Volodymyr O. Byelobrov

REPORT OF PROJECT WORK
during the stay at the George Green Institute for Electromagnetics Research
University of Nottingham, UK
in December 2011– February 2012, in the framework of the ESF Network “Newfocus”

Topic: Modeling of frequency-selective polarizing reflectors made of sub-wavelength wire grids for millimeter-wave and THz applications

The current term was the extension and continuation of my previous stay in July – October 2011. During this stay at the University of Nottingham, I continued my Ph.D.-thesis study into the modeling of electromagnetic wave scattering and emission by periodic reflectors made of dielectric or imperfect-metallic cylinders with sub-wavelength diameters.

According to the project plan, I investigated infinite linear chains of dielectric and silver cylinders (wires). In general formulation the considered geometry is set as a grating made of the parallel to the *z*-axis and periodic along the *x*-axis circular cylinders in free space. The grating period is p, the wire radius as a, and the refractive index is ν. We suppose that the field is time-harmonic with the wavelength λ. Although two alternative polarisations can be considered, based on my previous results the H-polarization is more interesting for both silver and dielectric-wire chains. The function that represents the field *z*-component must satisfy the Helmholtz equation with appropriate wavenumber inside and outside of cylinders, the Sveshnikov radiation condition at infinity, the condition of local integrability of power, and the boundary conditions demanding continuity of the tangential field components at the cylinder boundary. These conditions guarantee solution’s uniqueness. Mathematical approach consists of derivation of infinite matrix equation for the field expansion coefficients by the method of variable separation similar to [1,2] with involvement of the specific lattice sums [3]. However, unlike [1,2] we emphasize that, to ensure convergence, this equation must be cast to the so-called Fredholm second kind form. Further, although certain basic features of the scattering, such as periodicity-caused resonances, are present irrespectively of the wavelength range, the visible range offers additional phenomena such as plasmon resonances if the wire material is a noble metal such as silver. Therefore I made computations for the visible-light scattering by gratings containing silver wires. I have also studied the associated eigenvalue problems assuming that the dielectric wires are active. According to the approved proposal my main tasks were as follows:

1. Modification of existing mathematical model to treat the grids made of (1) several different wires in one period, (2) two-layer dielectric-coated metal wires, and/or (3) wires embedded into dielectric slab.
2. Generalization of the existing codes to the modified wire-grid reflectors and polarizers; analysis and characterization of their characteristics.
3. Analytical and numerical investigation of the near and far fields in the grating-type resonances of various orders.
4. Writing a joint paper for a technical journal and conference papers.

Modification of existing model to treat the grids made of several different wires in one period

Assume that there are two wires per period having the radii a_1 and a_2; their centre locations have coordinates $\vec{r}_1$ and $\vec{r}_2$ with respect to the local origin, and their refractive indices are ν_1 and ν_2, respectively. Firstly we investigated the structure where the two wires are placed in the plane of the grating (at $y = 0$) with $p = 400\,\mathrm{nm}$. The silver wire has fixed radius, $a_1 = 50\,\mathrm{nm}$, and is placed $100\,\mathrm{nm}$ to the right of the cell centre, i.e. $x_1 = 100\ \mathrm{nm}$. The dielectric cylinder with varying radius a_2 has its centre at $x_2 = 100\ \mathrm{nm}$, i.e. symmetrically to the left. The relief of the reflectance as a function of a_2 and wavelength λ is shown in Fig. 1 (a). Here, one can see that

