SCIENTIFIC REPORT

Summary

This is the scientific report for

- the master class on *Instantons, Knots and Khovanov* by Singer Professor of Mathematics at MIT, Tomasz S. Mrowka, from August 1 until August 5, 2011, `http://qgm.au.dk/cal/special/2011/instantons/`, and
- the *Århus Gauge Theory Workshop* from August 8 until August 12, 2011, `http://qgm.au.dk/cal/special/2011/gauge/`.

Both were held at the Centre for Quantum Geometry of Moduli Spaces at Aarhus University. The organizers were

- Jœrgen E. Andersen, Århus University, Denmark,
- Benjamin Himpel, Århus University, Denmark,
- Robert C. Penner, CalTech/Århus University,
- Nicolai Reshetikhin, UC Berkeley/Århus University.

There were 52 master class participants and 93 workshop participants from Australia, Brasil, Canada, Denmark, France, Germany, Hungary, India, Italy, Japan, Netherlands, Portugal, Scotland, South Korea, Spain, Sweden, Switzerland, Turkey, United Kingdom and the United States working in geometric topology or mathematical physics with a particular interest in gauge theory.

The master class was aimed at graduate students and junior researchers with a background in differential geometry, differential topology or mathematical physics. It gave an introduction and overview to one of the most important recent developments in gauge theory and served as a preparation for the workshop.

The workshop strengthened the interaction and exchange of ideas between these communities of researchers. There were a total of twenty-two 60-minute talks by internationally recognized experts. We left ample time for questions, problem sessions and informal discussion, and we organized two dinners in each week for social interaction. The workshop allowed European researchers with an interest in gauge theory to connect with each other and internationally.

1. Scientific content

Since the work of Donaldson in the 1980's, gauge theory has evolved as an indispensable tool in the study of smooth 4-manifolds. After the introduction of the Seiberg-Witten invariants in 1994, the theory was both simplified and extended, and the relation to Gromov-Witten invariants (for symplectic 4-manifolds) was established by Taubes. The equivalence of the Donaldson and Seiberg-Witten invariants, conjectured by Witten in 1994, now seems close to being proved by Feehan-Leness. Kronheimer-Mrowka have used part of the work of Feehan-Leness combined with that of Taubes to complete the proof of Property P for knots, which Kronheimer-Mrowka have later reproved more directly using sutured Floer homology. Recently, the Seiberg-Witten invariants together with the rational blow-down technique of

Fintushel-Stern have been applied to prove the existence of exotic smooth structures on $\mathbf{C}P^2\#k(-CP^2)$ for small values of k according to work of Akhmedov-Park after initial results of J. Park. The introduction of *finite-dimensional approximation* by Furuta signaled a new direction in gauge theory. He first used it to prove a weakened version of the 11/8 conjecture, the 10/8 theorem. Later, Bauer-Furuta defined the refined Seiberg-Witten invariants, which are stronger than the classical ones. However, Furuta-Kametani-Matsue-Minami have recently shown that they are not enough to show the 11/8 conjecture. In the theory of knots and 3-manifolds, the various kinds of Heegard Floer homologies introduced by Ozsvath-Szabo have become a highly active area of research. There is furthermore major work by Salamon-Wehrheim towards the classical Atiyah-Floer conjecture, which motivated Ozsvath-Szabo's construction. Last year, Kutluhan, Lee and Taubes presented a proof of the isomorphism between the long exact sequences for Heegaard Floer homology and Seiberg-Witten Floer homology—also known as monopole Floer homology—with some technical proofs announced for later. Also last year, Witten has presented an analytic continuation of Chern-Simons theory, which provides new links to $\mathrm{SL}(2,\mathbf{C})$–Chern-Simons theory and quantum gravity, and suggests possible connections between Chern-Simons theory and higher-dimensional gauge theories. Motivated from higher-dimensional gauge theory, Witten and Haydys have then independently discovered a five-dimensional gauge theory, which reduces to a holomorphic Chern-Simons theory on three-manifolds.

