INSTITUT D'ÉLECTRONIQUE ET DE TÉLÉCOMMUNICATIONS DE RENNES	SAPIENZA UNIVERSITÀ DI ROMA

PROJECT TITLE:

ANALITYCAL STUDY OF FOCUS WAVE MODES THROUGH A BESSEL BEAM LAUNCHER AT MILLIMETER WAVES

Scientific Report on the Research Activity within the framework
of the ESF program entitled "New Frontiers in Millimetre/Sub-Millimetre
Waves Integrated Dielectric Focusing Systems"

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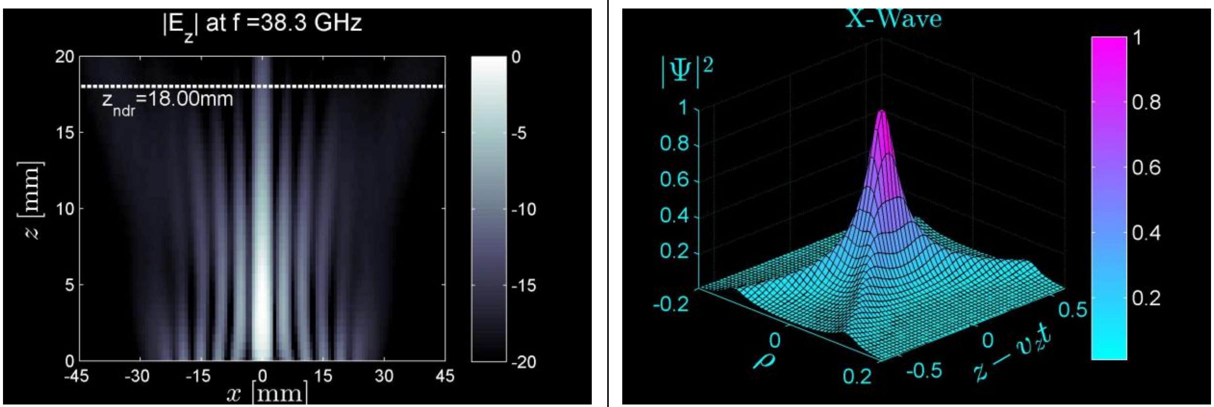
1. INTRODUCTION AND MOTIVATIONS

The ability to focus electromagnetic energy in a confined region of space is crucial for most of millimeter-wave applications, such as wireless power transfer, medical imaging, thermal ablation, and so on. However, the propagation of electromagnetic waves is affected by diffraction which clearly limits the collimating and focusing capabilities of any electromagnetic device. As a consequence, researchers have proposed different devices in order to produce localized beams (monochromatic solutions of the scalar wave equation) as well as localized pulses (non-monochromatic solutions). On one hand, the most known localized beams are represented by Bessel beams, whose first experimental generation at optical frequencies is due to Durnin [1]. Successively, more efficient implementations have been realized in both the microwave [2], [3] and millimeter-wave range [4]-[7]. On the other hand, localized pulses allow for a wider class of localized solutions known as Focus Wave Modes (FWM) [8]. Among them, the most interesting are represented by X-Waves (a suitable superposition of propagating Bessel beams) [9], whose first experimental generation in acoustics is due to Lu and Greenleaf [10]. However, most of the previous theoretical works on X-waves [11] are based on very strong assumptions, such as infinite aperture plane and a non-dispersive behaviour of the device. In this project, we have studied both analytically and numerically, the generation of X-waves from a Bessel-beam launcher working at f = 40 GHz with the first-order leaky-wave mode as in [7]. To this aim, we have taken into account the dispersive-like behaviour of the leaky mode under consideration and we have compared the results between an ideal X-wave and a dispersive one. The final results show that dispersion does not compromise the generation of an X-wave.



(a) (b)

Figure 1: (a) Typical Bessel beam profile generated by the Bessel beam launcher described in [7]. (b) Typical X-Wave profile as described in [9].

2. PROBLEM STATEMENT AND METHOD

An ideal Bessel beam can be thought as a superposition of plane waves with propagation constants lying on the surface of a cone with a given opening angle θ, sometimes referred as the axicon angle [2]. Being a superposition of Bessel beams, the analytical expression of an X-wave is given by [11]:

$$\psi(\rho,z,t)=\int_{-\infty}^{+\infty}F(\omega)J_0(k\sin(\theta)\rho)e^{jk\cos(\vartheta)(z-ct/\cos(\vartheta))}d\omega$$

where $F(\omega)$ is the frequency spectrum, k is the wavenumber, θ is the axicon angle and ω is the angular frequency. In such representation, some simplifying hypotheses are tacitly assumed:

1) The aperture field is a nondiffractive Bessel-beam over the entire frequency range. This means that an infinite aperture, or infinite energy is required.
2) The axicon angle θ is fixed, and thus it does not change with frequency. This means that no dispersion is taken into account.

