## ESF Research Networking Programme NEWFOCUS Final Scientific Report

## Advanced Mathematics for Antenna Analysis (EsoA Course)

## **1. SUMMARY**

The objective of the course "Advanced Mathematics for Antenna Analysis" was to explain the mathematical methods used in computational antenna analysis and to provide students with mathematical background necessary for advanced antenna engineering and electromagnetic software development.

This course covered the following topics:

- 1. Fundamental theorems in antenna analysis
- 2. Construction of solutions
- 3. Scalar wave equation
- 4. Vector wave equation
- 5. Complex Analysis Background
- 6. Green's functions
- 7. Asymptotic Evaluation of Integrals
- 8. Periodic Antenna Problems
- 9. Advanced Periodic Structures

The aim of this course was to help students gain a deeper understanding of which field representation is suited for a given complex electromagnetic problem. Bearing in mind many requests and comments after previous editions of the course to add extra time to the course (w.r.t. standard 5 days), while trying to keep the organizational costs at a reasonable level, the duration of the course was set to six working days. The course schedule comprised 40 hours of lectures and 10 hours of guided and self-study exercises. Lectures were held at the conference room of Hotel Uvala, while lodging was organized in the nearby Hotel Vis, both situated in the Lapad bay, not far from Dubrovnik city centre.

## **Course Info:**

Dates: 14/05/2012 - 19/05/2012

Venue: Hotel Uvala, Dubrovnik, Croatia

Organizing institutions: University of Zagreb (UNIZAG, Croatia), University of Siena (UNISI, Italy)

Participating institutions: University of Florence (UNIFI, Italy), Politecnico di Torino (POLITO, Italy), Ecole polytechnique federale de Lausanne (EPFL, Switzerland)

Course Coordinators: Prof. Zvonimir Sipus (UNIZAG), Prof. Stefano Maci (UNISI)

Organizing Committee: Sinisa Skokic (UNIZAG), Prof. Zvonimir Sipus (UNIZAG), Marko Bosiljevac (UNIZAG) Speakers:

1) Prof. Angelo Freni (UNIFI)

2) Prof. Stefano Maci (UNISI)

- 3) Prof. Zvonimir Sipus (UNIZAG)
- 4) Prof. Anja Skrivervik (EPFL)
- 5) Prof. Giuseppe Vecchi (POLITO)



## 2. DESCRIPTION OF THE EVENT

### 2.1. Lectures

The lectures were held from Monday 14<sup>th</sup> May thru Saturday 19<sup>th</sup> May 2012, in the morning and in the afternoon. The morning lessons focused more on theoretical background, while the afternoon lessons looked more into practical applications. Extra assignments were handed out on Friday morning and had to be solved in small groups. Each group got a random selection of three exercises related to the three main topics covered at the course. The results were presented on Saturday afternoon as a part of the final assessment tests.

### 2.2 Summary of the lectures

The analysis of many types of antennas and electromagnetic structures can be simplified if some of the electromagnetic theorems are applied. Therefore, the first lecture Fundamental theorems in antenna analysis gives the detailed overview of theorems in electromagnetics and discusses (with examples) how to implement them in the analysis procedure. One of them, the equivalence theorem (explained in the lecture **Construction of solutions**), is of particular interest since it enables the division of a complex electromagnetic problem into a set of simpler sub-problems. Electromagnetic structures are mathematically described with partial differential equations (PDE). Since the procedures for solving PDE are usually very complex, this topic is addressed with two lectures. First, the methods for solving scalar wave equation are explained, and then this approach is generalized for vector wave equation, both for the source-free case. The presence of the sources is taken into account by the Green's functions approach. The Green's functions approach leads to integral equations, which can be solved either numerically (e.g. by the method of moments) or by asymptotic evaluation of integrals. The latter approach is thoroughly explained in lectures Complex Analysis Background and Asymptotic Evaluation of Integrals. Finally, in advanced antennas and electromagnetic structures it is advantageous to use periodic structures, either to obtain artificial materials (such as magnetic materials at microwave frequencies) or to obtain frequency-dependent structures. The analysis methods for periodic structures are taught in the lecture Periodic Antenna Problems, while possible examples (such as metamaterials or EBG - electromagnetic bandgap structures) are explained in the lecture Advanced Periodic Structures.

### **2.3 Participants**

A total of twenty students attended the course. Most students came from Italy (25%), Spain (20%) and The Netherlands (20%), while the rest was distributed between Germany, Austria, Denmark, France, and even Israel. A photo of the class with students, teachers and local staff is shown in Fig. 1., while the demographic analysis with respect to the country of origin is shown graphically in Chart 1. The full list of all course participants, including teachers and technical staff, is given in Annex B.