The master class began with the basics of the study of the Anti-Self-Dual Yang-Mills equation. Mrowka emphasized the parts of the theory important for setting Floer homology for three manifolds and also for links in three manifolds. He showed how to extend the theory to sutured manifolds and cover the surgery exact sequences for these theories. We saw that these theories can be used to detect knot genus and the Thurston norm. Using this background he related Khovanov homology to a version of instanton Floer homology and deduced that Khovanov homology detects the unknot from the corresponding result in the instanton theory.

Assessment and impact

The master class and the workshop had created opportunities for PhD students, young researchers and experts to exchange ideas in gauge theory and collaborate, or to get introduced to this topic on the boundary between mathematical physics and geometric topology.

Video recordings of the master class and all the workshop talks as well as all the available slides are available via the respective websites.

List of speakers and final program

Here is a list of the speakers with the titles and abstracts of the corresponding talks.

Scott Baldridge: *On the rotation class of knotted Legendrian tori in* $\mathbf{R}^5$. In this talk I will discuss how to use Lagrangian hypercube diagrams to easily compute the "rotation number" for a large class of knotted Legendrian tori in $\mathbf{R}^5$ with respect to the standard contact form. This result is the first step in a program to understand the contact homology of such embeddings and is joint work with Ben McCarty.

Stefan Bauer: ***Differential equations and stable homotopy.*** The talk considers a space parametrizing certain differential equations. The topology of the space in question turns out to be related to spaces well known from stable homotopy theory: The classifying spaces of ko-theory and stable homotopy theory.

Inanc Baykur: ***Round handles and smooth four-manifolds.*** In this talk, we are going to unfold the strong affiliation of round handles with smooth four-manifolds. Various fundamental topics that appear in the study of four-manifolds, such as handlebodies, cobordisms, logarithmic transforms along tori, exotic smooth structures, and broken Lefschetz fibrations, will come into play as we discuss the relevant interactions between them.

Hans Boden: ***An $SU(N)$ Casson-Lin invariant for links $L \subset S^3$ with more than one component.*** In 1992, X.-S. Lin introduced a Casson-type invariant $h(K)$ for knots $K \subset S^3$ that counts conjugacy classes of irreducible $SU(2)$ representations of the knot group G_K with meridional trace equal to -2. Lin identified $h(K)$ with the signature of the knot, and his approach was generalized to give invariants for other trace conditions and for knots in homology 3–spheres independently by C. Herald in 1997 and by M. Heusener and J. Kroll in 1998. In [Pac. J. Math. 248 (2010), 139-154], E. Harper and N. Saveliev define a Casson-Lin type invariant $h(L)$ for 2–component links $L \subset S^3$ and show that $h(L)$ equals the linking number. This talk is a report on recent joint work with Eric Harper introducing analogous invariants of links $L \subset S^3$ with $n \geq 2$ components using the group $SU(N)$. The invariants are denoted $h_{N,a}(L)$, where $a = (a_1, \ldots, a_n)$ is a n–tuple of integer labels, one for each component of the link, and they are defined as a signed count of conjugacy classes of certain projectively flat $SU(N)$ invariants of the link group G_L. The talk will outline the compactness and irreducibility results needed to show that $h_{N,a}(L)$ is well-defined, a vanishing result for split links, and preliminary computations.

Celso Doria: ***Variational aspects about the Existence of Seiberg-Witten Monopoles on a Four Manifold.*** In abscence of an existence theorem for Seiberg-Witten Monopoles on a smooth four manifold a Variational set up is used to study the question. There exist smooth 4-manifolds admitting monopoles, whose existence is achieved by proving the Seiberg-Witten invariant is non-trivial. However, in all cases known by the author, the invariant's non-triviality depends on the fact that the invariant of a symplectic 4-manifold is non-trivial. Thus, by exploring the variational set up, the Witten's existence theorem on a Kähler manifold is proved and some question concerning the lowest eigenvalue of the differential operator $L_A = \nabla_A + \frac{k_g}{4}$, defined on sections of the positive spinors bundles, are discussed.