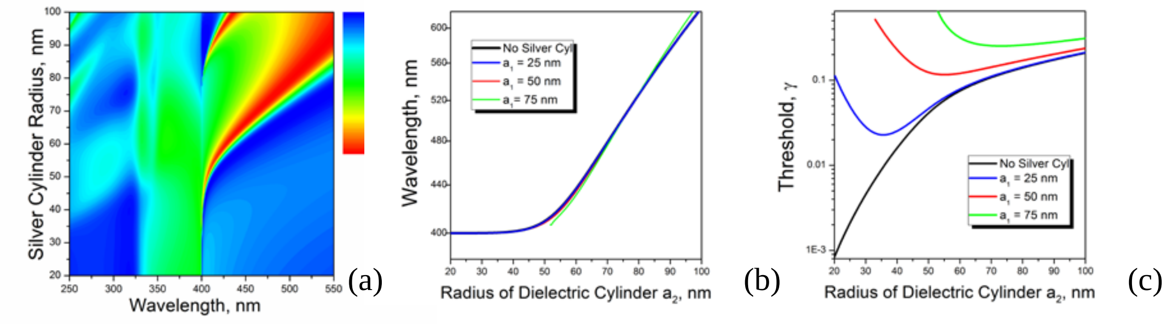


Fig. 1 Reliefs of reflectance (a) of the binary grating of silver and dielectric wires with $p = 400\,\text{nm}$. Silver wire is placed at $x_1 = 100$ nm and $a_1 = 50\,\text{nm}$. Dielectric wire with refractive index $\nu = 2.48$ has its centre at $x_2 = -100$ nm and varying period a_2. The LEP eigenpairs (b) lasing frequency and (c) lasing threshold versus the active wire radius: black line corresponds to the mode for the passive dielectric-wire grating. The colored lines are for two wires per period, one active and another of silver, where the first has the radius a_2 varying from $20\,\text{nm}$ to $100\,\text{nm}$ and the centre away $100\,\text{nm}$ from the origin and the silver wire has a constant radius $a_1 = 25, 50, 75\,\text{nm}$ and its centre is just symmetrically opposite from origin, the period is $400\,\text{nm}$.

the reflectance behavior inherits all the resonances for two different gratings of identical wires (see [4]). Figs. 1 (b) and (c) show the plots of eigenvalues (here they are the wavelength and the lasing threshold as we assume that dielectric wires are active [5,6]) for the same geometry as in Fig. 1 (a), i.e. for all wires placed in one plane. The curves in Fig. 1 (b) show that the lasing frequencies of the grating modes in the purely active-wire and the binary gratings have almost the same values – they are hardly distinguishable from each other. This is unlike the thresholds of the same modes shown in Fig. 1 (c): for the active-wire grating, they decay with shrinking of the wire radius (see [6]); however for the binary grating they reach a minimum value and then start growing. The explanation can be found in the fact that in the binary-grating case the pumping of the active dielectric cylinders has to overcome the losses in the silver wires in addition to the radiative losses. Interestingly enough, the minimum of the threshold is reached if the wires have roughly equal radii.

Modification of existing model to treat the grids made of wires in a dielectric slab.

Fabrication and use of the grating requires its embedding in a dielectric slab of different refractive index. Such slab with embedded grating was investigated using an approach similar to [7]; the resulting matrix obeys the Fredholm condition that guarantees convergence. The reflectance of a normally incident plane wave on the slab is shown in Fig. 2 (a).

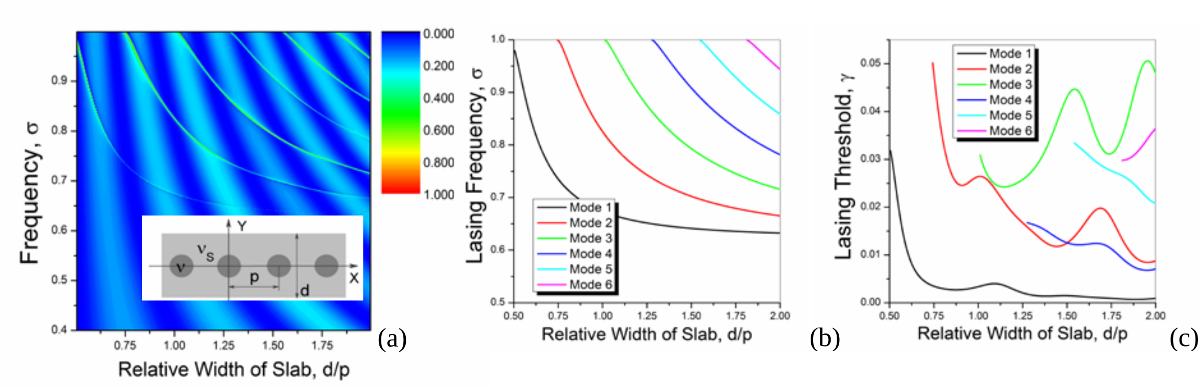
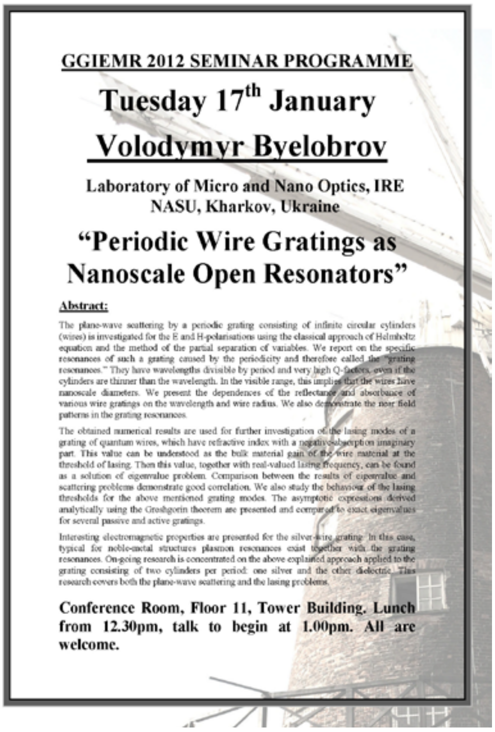


Fig. 2 Relief of reflectance in the H-polarization case versus the relative slab width and the frequency normalized by period (a). Refractive indices of slab and grating are $\nu_S = 1.6$ and $\nu = 2$, respectively. Eigenvalues for the corresponding active-wire resonator: lasing frequency (b) and lasing threshold (c).