Fig. 1. Photo of the course (students, teachers and course staff) with Lapad bay in the background

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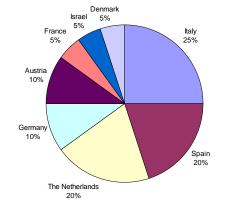


Chart 1. Distribution of students by country of origin

## **2.4.** Logistic arrangements

#### Venue

The course was held at Hotel Uvala in Dubrovnik, Croatia, situated in the tourist resort of Lapad bay, not far from the city centre. The conference room of the hotel was adapted to a classroom and fully equipped with desks, an overhead projector and an A0-size notepad/board. It was also air-conditioned. Some photos from the lectures are shown below.

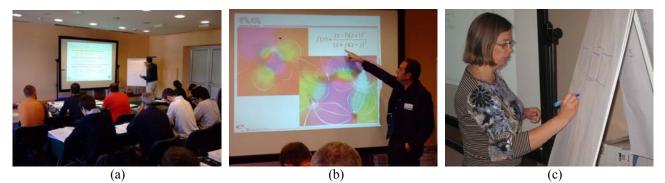


Figure 2. (a) Students during the lecture; (b) Prof. Maci showing the art of complex-valued functions; (c) Prof. Skrivervik demonstrating unit cell geometry.

#### Lodging

Lodging was organized at Hotel Vis (\*\*\*), only 50 m away from Hotel Uvala where classes were held. All students and teachers stayed at Hotel Vis in single and double rooms with private bathrooms. Internet connection was available for free in the hotel lobby. The standard half-board arrangement (breakfast + supper) was modified upon our request to breakfast + lunch. Everyone staying at the same location proved to be an excellent idea because it enabled stronger social interaction among students as well as between students and teachers.



Figure 3. Coffee break (left); Volleyball with prof. Freni (middle); Social dinner (right)

#### Coffee breaks and social events

Two coffee breaks with coffee, tea, juices and different snacks were prepared daily: one between two morning lessons and the other between two afternoon lessons. The coffee breaks were held on the terrace of Hotel Uvala overlooking Lapad bay. The official course dinner was organized on Wednesday 16<sup>th</sup> May at restaurant "Buono" not far away from

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hotel "Vis". Among informal social events, we should mention an excursion to Mount Srd, led by prof. Šipuš on Thursday 17<sup>th</sup> May and some recreational volleyball with participation of prof. Freni, as shown in Figs. 3.

### 2.5. Course management

#### **Course Material**

All students were given the handouts of the lecture prior to each lesson. Most of the photocopying was done at University of Zagreb before the course, while the rest was done in Dubrovnik at special discount prices. At the end of the course, all participants were also given CDs with all lectures in PDF format, as well as exercises and questions from the final exam. A selection of photos taken during the course by both students and organisers was also included.

All students were also given a photocopy of the draft version of the course book, which had been prepared by joint efforts of University of Siena, University of Zagreb and University of Florence. For this year's course, a number of typing errors in the manuscript were corrected and parts of the text were significantly improved. The work on the final version of the course book is still ongoing.

## **3. ASSESSMENT OF THE RESULTS**

#### **3.1. Credits and Exams**

In accordance with the number of hours of lectures and exercises, the course has been valued **3 ECTS credits** for all students who followed and successfully passed the final exam and **2 ECTS credits** for students who attended the course but did not take the final exam. The exam exercises were composed by the course teachers and included three exercises for each major course topic (fundamental theorems, construction of solutions, Green's functions/asymptotic evaluation). Students were divided in groups and were assigned one random exercise for each topic. During the last lesson, the best of the submitted solutions were presented by their respective authors.

#### 3.2. Evaluation of the course from the students, detected strong and weak points

On the last day of the course, the students were given the standard ESoA Evaluation Form in order to provide feedback on the quality and balance between the lessons. The rating system was from 1 to 5, with 1 being the worst note and 5 being the best. The average scores are reported in Tables 1.a and 1.b.

	Prof. Angelo Freni	Prof. Stefano Maci	Prof. Zvonimir Šipuš	Prof. Anja Skrivervik	Prof. Giuseppe Vecchi
Instructor is knowledgeable about the subject	4,95	5,00	4,89	4,83	5,00
Instructor is prepared	4,89	4,89	4,95	4,89	4,89
Instructor encourages participation	4,63	4,58	4,84	4,28	4,47
Instructor answers students' questions	4,89	4,89	4,89	4,67	4,79
Instructor is enthusiastic about teaching	4,53	4,79	5,00	4,78	4,79
Instructor's fluency in English	4,67	4,72	4,83	4,82	4,89
Interest of material	4,84	4,79	4,74	4,50	4,53
Relevance of material	4,95	4,89	4,94	4,50	4,68
Using teaching aids (overheads, videos, handouts)	4,63	4,47	4,47	4,39	4,32
How pertinent were the course objectives to the target audience?	4,56	4,44	4,67	4,47	4,44
How well do you feel the course objectives were met?	4,84	4,26	4,84	4,44	4,47
How do you feel about the level of detail of the course documents?	4,63	4,37	4,68	4,33	4,37
Average note for the teacher	4,75	4,68	4,81	4,58	4,64