Gabor Etesi: ***On the existence of a complex structure on the six-sphere.*** Existence of a complex structure on the six dimensional sphere is re-examined in the talk. The approach is based on re-interpreting a hypothetical complex structure as a classical vacuum solution of a non-linear Higgs scalar field theory (with gauge symmetry) formulated on S^6. This classical vacuum solution is then constructed by Fourier expansion from the obvious one of a similar theory on the exceptional compact Lie group G_2.

Ron Fintushel: ***Surgery on nullhomologous tori.*** Abstract: I will discuss joint work with Ron Stern which attempts to identify, in a connect sum of projective spaces with either orientation, a single nullhomologous torus upon which surgery changes the smooth structure. This will be described more or less explicitly for the case of $\mathbf{C}P^2\#3(-\mathbf{C}P^2)$.

Kim Frøyshov: ***Smooth four-manifolds and intersection forms with local coefficients.*** Let X be a closed, oriented, smooth 4–manifold. We consider singular cohomology of X with coefficients in a bundle L of infinite cyclic groups over X. The cup product gives rise to a unimodular quadratic form Q_L on $H^2(X;L)/\text{torsion}$. If L is trivial then this is just the usual intersection form Q of X. In the 1980's Donaldson proved, using instanton moduli spaces, that if Q is definite then it must be diagonal. In this talk I will extend this result to Q_L for non-trivial L, under some constraints. This yields new examples of non-smoothable topological 4–manifolds.

Andriy Haydys: ***Fukaya-Seidel category and gauge theory.*** I will outline a new construction of the Fukaya-Seidel category, which is associated to a symplectic manifold equipped with a compatible almost complex structure J and a J–holomorphic Morse function. Then this construction will be applied in an infinite dimensional case of the complex Chern-Simons functional. The corresponding construction, which is based upon a five-dimensional gauge theory, conjecturally associates a Fukaya-Seidel-type category to a smooth three-manifold.

Matthew Hedden: ***The Khovanov module and unlink detection.*** Kronheimer and Mrowka recently showed that Khovanov homology detects the unknot using connections with an instanton Floer homology invariant for knots. I'll discuss a module structure on Khovanov homology, and a theorem that connects this structure with the Heegaard Floer module of the branched double cover of a link. Using this connection, together with a result indicating that the Heegaard Floer module detects homologically essential spheres in 3–manifolds, we can show that the Khovanov module detects (n–component) unlinks. This is joint work with Yi Ni.

Christopher Herald: ***An $SU(3)$ Casson invariant of rational homology spheres.*** In this talk I will describe joint work in progress with Hans Boden, which develops an $SU(3)$ Casson invariant of rational homology spheres. This invariant extends the integer-valued invariant of integral homology spheres defined in earlier work by the authors together with Paul Kirk. After perturbing the flatness equation to obtain a flat moduli space consisting of finitely many points, we begin with an algebraic count of the irreducible points, and then add suitable correction terms for the reducible points and two types of non-central abelian points. The correction terms are defined in terms of spectral flow of the odd signature operator, and have the property that the algebraic count of irreducible points plus the correction terms is perturbation independent.

Paul Kirk: ***Instantons, concordance, and Whitehead doubling.*** We use moduli spaces of $SO(3)$ instantons and Chern-Simons invariants of flat connections to show that the whitehead doubles of certain infinite families of torus knots are linearly independent in the kernel of the homomorphism from the smooth knot concordance group to the topological concordance group. (Joint with Matthew Hedden)

Maxim Kontsevich: ***Symplectic topology of complex integrable systems.***

Brendan McLellan: ***Non-Abelian Localization and*** $U(1)$ ***Chern-Simons Theory.*** Our goal in this talk is to describe the method of non-abelian localization and how this method yields some new results in $U(1)$ Chern-Simons theory. Starting from a beautiful localization result of Duistermaat and Heckman, we review localization on a finite dimensional Hamiltonian G–space and recall how this result is generalized to path integrals in quantum field theory. In particular, we briefly recall Edward Witten's 1992 paper, "Two dimensional Gauge Theories Revisted", where non-abelian localization is introduced and applied to two dimensional quantum Yang-Mills theory, and Chris Beasley's and Edward Witten's 2005 paper, "Non-Abelian Localization for Chern-Simons Theory". We will then present our results (joint with Lisa Jeffrey), which follow by adapting the method of non-abelian localization to $U(1)$ Chern-Simons theory.