Here there are visible the broad resonances on the modes of the slab at the background and six sharper grating resonances. The eigenvalue problem assumes adding imaginary part to the wire refractive index [5,6]. The lasing frequencies of the associated eigenmodes in Fig. 2 (b) exactly follow the resonances in Fig. 2 (a). However their lasing thresholds in Fig. 2 (c) are influenced by the slab modes and may show local minima. Solution of the mentioned two problems demanded generalization of the earlier codes to the modified wire-grid reflectors and polarizers. I have also analyzed analytically and numerically the near and far fields in the grating-type resonances.

Thanks to the ESF Newfocus Exchange Travel Grant, I had a brilliant opportunity to have scientific meetings and discussions with Prof. Trevor Benson, Dr. Ana Vukovic and other academics and students of the George Green Institute for Electromagnetics Research, participate and present my results on their seminar (see poster), and had access to the library of the University of Nottingham.



GGIEMR 2012 SEMINAR PROGRAMME

Tuesday 17th January

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"Periodic Wire Gratings as Nanoscale Open Resonators"

Abstract:

The plane-wave scattering by a periodic grating consisting of infinite circular cylinders (wires) is investigated for the E and H-polarisations using the classical approach of Helmholtz equation and the method of the partial separation of variables. We report on the specific resonances of such a grating caused by the periodicity and therefore called the "grating resonances." They have wavelengths divisible by period and very high Q-factors, even if the cylinders are thinner than the wavelength. In the visible range, this implies that the wires have nanoscale diameters. We present the dependences of the reflectance and absorbance of various wire gratings on the wavelength and wire radius. We also demonstrate the near field patterns in the grating resonances.

The obtained numerical results are used for further investigation of the lasing modes of a grating of quantum wires, which have refractive index with a negative-absorption imaginary part. This value can be understood as the bulk material gain of the wire material at the threshold of lasing. Then this value, together with real-valued lasing frequency, can be found as a solution of eigenvalue problem. Comparison between the results of eigenvalue and scattering problems demonstrate good correlation. We also study the behaviour of the lasing thresholds for the above mentioned grating modes. The asymptotic expressions derived analytically using the Groshgorin theorem are presented and compared to exact eigenvalues for several passive and active gratings.

Interesting electromagnetic properties are presented for the silver-wire grating. In this case, typical for noble-metal structures plasmon resonances exist together with the grating resonances. On-going research is concentrated on the above explained approach applied to the grating consisting of two cylinders per period: one silver and the other dielectric. This research covers both the plane-wave scattering and the lasing problems.

Conference Room, Floor 11, Tower Building. Lunch from 12.30pm, talk to begin at 1.00pm. All are welcome.

References

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3. C. M. Linton, "The Green's function for the two-dimensional Helmholtz equation in periodic domains", J. *Engineering Mathematics*, **33**, 377-402 (1998).
4. R. Gomez-Medina, M. Laroche, J. J. Saenz, "Extraordinary optical reflection from sub-wavelength cylinder arrays," *Opt. Exp.*, **14**, 3730-3737 (2006).
5. E.I. Smotrova, V.O. Byelobrov, T.M. Benson, J. Ctyroky, R. Sauleau, A.I. Nosich, "Optical theorem helps understand thresholds of lasing in microcavities with active regions," *IEEE J. Quant. Electron*, **47**, 20-30 (2011).
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7. H. Jia, *et al.*, "Reflection and Transmission Properties of Layered Periodic Arrays of Circular Cylinders Embedded in Magnetized Ferrite Slab" *IEEE Trans. Antennas Propag.*, **53**, no 3 1145-1152 (2005).

Journal and conference papers related to the project and crediting ESF-Newfocus:

1. V.O. Byelobrov, T.M. Benson, A.I. Nosich, "Binary Grating of Sub-Wavelength Silver and Quantum Wires as a Photonic-Plasmonic Nanoscale Lasing Platform," submitted to the special issue of the *IEEE Journal of Selected Problems in Quantum Electronics*, 2012.
2. V.O. Byelobrov, T.M. Benson, A.I. Nosich, "Impact of Hosting Media on the Specific Resonances of Periodic Grating Embedded in Dielectric Slab," submitted to the international conference *OWTNM2012*, Barcelona, 2012.