### **Table 1.a Teachers Evaluation – Average Scores**



## Table 1.b Logistical Support – Average Scores

Overall Ave	erage 4,61
Quality and variety of the meals served	3,95
Desks and tables provide adequate working space	4,21
Classroom is comfortable and inviting	4,58
Accomodation	4,47
Support during the course (materials, handouts,)	4,84
Registration / Check-in	4,74
Support before coming to the course (travel information, accomodation, visa)	4,89
Invitation letter about the course	4,65

The course was in general rated very positively. The choice and the order of lectures were found satisfactory, and teachers and most lectures had very high average notes. The general opinion is that the course is very good, but in some parts hard to follow. The hardest lesson to follow, according to students, was the lesson on Asymptotic evaluation of integrals, given by Prof. Maci. We will try to attribute more time to that lecture in the next edition of the course.

The organization of the course has been rated excellent, except for the quality and variety of the meals served and, to some extent, the quality of working space provided. Unfortunately, the price/quality ratio in Dubrovnik restaurants does not leave room for much improvement in that area, despite all efforts to bring that part of the organization to a higher level. As for the classroom, a bigger and better-equipped classroom will be sought for the next edition of the course.

Though some students point out that, in order to have enough time to accept all the mathematical concepts, the course should last two weeks, the six-day format of the course, introduced for the first time for this edition of the course, has been found satisfactory by teachers and most students, and is likely to be kept in the future, possibly with slight reorganisation of the course schedule.

## 4. ACKNOWLEDGEMENTS

This course was financially supported by European Science Foundation project "New Frontiers in mm/sub-mm waves integrated dielectric focusing systems" (ESF RNP NewFocus), and COST action IC1102 "Versatile, Integrated, and Signal-aware Technologies for Antennas" (COST VISTA).

## ANNEX A: COURSE PROGRAMME

Monday 14/0	5/2012: FUND	AMENTAL THEOREMS FRAMED IN THE ANTENNA ANALYSIS
<b>Z. Sipus</b> (UNIZAG)	09.30-10.00	<ul><li>The European School of Antennas</li><li>Introduction to the course</li></ul>
<b>A. Freni</b> (UNIFI)	10.00-13.00	Fundamental theorems (1) <ul> <li>Uniqueness</li> <li>Energy</li> </ul>
	15.30-19.30	<ul> <li>Fundamental theorems (2)</li> <li>Equivalence and induction theorems</li> <li>Integral equation and Computational Electromagnetics (CEM)</li> <li>HF approximations for scattering and diffraction</li> <li>Examples and exercises</li> </ul>

Tuesday 15/05/2012: FUNDAMENTAL THEOREMS / SCALAR AND VECTOR WAVE EQUATION				
<b>A. Freni</b> (UNIFI)	09.00-13.00	<ul> <li>Fundamental theorems (3)</li> <li>Reciprocity Theorem</li> <li>Implication of reciprocity in IE-MoM and application in CEM</li> </ul>		
	Lunch break			

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<b>Z. Sipus</b> (UNIZAG)	15.30-17.30	Fundamental theorems         • Examples and exercises
(UNIZAO)	17.30-19.30	<ul> <li>Scalar wave equation</li> <li>Introduction to waves</li> <li>Definition of scalar wave equations; basic properties</li> <li>Solving scalar wave equation using separation of variables</li> </ul>

Wednesday	16/05/2012: SC	ALAR AND VECTOR WAVE EQUATION / COMPLEX ANALYSIS
<b>Z. Sipus</b> (UNIZAG)	09.00-13.00	Scalar wave equation         • Solving scalar wave equation using separation of variables         • Eigenvectors, eigenmodes and completeness relations         • Examples and exercises         Vector wave equation         • Helmholtz' and Deby's potentials         • Open and closed domains         • Green's identities         • Examples and exercises
	Lunch break	
<b>S. Maci</b> (UNISI)	15.30-17.30	<ul> <li>Examples, open discussion and exercises</li> <li>Scalar and vector wave equation</li> </ul>
	17.30-19.30	<ul> <li>Complex analysis fundamentals</li> <li>Residue theorem – Jordan's lemma</li> <li>Transformation of complex variables, branch cuts</li> <li>Examples and exercises</li> </ul>

