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REPORT OF PROJECT WORK
during the stay at the George Green Institute for Electromagnetics Research, University of Nottingham, UK in July – October 2011, in the framework of the ESF Network "Newfocus"

Topic: Modeling of frequency-selective polarizing reflectors made of sub-wavelength wire grids for millimeter-wave and THz applications

During my stay in Nottingham I continued my Ph.D.-thesis study into the modeling of electromagnetic wave scattering by periodic reflectors made of dielectric or imperfect-metallic cylinders with sub-wavelength diameter. The interest in this study is explained by numerous applications of the grid-based components in quasi-optics, from short centimeter to THz waves.

According to the project plan, I continued the investigation of infinite linear chain of dielectric and silver cylinders (wires). Considered was a grating made of the parallel to the *z*-axis and periodic along the *x*-axis circular cylinders in free space. The grating period is p, the wire radius as a, and the refractive index is ν. We suppose that the field is time-harmonic with wavelength λ. Although two alternative polarisations can be considered, based on my previous results the H-polarization is more interesting for both silver and dielectric-wire chains. The function that represents the field *z*-component must satisfy the Helmholtz equation with appropriate wavenumber inside and outside of cylinders, the Sveshnikov radiation condition at infinity, condition of local integrability of power, and the boundary conditions demanding continuity of the tangential field components at the cylinder boundary. These conditions guarantee of solution uniqueness.

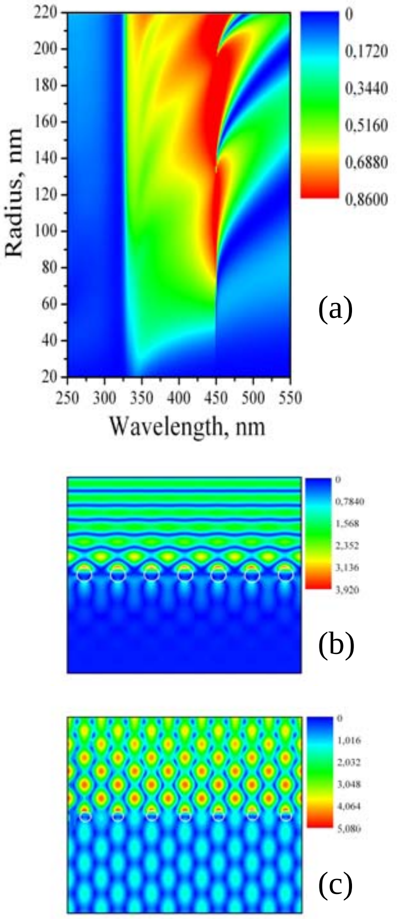

Fig.1 Reflectance of infinite grating of silver wires as function of the wavelength and wire radius for the normal incidence of the H-polarized plane wave (a). Period $p = 450$ nm. Near-field patterns for a = 90 nm: in P-resonance ($\lambda = 335.8$ nm) (b) and in G-resonance ($\lambda = 447.6$ nm) (c)

Performing mathematical operations similar to [1,2] with involvement of the specific lattice sums [3] we derive an infinite matrix equation for the field expansion coefficients. However, unlike [1,2] we emphasize that this equation has to be cast to the so-called Fredholm second kind form. Further, although basic features of scattering such as periodicity-caused resonances are present irrespectively of the wavelength range, the visible range offers additional phenomena such as plasmon resonances. Therefore I made computations for the visible-light scattering.

In Fig. 1 (a), I present the relief of reflectance as a function of two variables (wavelength and cylinder radius) for the grating made of silver nanowires. One can see the existence of two kinds of resonances. The first, observed near the wavelength of $\lambda \approx 340$ nm, is plasmon (P-resonance). Another one appears at the wavelength slightly larger than the grating period, $p = 450$ nm (G-resonance) [4]. Together they lead to strong reflectance in wide band. It is interesting to visualise the in-resonance field patterns. In Figs. 1(b),(c), I present the total near fields in the P and G resonances, respectively, for the same parameters. One can see common features of the patterns in well-separated P and G-resonances: shadows in the lower half-space and standing waves caused by the intensive reflection in the upper half-space. However in the P-resonance the field hot spots are seen at the illuminated faces of each cylinder, and far above the grating the field is a superposition of the 0-th and the ±1-st Floquet spatial harmonics. In the case of isolated G-type resonance the grating vicinity is dominated by the very intensive standing wave formed by two ±1-st Floquet spatial harmonics. Those results were reported in the paper published in the *Optics Express* journal.

The G-type resonances are much clearer in the case of dielectric wire gratings. Fig 2 (a) shows the reflectance relief depending on the normalized frequency and the relative separation ($\xi = p/a$). As one can see, the G-resonance frequency tends to the branch point at $\sigma = 1$. Note that here the normalized frequency $\sigma = p/\lambda$ is used instead of the wavelength λ. G-resonance becomes even sharper coming closer to the branch point.

Approximate analytical expressions for eigenvalues for passive and active problems are deduced using the Gershgorin theorem and the fact that the derived matrix generates a Fredholm 2-nd kind operator. The expressions for the real part of eigenfrequency and Q-factor for the lowest G-type *x*-even-mode agree well with the reflectance maxima location and sharpness in the scattering problem

$$\sigma = 1 - \frac{\pi^8\left(\alpha^2-1\right)^2}{2\xi^8}, \quad Q = \frac{(\pi-1)\xi^4}{2(\alpha^2-1)\pi^4} \qquad (1)$$

Fig. 2 (b) shows the total field cuts along the axis of propagation, i.e. the intensity of the total field depending on the distance from the grating plane normalized by the period, in the lowest G-type *x*-even-mode resonance under the normal incidence of plane wave. The case of extremely sparse grating with $\xi = 20$ displays that the total field is 1000 times stronger than the incident one near the grating and still 100 times stronger at the distance of 100 periods from it.

