# Scientific report of ESF Exchange Grant

## Topic: Surface plasmon lenses based on Transformation Optics.

The main goal of this short visit (five weeks) was to start a new collaborative project between my theoretical group at Universidad Autónoma de Madrid with the groups directed by Prof. Stefan Maier and Prof. John B. Pendry, both working at Imperial College of London. The general idea of this project has been the application of Transformation Optics (TO) recipes to mould the flow of surface plasmon (SPs) in nanostructured metal surfaces. In particular, we have worked on developing feasible strategies to concentrate the energy (light) carried out by SPs in subwavelength volumes located on a metal surface, aiming for bio-sensing applications.

During these five weeks our main work has been concentrated on testing different strategies for building up lenses for SPs based on TO. At the beginning, we developed several TO recipes for both the permittivity (ε) and permeability (μ) of the film deposited on top of the metal surface. The permittivity and permeability tensors required by the TO framework are usually implemented using metamaterials, artificial materials that allow to effectively mimic electromagnetic (EM) responses by repeating structured unit cells at a subwavelength scale. However, we found that our values for the EM parameters (ε and μ) were highly anisotropic and inhomogeneous and, therefore, it would be extremely difficult to put them into practice. After these first attempts, we realized that there was a more feasible way to design functionalities for SPs: using two-dimensional (2D) conformal mapping. This mapping is a particular case of TO for 2D structures in which some mathematical requirements for the coordinate transformation are imposed. The important asset of these conformal transformations is that they give rise to isotropic material parameters, yielding easier implementations of transformation media based on isotropic dielectrics. The limitation of conformal mapping is that their applicability is restricted to 2D systems and for a particular polarization of the EM wave. **However, SPs are quasi-2D surface waves and its polarization is well defined**. Therefore, it seemed a good idea to apply conformal mapping techniques to develop lenses for SPs.

In particular, we have used Schwartz-Christoffel transformations (SCTs). SCTs are conformal maps, which map 2D polygons onto the unit disk or the upper complex half plane (see Figure 1).

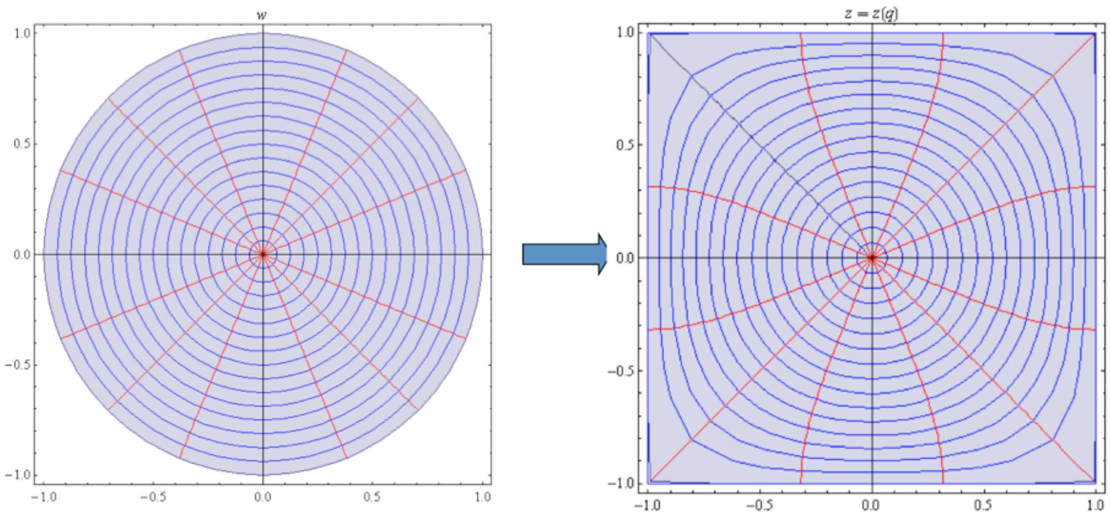

**Figure 1**. Conformal transformation of a radial and an azimuthal grid in a unit disk onto a square.

The 2D refractive index map associated with this conformal transformation is displayed in Figure 2:

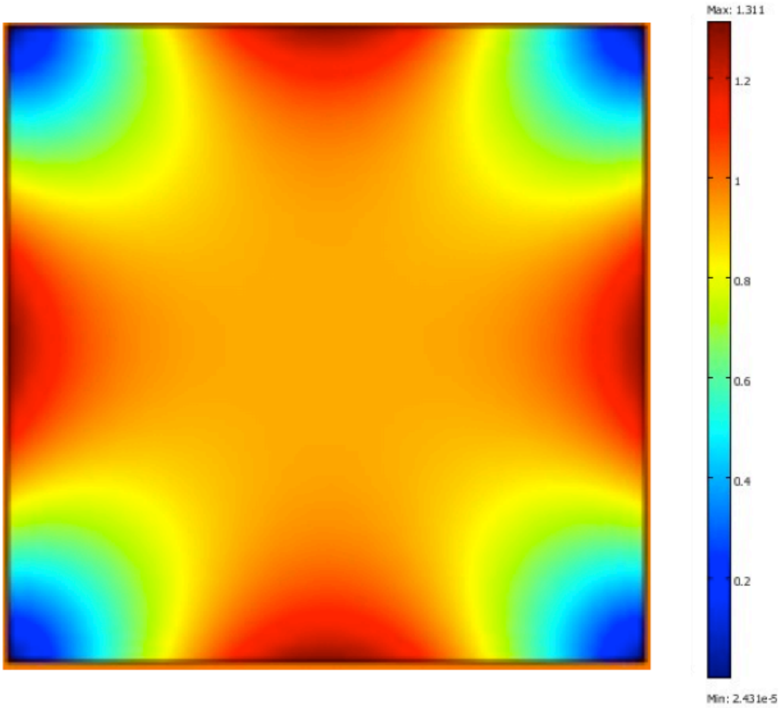

**Figure 2**. The corresponding 2D map for the refractive index (ranging from 1.3 to 0) associated with the 2D Schwartz-Christoffel transformation shown in Figure 1.

In the right panel of Figure 3 we show the results of a 2D simulation in which an incident Gaussian beam (with a waist of 2 microns, coming from a medium with refractive index 1.5 and characterized by a polarization in which the electric field points in the perpendicular direction to the 2D plane) impinges into a dielectric layer of thickness 2.5 microns and width 5 microns whose refractive index map is rendered in the left panel of Figure 3.

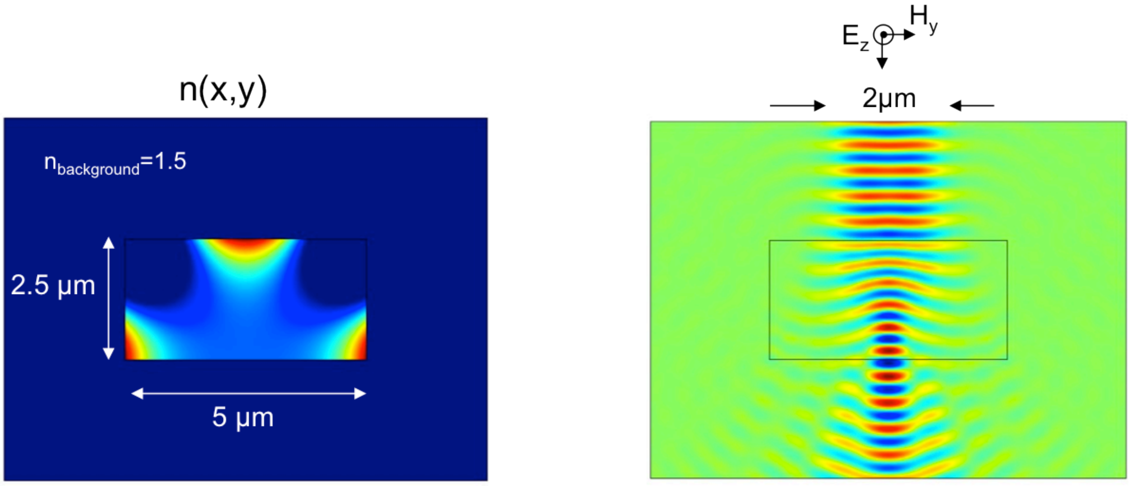

**Figure 3**. 2D numerical simulation of a Gaussian beam impinging into a dielectric film (see details in the main text).

Thanks to the dependence of the refractive index with x and y within the dielectric film, most of the EM energy carried out by the incident Gaussian beam is focused on a tiny spot placed at the output interface of the dielectric film.

