clouds, precipitations and clear air turbulence. It consists of a parabolic reflector and a conical shield. One of the major antenna design requirements is a very low level of the sidelobes around $\theta = 90$ degrees, less than -70 dB. This is the purpose of including the shield in the design of antenna.

The TARA parabolic reflector has the diameter $D = 2f$ of 33 wavelengths (3 meters) and the shield has the length L of 22 wavelengths (2 meters) and the angle of inclination φ_0 of 30 degrees. For modeling the TARA in the transmission case the feed is simulated using a Complex Huygens Element (CHE) placed in the geometric focus of the parabolic reflector. For modeling the TARA in the reception case we consider plane wave diffraction and focusing.

The CHE is a convenient simplified model of a realistic corrugated-horn or horn-lens antenna. Its field function has some parameter "*b*" that is formally the imaginary part of the source location point. If $b = 0$ then the field function coincides with the field of the classical Huygens Element (HE) which consists of orthogonal to each other elementary electrical and magnetic dipoles. As known, HE has fixed directivity. If b is increased, then the directivity of such a modified source can be made larger and, correspondingly, the reflector edge illumination lower. Therefore such a feed is convenient for simulating the incident fields in the modeling of reflector antennas. In Fig. 2 we show the directivity of the TARA-size paraboloidal reflector illuminated by CHE as a function of parameter kb. One can see that the optimal parameter is $kb = 2.37$. Further we use CHE with this optimal parameter.

There is an angle on the contour of TARA reflector between the paraboloid and the shield. However, the method considered needs contour smoothness. Suppose that $\rho(s), z(s)$ is the natural parameterization of the shielded paraboloidal surface rotation contour C (the expressions of this parameterization have been established), l_{Par} is the parabolic-part length, l_{Sh} is the shield-part length. Now, fix the points A and B on the rotation contour (Fig.3).

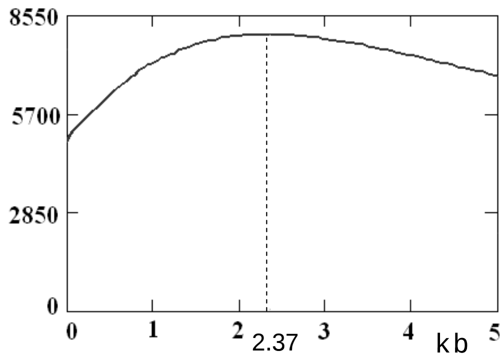

Fig. 2. The directivity of a stand-alone paraboloidal reflector illuminated by CHE as a function of parameter kb

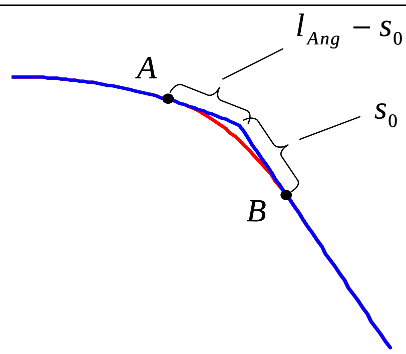

Fig. 3. Smoothening of the shielded paraboloidal reflector contour.

Suppose that l_{Ang} is the angle between the points A and B. Then the contour parameter value $l_A = l_{Par} - (l_{Ang} - s_0), s_0 \in (0, l_{Ang})$ corresponds to the point A and the parameter value $l_B = l_{Par} + s_0, s_0 \in (0, l_{Ang})$ corresponds to the point B. To approximate the angle between points A and B one can use a spline. There is a set of splines with parameter s_0. We have chosen the spline with the smallest length. The numerical experiments for TARA have shown that the far- and near-field patterns for $l_{Ang} = \lambda$ and smaller values almost identical. Therefore we have further considered the shielded paraboloidal reflector with a smoothed angle characterized with $l_{Ang} = \lambda$.

In Fig. 4, the H- and E-plane far-zone radiation patterns of a TARA paraboloidal reflector without the conical shield and that of the full TARA system are compared on a logarithmic scale.