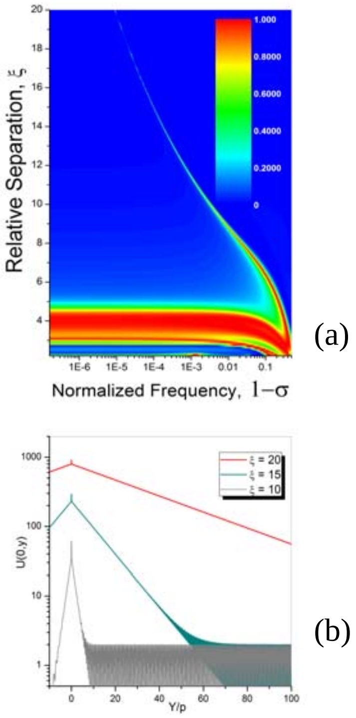

Fig.2 Reflectance of infinite grating of dielectric wires with $\nu = 2.48$ as a function of the normalized frequency and relative separation between cylinders (a). In-resonance total field cut along the propagation axis (b).

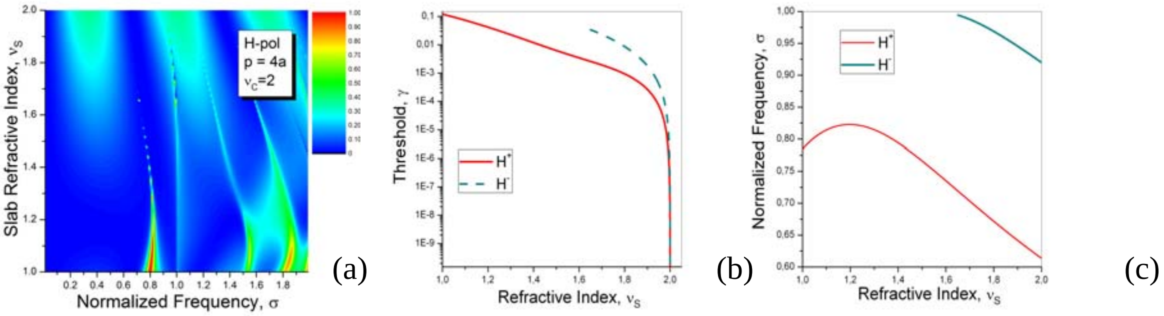

Fig.3 Reflectance of infinite grating of dielectric wires of $\nu = 2.0$ embedded in a slab with a variable refractive index as function of the normalized frequency (a). The eigenvalues for the first two bright ridges vs. refractive index: (b) threshold and (c) lasing frequency.

Additionally, a modified problem has been investigated where the cylinders medium "material gain" or threshold γ is introduced as a negative imaginary part added to refractive index, $\alpha - i\gamma$, to compensate for the radiation losses. As a result, we obtain a model where two parameters, the lasing frequency and the material threshold, are sought as eigenvalues ($\gamma, \sigma > 0$)

that is convenient for implementation [5]. The asymptotic solution for the lowest G-type *x*-even-mode lasing frequency in the H-polarization case is the same as in (1) while the threshold is

$$\gamma = \pi^4 \left(1-\alpha^2\right)^2 \left(\pi - 1\right) \Big/ 8\alpha\xi^4 \quad . \tag{2}$$

Here one can see inverse dependence of the threshold on the relative separation ξ, but it is also in direct dependence on the value $\alpha^2 - 1$, meaning that a smaller contrast between the refractive indexes of the host and cylinder media yields even smaller threshold. That is why I tried to enhance the G-resonance by embedding the grating of cylinders into a dielectric slab.

Firstly, I investigated the reflectance from a dielectric slab with embedded dielectric grating within of varying refractive index Fig. 3 (a). The limiting cases are well known: the first is a grating in free space and the second is a uniform slab. Intermediate case is more complicated. With increasing the slab refractive index the G-resonances become sharper and vanish because of the finite precision. The eigenvalues calculated for associated eigenproblem are shown in Fig.3 (b),(c). The lasing frequency accurately follows the traces of the grating resonances from Fig.3 (a). However the lasing threshold shows extremely rapid fall if coming to the case of the slab having only a small gain-induced contrast; then its threshold quickly reaches the previously not seen level of machine precision 10^{-12}.

Thanks to the ESF Newfocus Exchange Travel Grant, I had a brilliant opportunity to have scientific meetings and discussions with Prof. Trevor Benson, Dr. Ana Vukovic and many other academicians and researchers from the George Green Institute for Electromagnetics Research, had access to reach library of the University of Nottingham, and attended seminars of GGIEMR. The Exchange Grant gave me a possibility to take part in two international conferences where I presented my results: IONS-10 in Southampton and EuMC-2011 in Manchester.

References

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Journal paper related to the project and crediting ESF-Newfocus:

D.M. Natarov, V.O. Byelobrov, R. Sauleau, T.M. Benson, A.I. Nosich, “Periodicity-indused effects in the scattering and absorption of light by infinite and finite grating of circular silver nanowires,” Optical Express, vol. 19, no 22, pp. 22176-22190, 2011.

Conference papers related to the project and crediting ESF-Newfocus:

V.O. Byelobrov, T.M. Benson, A.I. Nosich, “Ultra low-threshold lasing modes of a periodic grating of nanocylinders,” *Proc. International OSA Network of Students Conf. (IONS-10),* Southampton, 2011.

V.O. Byelobrov, T.M. Benson, A.I. Nosich, “Near and far fields of high-quality resonances of an infinite grating of sub-wavelength wires,“ *Proc. European Microwave Conf. (EuMC-11),* Manchester, 2011, pp. 858-861

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