The purpose of this project is to analyze the focusing properties of an X-wave, firstly under the general hypotheses (1) and (2) and then by replacing hypothesis (2) with a more realistic assumption in which the axicon angle is a function of frequency. In particular we will consider the dispersion of a leaky mode whose normalized longitudinal wavenumber $k_z/k= cos(\theta)$ is linear function of the frequency. As a consequence, the axicon angle would also vary with frequency.

The analytical study of the generation of an X-wave generation with a Bessel beam launcher has been organized in the following three main steps: (1) evaluation of the focusing features of an X-wave by defining suitable and measurable parameters; (2) analytical study of the properties of an ideal X-wave; (3) analytical study of the properties of a dispersive X-wave.

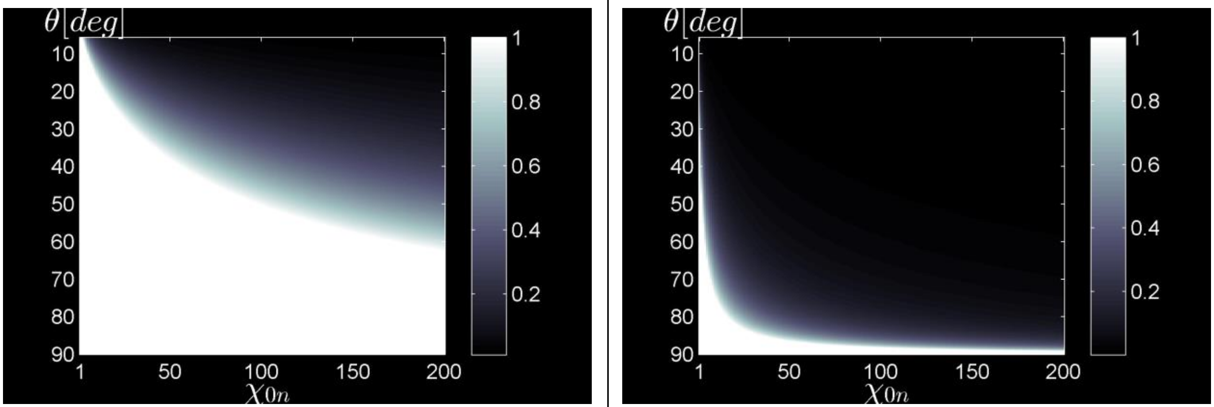
For the first point (1), we need a parameter describing the focusing properties of an X-wave generated from a Bessel beam launcher. We have defined an aperture confinement η as the product of two terms: i) a confinement η_ρ along ρ, given by the ratio between BW_ρ (the peak-to-null beamwidth along ρ) and the radius of a given aperture plane ρ_{ap}; b) a confinement η_z along z, given by the ratio between BW_z (the peak-to-null beamwidth along z) and the non-diffractive range z_{ndr} (the propagating distance at which a Bessel beam starts to widen along the transverse direction) achievable from a launcher with the same aperture. From the definition given above, it appeared that a focusing behavior is reached only for η <1. In the following we will provide the conditions needed to satisfy this constraint.

For the point (2), we have found analytical close-form expressions for both the transverse (Intensity along ρ -axis) and longitudinal (Intensity along z-axis) profile of an ideal X-wave. Without loss of generality, we have assumed $F(\omega)=\mathrm{rect}_{\Delta\omega}(\omega-\omega_0)$, i.e. a rectangular spectrum centered around a carrier frequency ω_0 and with a fractional bandwidth $\Delta\omega/\omega_0$.

This analysis has revealed that η_ρ is independent from the $\Delta\omega/\omega_0$, but it depends mainly on ω_0. Moreover, η_ρ is always less than one, and it remains constant as the wave propagates Conversely, η_z depends on the bandwidth, the axicon angle, and χ_{0n} that is the number of zeros of J_0 (Bessel function of zeroth-order and first-kind) supported by the radiating aperture. As shown in Fig. 2, the bandwidth strongly affects η_z. In particular, for a fractional bandwidth of 5% (see Fig. 2(a)), we have a focusing behavior on the z-axis only for very small axicon angles, or for very large apertures (in terms of zeros of the Bessel distribution). On the other hand, for a fractional bandwidth of 20% (see Fig. 2(b)), it is possible to achieve a good confinement even for wider angles and larger apertures.

It is worth noting that since no dispersion is taken into account, the wave propagates rigidly along the axis of propagation z, and thus the beamwidths over the two directions BW_ρ and BW_z remain constant in time.