Timothy Nguyen: ***Lagrangians from Seiberg-Witten Theory and Donaldson's TQFT.*** We discuss how boundary values of the space of solutions to the Seiberg-Witten equations, both on compact 3–manifolds and on 3–manifolds with cylindrical ends, yield Lagrangian submanifolds within the corresponding boundary con figuration space. In the case of cylindrical ends, this construction provides a Lagrangian correspondence between the vortex moduli spaces at infinity. As an application, we discuss work in progress for supplying the analytic details of Donaldson's TQFT construction of the Seiberg-Witten invariants of a closed 3–manifold.

Brendan Owens: ***A concordance group of links.*** I will discuss a notion of sliceness for links based on Euler characteristic and show that it leads to an equivalence relation generalising knot concordance. The equivalence classes form a group L containing the knot concordance group as a direct summand with infinitely generated direct complement. I will also exhibit some homomorphisms from L. This is joint work with Andrew Donald.

Burak Ozbagci: ***Milnor fillable contact structures are universally tight.*** We show that the canonical contact structure on the link of a normal complex singularity is universally tight. As a corollary we show the existence of closed, oriented, atoroidal 3-manifolds with infinite fundamental groups which carry universally tight contact structures that are not deformations of taut (or Reebless) foliations. This is a joint work with Y. Lekili.

Jacob Rasmussen: ***Khovanov homology of torus knots.*** Khovanov homology and its generalizations have proven to be very interesting invariants of knots in S^3, but their geometrical meaning remains mysterious. I'll discuss some conjectures relating Khovanov homology of a very special class of knots (torus knots) with algebraic geometry (Hilbert schemes of singular plane curves) and representation theory.

Henrique Sá Earp: ***Perspectives on*** G_2***–instantons.*** Solutions to the Hermitian Yang-Mills problem over A. Kovalev's asymtotically cylindrical Calabi-Yau 3–folds induce instantons over compact 7–manifolds with holonomy group G_2, obtained by a twisted gluing procedure. Moreover, algebraic-geometric monad constructions can be used to generate numerous concrete examples of such G_2–instantons. I will present a survey of that study, punctuated by some open questions ranging from naive to quite ambitious.

Nikolai Saveliev: ***An index theorem for end-periodic operators.*** We extend the Atiyah, Patodi, and Singer index theorem for first order differential operators from the context of manifolds with cylindrical ends to that of manifolds with general periodic ends. This theorem provides a natural complement to Taubes' Fredholm theory for general end-periodic operators. It expresses the index in terms of a new periodic eta-invariant that equals the Atiyah-Patodi-Singer eta-invariant in the cylindrical setting. As an application, we use these index-theoretic techniques to make a count of Seiberg-Witten monopoles on a homology $S^1 \times S^3$ into a topological invariant. This is a joint project with Tomasz Mrowka and Daniel Ruberman.

Andras Stipsicz: ***Surface singularities with rational homology disk smoothings.*** We indicate the classification of weighted homogeneous singularities with rational homology disk smoothings, and show a few examples of such singularities. We examine the similar question in the symplectic category, and (in certain particular) cases show uniqueness for the symplectic fillings (up to symplectic deformation).

Jonathan Weitsman: ***Chern-Simons Gauge Theory, Supersymmetry, and Open Book Decompositions.*** Preliminary report, joint with Chris Beasley.

Program

	Monday	Tuesday	Wednesday	Thursday	Friday
09.30–10.30	Bauer	Rasmussen	Stipsicz	Fintushel	Weitsman
11.00–12.00	Saveliev	Baldridge	Baykur	Boden	Doria
13.30–14.30	Frøyshov	Kirk	Ozbagci	Kontsevich	
14.45–15.45	Herald	Hedden	Owens	Haydys	
16.15–17.15	McLellan	Nguyen	Etesi	Sá Earp	

Appendix: Full list of participants and their affiliations

Master class.