We have found that this recipe, which operates for a 2D problem, can be used to create a lens for a 3D SP. The idea is very simple. Since TO relies on Maxwell's equations, it applies to all kinds of EM waves and in particular to SPs. However, we need to bear in mind the fact that SPs are 3D surface waves propagating along a metal-dielectric interface with a field distribution that is evanescent in the direction perpendicular to the interface. Therefore, in principle, an infinite number of 2D coordinate transformations should be performed in planes parallel to the metal-dielectric interface in order to operate over the whole plasmonic field. As the SPP field extends into the dielectric and the metal, these 2D coordinate transformations should be implemented at both sides of the interface. Due to the fact that the SP decay length in the metal (skin depth) is of a few tens of nanometers at optical wavelengths, a manipulation of the metal properties would be needed at a nanometer scale. However, we have found that a more feasible approach that consists of modifying the EM properties of the dielectric side only indeed gives very accurate results, while control over the metal is not actually required. In practice, the thickness of the transformed media in the dielectric side should be a bit larger than the decay length of the SP in the dielectric that, at optical frequencies, is of the order of hundreds of nm.

Our final design for a SP lens is illustrated in Figure 4(a). Basically, the 2D profile for the refractive index, as shown in panel (c), is repeated in the third dimension up to a thickness of 500 nm in the dielectric side. Panel (b) shows how the z-component of the electric field behaves when a SP is impinging into the dielectric slab from the left side. As in the 2D case displayed in Figure 3, the EM energy carried out by the SP is concentrated on the output exit of the film. Panel (d) renders the z-component of the E-field evaluated just at the interface between the metal and the *textured* dielectric film.

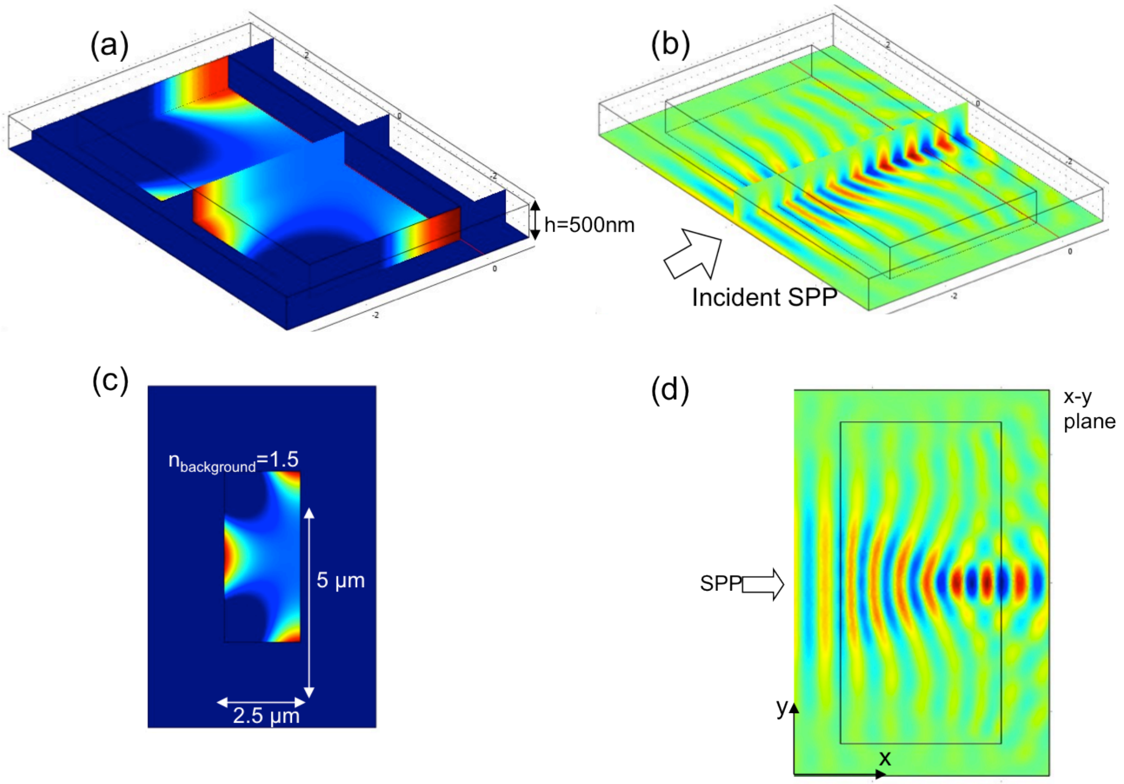

**Figure 4**. A lens for a surface plasmon (see details in the main text). The operating wavelength is 800 nm.

Once we have designed a lens for a surface plasmon, the next step is to put it into practice. This is going to be the subject of our collaborative project for the next few months, namely, to find the best way to engineer a metamaterial such that its optical response could be well reproduced by the 3D map of the refractive index displayed in Figure 4(a). If successful, we could use the fabrication facilities of Imperial College to build up the first prototype and test it.