One can see that in the direction which is orthogonal to the axis of rotation (90 degrees), and near to it, the TARA radiation pattern has sidelobes lower than for the stand-alone parabolic reflector, by some 20 to 30 dB. The CHE source here has been taken in such a way that it provides the maximum directivity for the stand-alone parabolic reflector.

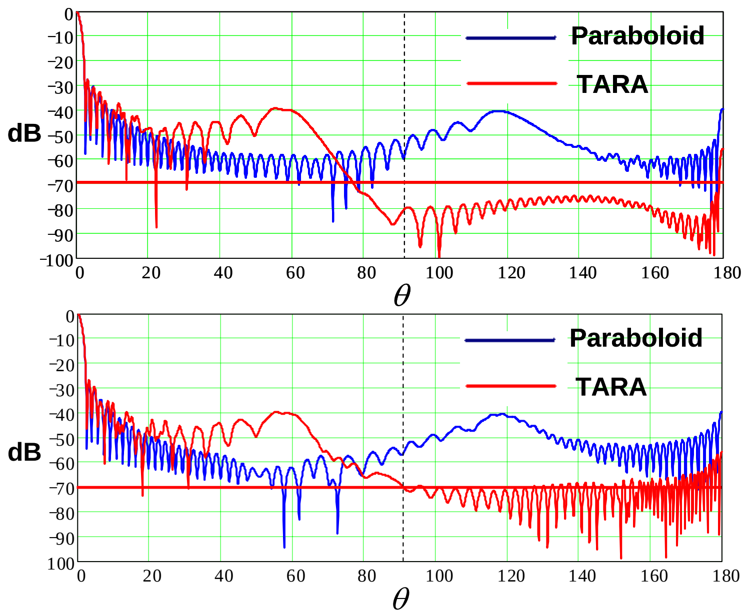

Fig. 4. The total far-zone radiation patterns of the TARA-like parabolic reflector without the conical shield and the full TARA system in the H-plane (top) and in the E-plane (bottom)

In Fig. 5, we show, on a logarithmic scale, the near-zone field of the full TARA reflector illuminated by the optimal CHE in the E- and H-planes. For comparison, the $|\vec{E}^{tot}/\vec{E}^{0}|$ pattern of the full TARA illuminated by the plane wave in the E- and H-planes is shown in Fig. 6.

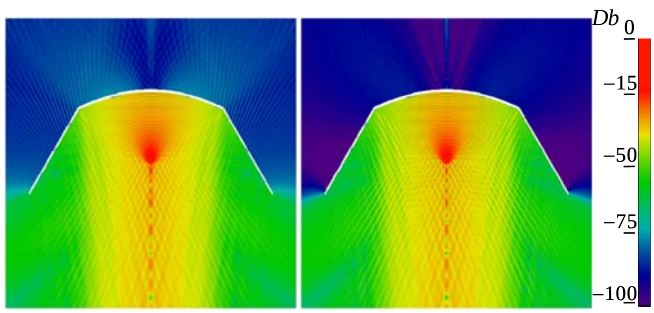

Fig.5. The near-field of the full TARA configuration illuminated by the optimal-*kb* CHE source placed into geometrical focus in the H-plane (left) and the E-plane (right)

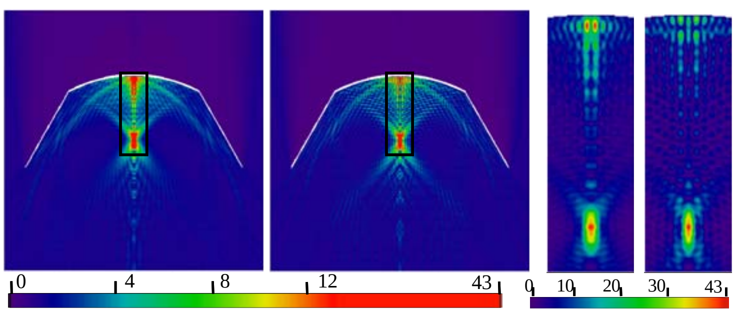

Fig. 6. The near-field of the full TARA configuration illuminated by the plane wave propagating along the axis of rotation in the H-plane (left) and the E-plane (right)

In Fig. 6 we can see an interesting phenomenon that escapes geometrical-optics descriptions. Besides of the main focal spot (area of the field concentration) there is another split "focus" near the paraboloid bottom. The latter areas of the field concentration appear because of combined diffraction by the conical shield and paraboloidal reflector.