(1) Akyar Moller, Bedia (Dokuz Eylül University)
(2) Andersen, Jørgen Ellegaard (AU)
(3) Anvari, Nima (McMaster University)
(4) Bjerre, Mette (AU)
(5) Blaavand, Jakob (AU)
(6) Boden, Hans (McMaster University)
(7) Bowden, Jonathan (Universität Augsburg)
(8) Bökstedt, Marcel (AU)
(9) Callies, Martin (Georg-August Universität Göttingen)
(10) Chan Kang, Young (Seoul National University)
(11) Choi, Hakho (Seoul National University)
(12) Choi, Ka (UC Berkeley)
(13) De, Amit (AU)
(14) Doria, Celso (UFSC, Brasil)
(15) Dupont, Johan (AU)
(16) Egsgaard, Jens Kristian (AU)
(17) Frøyshov, Kim (AU)
(18) Ghiggini, Paolo (CNRS)
(19) Gothen, Peter (Universidade do Porto)
(20) Halacheva, Iva (University of Toronto)

(21) Haydys, Andriy (Imperial College of London)
(22) Hedden, Matthew (Michigan State University)
(23) Herbig, Hans-Christian (AU)
(24) Himpel, Benjamin (AU)
(25) Horvath, Ramon (Uppsala University)
(26) Ito, Noboru (Waseda University)
(27) Johnson-Freyd, Theo (University of California, Berkeley)
(28) Jørgensen, Søren Fuglede (AU)
(29) Khandhawit, Tirasan (MIT)
(30) Kirillov, Anatol (RIMS, Japan)
(31) Kragh, Thomas (MIT)
(32) Lee, Ju A (Seoul National University)
(33) Lewallen, Sam (Princeton University)
(34) Mares, Ben (McMater University)
(35) Martens, Johan (AU)
(36) Mrowka, Tomasz (MIT)
(37) Nissen, Jens-Jakob Kratmann (AU)
(38) Park, Heesang (Korean Institute for Advanced Study)
(39) Pavlov, Dmitri (UC Berkeley)
(40) Penner, Bob (AU)
(41) Ravelomanana, Huygens C. (Université de Quebec, Montreal)
(42) Prat-Waldron, Arturo (Max Planck Institut, Bonn)
(43) Romao, Nuno (University of Barcelona)
(44) Shin, Dongsoo (Chungnam National University, South Korea)
(45) Sikander, Shehryar (AU)
(46) Sleigh, Callum (Melbourne University)
(47) Stiller, Michael (Hamburg University)
(48) Swann, Andrew (AU)
(49) Tanaka, Hiro (Northwestern University)
(50) Torres, Rafael (University of Oxford)
(51) Vertesi, Vera (MIT)
(52) Zentner, Raphael (Universität Münster)

Workshop.

(1) Andersen, Jørgen Ellegaard (AU)
(2) Andersen, Henning Haahr (AU)
(3) Anvari, Nima (McMaster University)
(4) Baldridge, Scott (Lousiana State University)
(5) Bauer, Stefan (Universität Bielefeld)
(6) Baykur, Inanc (Brandeis University)
(7) Behrens, Stefan (Max Planck Insitute, Bonn)
(8) Berwick, Dan (UC Berkeley)
(9) Bjerre, Mette (AU)
(10) Blaavand, Jakob (AU)
(11) Blau, Mathias (Universität Bern)
(12) Boden, Hans (McMaster University)
(13) Bökstedt, Marcel (AU)
(14) Callies, Martin (Georg-August Universität Göttingen)
(15) Chekeres, Olga (University of Geneva)