Thursday 17/0	Thursday 17/05/2012: GREEN'S FUNCTIONS			
<b>S. Maci</b> (UNISI)	09.00-13.00	Green's functions         • Generalities         • Scalar and Dyadic GF         Scalar GF representation         • 1D, 2D and 3D GF for the wave eqs         GF in cylindrical spectral coordinates open-domain         • p-Tx line and z-Tx line         GF in closed –domains         • Waveguides and cavities		
	Lunch break			
	15.30-19.30	<ul> <li>Examples and Exercises:</li> <li>Parallel plate waveguide</li> <li>Horizontal dipole in stratified media,</li> <li>Dipoles close to cylindrical multistrates</li> <li>Dipoles close to a wedge</li> <li>Dipole close to an impedance surface</li> </ul>		

Friday 18/05/	Friday 18/05/2012: ASYMPTOTIC EVALUATION OF INTEGRALS				
<b>S. Maci</b> (UNISI)	09.00-13.00	Basics of asymptotic evaluation of integrals:         • Typical integral representation         • Saddle points and SDP         • Examples and exercises			
	Lunch break				
	15.30- 19.30	<ul> <li>Non Uniform and Uniform asymptotics:</li> <li>Saddle-point – pole interaction (wedge problem)</li> <li>Saddle-point – complex pole (evanescent wave diffraction, slab Green's function)</li> <li>End-point – saddle-point (PO-diffraction, Shadow boundary diffraction)</li> </ul>			



9.00-12.00	<ul> <li>Periodic structures:</li> <li>Floquet Theorem</li> <li>Green's functions for periodic structures in space domain</li> </ul>
	<ul> <li>Green's functions for periodic structures in spectral domain</li> <li>Convergence problems</li> </ul>
2.00-13.00	<ul> <li>Periodic structures – advanced topics:</li> <li>Acceleration of calculation of Green's functions - introduction</li> </ul>
unch break	
	<ul> <li>Periodic structures – advanced topics:</li> <li>Acceleration of calculation of Green's functions (summation by parts, Shanks' transform, Ewald's transform,)</li> <li>Dispersion of Green's functions for periodic structures (dispersion diagrams, bandgaps, EBG structures, metamaterials)</li> <li>Final assessment tests</li> </ul>
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# **ANNEX B: LIST OF PARTICIPANTS**

NAME	LEVEL	INSTITUTION	COUNTRY	ROLE
Stefano Maci	Prof.	University of Siena	Italy	coord. / teacher
Zvonimir Sipus	Prof.	University of Zagreb	Croatia	coord. / teacher
Angelo Freni	Prof.	University of Florence	Italy	teacher
Anja Skrivervik	Prof.	Ecole polytechnique federale de Lausanne	Switzerland	teacher
Giuseppe Vecchi	Prof.	Politecnico di Torino	Italy	teacher
Sinisa Skokic	Post-doc.	University of Zagreb	Croatia	organizer
Marko Bosiljevac	Post-doc.	University of Zagreb	Croatia	organizer
Antonio Berenguer	Ph.D.	Istituto de Telecomunicaciones y Aplicaciones Multimedia	Spain	student
Darwin Blanco	Ph.D.	Universidad Carlos III de Madrid	Spain	student
Edgar Diaz Tapia	Ph.D.	Universitat Politecnica de Catalunya	Spain	student
Valentina Di Mattia	Ph.D.	Università Politecnica delle Marche	Italy	student
Ariel Epstein	Ph.D.	Technion - Israel Institute of Technology	Israel	student
Marco Faenzi	Ph.D.	University of Siena	Italy	student
Markus Gardill	Ph.D.	FAU Erlangen-Nuremberg	Germany	student
Andreas Haderer	Post-doc.	NTHFS	Austria	student
Heinz Haderer	Ph.D.	NTHFS	Austria	student
Marco Righero	Post-doc.	Istituto Superiore Mario Boella	Italy	student
Daniel Rodrigo	Ph.D.	Universitat Politecnica de Catalunya	Spain	student
Eugen Safin	Ph.D.	University of Kiel	Germany	student
Josip Seljan	Ph.D.	University of Rennes 1	France	student
Valentina Sozio	Ph.D.	University of Siena	Italy	student
Waqas Hassan Syed	Ph.D.	TU Delft	The Netherlands	student
Marco Tannino	Ph.D.	La Sapienza Università di Roma	Italy	student
Alexandru Tatomirescu	Ph.D.	Aalborg University	Denmark	student
Nurhan Turker Tokan	Post-doc.	TU Delft	The Netherlands	student
Kalyan Vaddagiri	M.Sc.	TU Delft	The Netherlands	student
Ozan Yurduseven	Ph.D.	TU Delft	The Netherlands	student