Our computations have shown that the TARA directivity can be improved, and the <u>length of the conical shield can be reduced by half a meter,</u> by changing the shield inclination φ_0 from 30 deg to 5 deg. In Fig. 5 we compare, on logarithmic scale, the far-field patterns for a shielded paraboloidal reflector with the inclination angle $\varphi_0 = 5^0$ and shield length $L = 1.5$ m and similar dependences for the real TARA with $\varphi_0 = 30^0$ and $L = 2$ m.

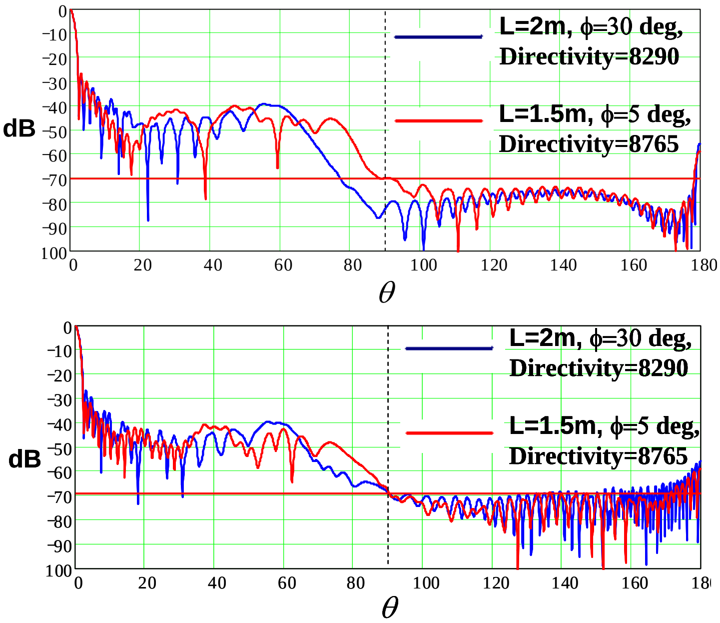

Fig. 5. The total far-zone radiation patterns of the TARA-like configurations in the H-plane (top) and in the E-plane (bottom) for $\varphi_0 = 5^0, L = 1.5$ m and $\varphi_0 = 30^0, L = 2$ m

From Fig. 5 it follows that the shielded reflector with $\varphi_0 = 5^0$ satisfies the major antenna design requirement on the sidelobes as mentioned above. It should be noted that the directivity for the shielded paraboloid with $\varphi_0 = 5^0$, $L = 1.5$ m is larger than for TARA ($\varphi_0 = 30^0, L = 2$ m).

<u>Publications based on the project work:</u>

Conference papers

1. V.S. Bulygin, Y.V. Gandel, A.I. Nosich, "Fast and accurate numerical modeling of a TARA-like shielded paraboloidal reflector antenna," *Proc. Int. Workshop Microwaves, Radar and Remote Sensing (MRRS-2011)*, Kiev, 2011, pp. 86-88.
2. V.S. Bulygin, Y.V. Gandel, T.M. Benson, A.I. Nosich, "Numerical optimization of a TARA-like shielded paraboloidal reflector antenna," *Proc. European Conf. Antennas and Propagation (EuCAP-2012)*, Prague, 2012 (accepted).

Journal paper

1. V.S. Bulygin, T.M. Benson, Y.V. Gandel, A.I. Nosich, "Full-wave analysis and optimization of a TARA-like shielded paraboloidal reflector antenna using a Nystrom-type method," *IEEE Trans. Antennas and Propagation*, in preparation.