(16) Choi, Hakho (Seoul National University)
(17) Chatterjee, Saikat (Tata Institute)
(18) Dancer, Andrew (Oxford University)
(19) Donald, Andrew (University of Glasgow)
(20) Doria, Celso (UFSC, Brasil)
(21) Dupont, Johan (AU)
(22) Earp, Henrique Sá (Unicamp, Brasil)
(23) Egsgaard, Jens Kristian (AU)
(24) Etesi, Gabor (Budapest University of Technology and Economics)
(25) Fernandez, Mario Garcia (AU)
(26) Fintushel, Ron (Michigan State University)
(27) Frøyshov, Kim (AU)
(28) Gothen, Peter (Universidade do Porto)
(29) Hahn, Atle (Universidade de Lisboa)
(30) Halacheva, Iva (University of Toronto)
(31) Harris, Chris (University of Miami)
(32) Haydys, Andriy (Imperial College of London)
(33) Hedden, Matthew (Michigan State University)
(34) Herald, Chris (University of Nevada, Reno)
(35) Herbig, Hans-Christian (AU)
(36) Himpel, Benjamin (AU)
(37) Johnson-Freyd, Theo (University of California, Berkeley)
(38) Jørgensen, Søren Fuglede (AU)
(39) Kang, Young Chan (Seoul National University)
(40) Khandhawit, Tirasan (MIT)
(41) Kirillov, Anatol (RIMS, Japan)
(42) Kirk, Paul (Indiana University)
(43) Kontsevich, Maxim (IHÉS)
(44) Kragh, Thomas (MIT)
(45) LaFountain, Douglas (AU)
(46) Lee, Ju A (Seoul National University)
(47) Lewallen, Sam (Princeton University)
(48) Madsen, Thomas Bruun (University of Southern Denmark)
(49) Mares, Ben (McMaster University)
(50) Martens, Johan (AU)
(51) McLellan, Brendan (AU)
(52) Meaze, A. K. M. Moinul Haque (Bari University)
(53) Moller, Bedia Akyar (Dokuz Eylül University)
(54) Muhamed, Abera Ayalew (University of Kent)
(55) Nguyen, Timothy (MIT)
(56) Nissen, Jens-Jakob Kratmann (AU)
(57) Ott, Andreas (Isaac Newton Institute Cambridge)
(58) Owens, Brendan (University of Glasgow)
(59) Ozbagci, Burak (Koc University, Istanbul)
(60) Park, Heesang (Korean Institute for Advanced Study)
(61) Pavlov, Dmitri (UC Berkeley)
(62) Pedersen, Helge Møller (Universität Heidelberg)
(63) Penner, Bob (AU)

(64) Arturo Prat-Waldron (Max Planck Institute, Bonn)
(65) Rasmussen, Jacob (University of Cambridge)
(66) Ravelomanana, Huygens C. (Université de Quebec, Montreal)
(67) Reshetikhin, Nicolai (AU/UC Berkeley)
(68) Rezazadegan, Reza (AU)
(69) Romao, Nuno (University of Barcelona)
(70) Röser, Markus (Oxford University)
(71) Saveliev, Nikolai (University of Miami)
(72) Shin, Dongsoo (Chungnam National University, South Korea)
(73) Sikander, Shehryar (AU)
(74) Sleigh, Callum (Melbourne University)
(75) Steenstra, Johannes (Utrecht University)
(76) Stiller, Michael (Hamburg University)
(77) Sunukjian, Nathan (Stony Brook University)
(78) Stern, Ronald (University of California, Irvine)
(79) Stipsicz, András (Alfréd Rényi Institute)
(80) Street, Ethan (Harvard University)
(81) Swann, Andrew (AU)
(82) Szilard, Szabo (Budapest univ. Of Technology and Economics)
(83) Tanaki, Hiroaki (Northwestern University)
(84) Thompsen, George (ICTP Trieste)
(85) Tornehave, Jørgen (AU)
(86) Torres, Rafael (University of Oxford)
(87) Veloso, Diogo (Université de Provence)
(88) Villemoes, Rasmus (AU)
(89) Venkov, Alexei (AU)
(90) Weitsman, Jonathan (Northeastern University)
(91) Young, Matthew (Stony Brook)
(92) Yun, Ki-Heon (Sungshin Women's University, South Korea)
(93) Zentner, Raphael (Universität Münster)