ESF Research Networking Programme NEWFOCUS

Analysis of oblique incidence on multilayer cylindrical lenses

scientific report

This report describes the work carried out during the short visit to Dipartimento di Ingegneria dell'Informazione, Università degli Studi di Siena from June 23 to July 5. The purpose of the visit was to start working on a oblique incidence on multilayer cylindrical lenses.

Main results

Oblique incidence is more complicated to analyze than perpendicular incidence because in oblique incidence each component of incoming electromagnetic wave ($\mathbf{E}$ or $\mathbf{H}$) doesn't have component parallel to z axis (regardless of polarization). Because of this equations will be coupled. Rigorous treatment of scattering from anisotropic objects is closely related with the possibility to generate suitable electromagnetic field representations. In most cases the considered structure can be described with diagonal permittivity and permeability tensors, which simplifies otherwise extremely complex problem. On Fig. 1 we can see the described situation.

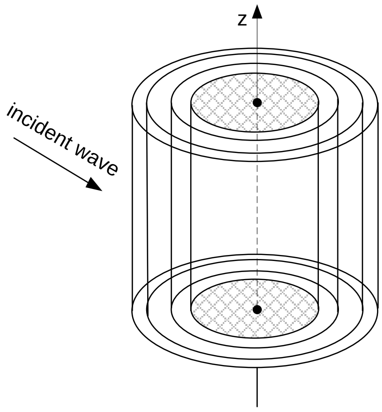

Figure 1 Oblique incidence on the circular cylinder

Starting point for modeling described situation are Maxwell equations. Combining those equations we get system of partial differential equations:

$$E_\rho = -\mathrm{j}\frac{1}{\omega\varepsilon_0\varepsilon_{\rho\rho}}\frac{1}{\rho}\frac{\partial H_z}{\partial\phi} + \mathrm{j}\frac{1}{\omega\varepsilon_0\varepsilon_{\rho\rho}}\frac{\partial H_\phi}{\partial z} \tag{1}$$

$$E_\phi = -\mathrm{j}\frac{1}{\omega\varepsilon_0\varepsilon_{\phi\phi}}\frac{\partial H_\rho}{\partial z} + \mathrm{j}\frac{1}{\omega\varepsilon_0\varepsilon_{\phi\phi}}\frac{\partial H_z}{\partial\rho} \tag{2}$$

$$H_\rho = \mathrm{j}\frac{1}{\omega\mu_0\mu_{\rho\rho}}\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} - \mathrm{j}\frac{1}{\omega\mu_0\mu_{\rho\rho}}\frac{\partial E_\phi}{\partial z} \tag{3}$$

$$H_{\phi} = \mathrm{j}\frac{1}{\omega\mu_0\mu_{\phi\phi}}\frac{\partial E_{\rho}}{\partial z} - \mathrm{j}\frac{1}{\omega\mu_0\mu_{\phi\phi}}\frac{\partial E_z}{\partial \rho} \tag{4}$$

This system of partial differential equations is difficult to solve, so we have to find some transformation which will simplify it. We anticipate symmetry in ϕ and z direction from which we get:

$$E(\rho,\phi,z) = \sum_{n=-\infty}^{+\infty} \hat{E}^n(\rho) e^{-jk_z z} e^{jn\phi} \tag{5}$$

$$H(\rho,\phi,z) = \sum_{n=-\infty}^{+\infty} \hat{H}^n(\rho) e^{-jk_z z} e^{jn\phi} \tag{6}$$

and from that we get system of ordinary differential equations:

$$\hat{E}_{\rho} = \frac{\omega\mu_0\mu_{\phi\phi}}{m_{\phi\rho}^2 - k_z^2}\frac{1}{\rho} n\hat{H}_z - j\frac{k_z}{m_{\phi\rho}^2 - k_z^2}\frac{d\hat{E}_z}{d\rho} \tag{7}$$

$$\hat{E}_{\phi} = \frac{k_z}{m_{\rho\phi}^2 - k_z^2}\frac{1}{\rho} n\hat{E}_z + j\frac{\omega\mu_0\mu_{\rho\rho}}{m_{\rho\phi}^2 - k_z^2}\frac{d\hat{H}_z}{d\rho} \tag{8}$$

$$\hat{H}_{\rho} = -\frac{\omega\varepsilon_0\varepsilon_{\phi\phi}}{m_{\rho\phi}^2 - k_z^2}\frac{1}{\rho} n\hat{E}_z - j\frac{k_z}{m_{\rho\phi}^2 - k_z^2}\frac{d\hat{H}_z}{d\rho} \tag{9}$$

$$\hat{H}_{\phi} = \frac{k_z}{m_{\phi\rho}^2 - k_z^2}\frac{1}{\rho} n\hat{H}_z - j\frac{\omega\varepsilon_0\varepsilon_{\rho\rho}}{m_{\phi\rho}^2 - k_z^2}\frac{d\hat{E}_z}{d\rho} \tag{10}$$

From this equations we get system of two coupled ordinary differential equations for E_z and H_z field:

$$\begin{gathered}\rho^2\frac{d^2\hat{E}_z^n}{d\rho^2} + \rho\frac{d\hat{E}_z^n}{d\rho} + \left(\rho^2\left(m_{\phi\rho}^2 - k_z^2\right)\frac{\varepsilon_{zz}}{\varepsilon_{\rho\rho}} - n^2\frac{m_{\phi\rho}^2 - k_z^2}{m_{\rho\phi}^2 - k_z^2}\frac{\varepsilon_{\phi\phi}}{\varepsilon_{\rho\rho}}\right)\hat{E}_z^n + \\ \rho\left(\frac{jnk_z}{\omega\varepsilon_0\varepsilon_{\rho\rho}} - \frac{m_{\phi\rho}^2 - k_z^2}{m_{\rho\phi}^2 - k_z^2}\frac{jnk_z}{\omega\varepsilon_0\varepsilon_{\rho\rho}}\right)\frac{d\hat{H}_z^n}{d\rho} = 0\end{gathered} \tag{18}$$

$$\begin{gathered}\rho^2\frac{d^2\hat{H}_z^n}{d\rho^2} + \rho\frac{d\hat{H}_z^n}{d\rho} + \left(\rho^2\left(m_{\rho\phi}^2 - k_z^2\right)\frac{\mu_{zz}}{\mu_{\rho\rho}} - n^2\frac{m_{\rho\phi}^2 - k_z^2}{m_{\phi\rho}^2 - k_z^2}\frac{\mu_{\phi\phi}}{\mu_{\rho\rho}}\right)\hat{H}_z^n + \\ \rho\left(\frac{m_{\rho\phi}^2 - k_z^2}{m_{\phi\rho}^2 - k_z^2}\frac{jnk_z}{\omega\mu_0\mu_{\rho\rho}} - \frac{jnk_z}{\omega\mu_0\mu_{\rho\rho}}\right)\frac{d\hat{E}_z^n}{d\rho} = 0\end{gathered} \tag{19}$$

$$m_{\rho\phi}^2 = \omega^2 \mu_0 \mu_{\rho\rho} \varepsilon_0 \varepsilon_{\phi\phi} \tag{20}$$

$$m_{\phi\rho}^2 = \omega^2 \mu_0 \mu_{\phi\phi} \varepsilon_0 \varepsilon_{\rho\rho} \tag{21}$$

Every other field component can be calculated from equations (7)-(10).

In future work we have to find out how to solve this system for differenth boundary conditions, and how to efficiently calculate those fields.