Finally, for the point (3), we have found analytical close-form expressions for the longitudinal profile and numerical results for the transverse profile of a dispersive X-wave. The dispersive behavior was taken into account by considering a first-order Taylor expansion of the normalized transverse wavenumber k_z/k. Hence, k_z/k has been varied linearly with frequency, as typical for leaky waves. The results that we have obtained are very similar to those of an ideal X-wave. In particular, η_ρ and η_z still depend on the same parameters (ω_0, and $\Delta\omega$, θ, χ_{0n}, respectively) but in a more complicated fashion. However, due to dispersion behavior, the X-wave no longer propagates rigidly, and hence the beamwidths could not remain constant in time. As an example, in the next section, we will show the variation of the field intensity on the ρz planes and the beamwidths of an ideal and dispersive X-wave for a given set of parameters.



(a) (b)

Figure 2: η_z Confinement on *z*-axis for (a) $\Delta\omega/\omega_0$=0.05 and (b) $\Delta\omega/\omega_0$=0.2. The X-wave is more confined as η_z tends to 0, whereas η_z means no confinement. Note that for a wider fractional bandwidth lower values of η_z are obtained for a given pair χ_{0n}, θ.

3. NUMERICAL RESULTS

An ideal X-wave has been simulated over a plane $\rho \in [-\rho_{ap}, \rho_{ap}]$, $z \in [-5\rho_{ap}, 5\rho_{ap}]$ with:

- ρ_{ap} = 22.3 mm
- θ = 30°
- f_0 = 40 GHz

Fig. 3a shows the field plots r for $\Delta\omega/\omega_0$ = 0.05and the $\Delta\omega/\omega_0$ = 0.2 (see Fig. 3(b)) at time t=0. These results clearly show that the field confinement along z-axis improves as $\Delta\omega/\omega_0$ increases.

A dispersive X-wave has been simulated with the same parameters except for the axicon angle that now varies with frequency as presented in the previous section and in [7]. As expected the focusing behaviour along *z* improves as the bandwidth increases (see Fig. 3(c)-(d)).

Finally, the -3dB beamwidth along both axes versus time, were calculated for the case of a dispersive X-wave (note that for an ideal X-wave the beamwidths are constant in time). The results are shown in Fig. 4(a)-(b) where we can notice that that the beamwidth along ρ remains constant (since the truncation of the aperture is not considered, and thus the nondiffractive range is infinite), whereas the beamwidth along *z* starts to widen.

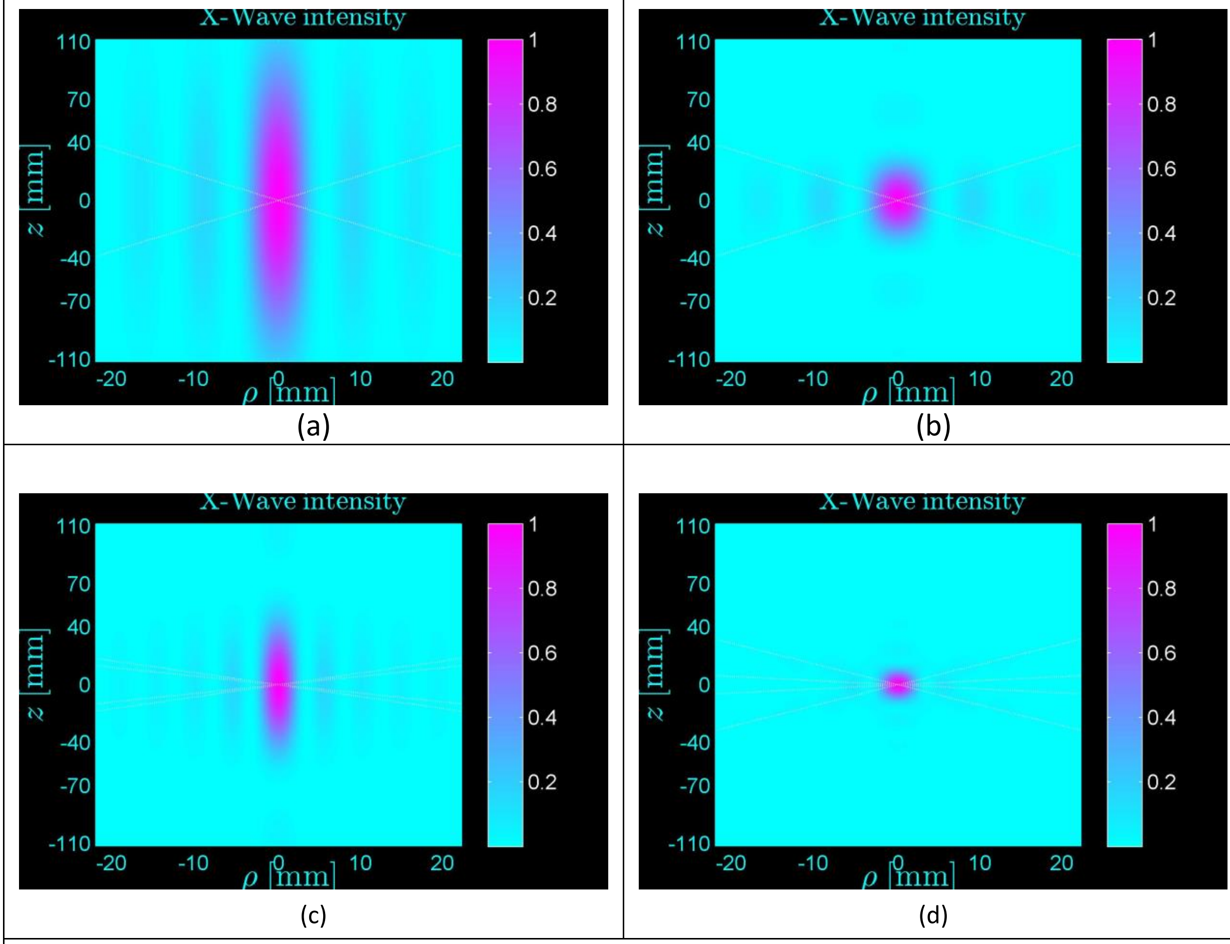


Figure 3: Contour plot of the ideal X-Wave for (a) $\Delta\omega/\omega_0$ = 0.05 and (b) $\Delta\omega/\omega_0$ = 0.2. Contour plot of the dispersive X-Wave for (c) $\Delta\omega/\omega_0$ = 0.05 and (d) $\Delta\omega/\omega_0$ = 0.2.

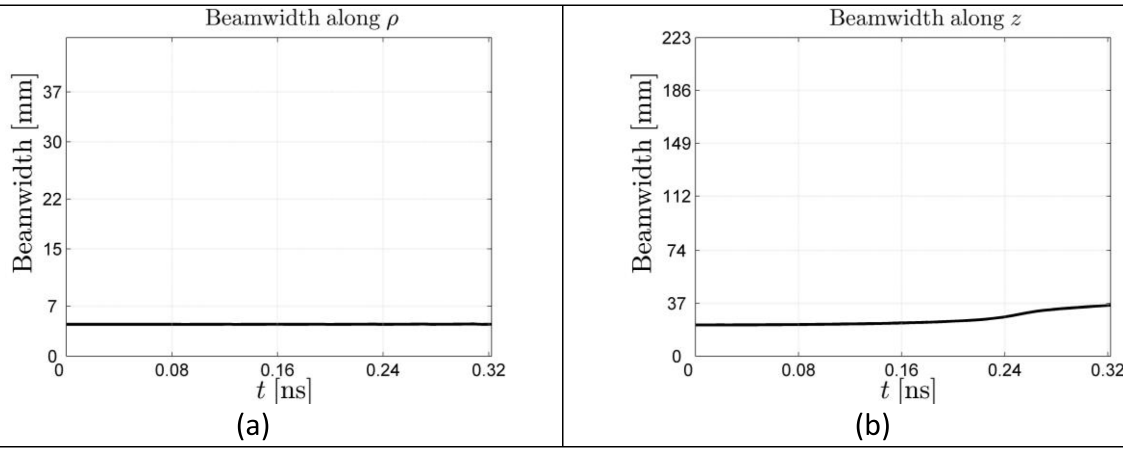


Figure 4: (a) -3dB beamwidth along ρ versus time. (b) -3dB beamwidth along *z* versus time.

4. CONCLUSIONS

During this project we have developed a very useful analytical framework and numerical tool for the understanding of the focusing capabilities of ideal and dispersive X-waves. Further studies will consider the effects of the truncation of the aperture. The results obtained up to now provide useful guidelines for using Bessel-beam radiators based on leaky-wave modes for the generation of localized waves in general and X-waves in particular. In addition, during the time frame of this activity, a full paper [P1] has been submitted in the IEEE Transactions on Antennas and Propagation.

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6. SUBMITTED PAPERS & CONFERENCE COMMUNICATIONS

[P1] W. Fuscaldo, G. Valerio, A. Galli, R. Sauleau, A. Grbic and M. Ettorre, "Higher-order leaky-mode Bessel-beam Launcher", Trans. On Antenna and Propag., under review .