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Scientific Report for ESF Short Visit Grant

Visit:
Place: CAMGSD, Instituto Superior Técnico in Lisbon, Portugal
Dates: Jan. 6 - 11, 2012
Host: José Mourão (Professor, Instituto Superior Técnico), jmourao@math.ist.utl.pt

Actual Travel Expenses:
Airfare: €324, 07

Project: Coherent state transforms and geometric quantization

The purpose the proposed short visit to the Center for Mathematical Analysis, Geometry and Dynamical Systems in Lisbon, Portugal, was to continue undergoing collaborative work with J. Mourão and J. Nunes regarding the relationship between the coherent state transform (also called the Hall-Segal-Bargmann transform, which is a unitary isomorphism between the space of square-integrable functions on a compact Lie group and a certain weighted space of holomorphic functions on the complexification of the group) and geometric quantization. In particular, we are studying how the coherent state transform can be explained in terms of geometric quantization with respect to adapted complex structures (using ideas of Thiemann). A significant portion of the work is generalizing the construction to a larger class of adapted-type complex structures, which not only leads to new coherent-state-type transforms, but also to a better understanding of the properties of the coherent state transform itself. Our work has also enabled us to answer a question of Hall, who discovered the coherent state transform, asking for a Stone-von Neumann-type justification for the existence of the Hall-Segal-Bargmann transform. We are not only able to provide such a justification (using the Stone-von Neumann-Mackey theorem), we can even apply our arguments to the generalized construction to explain the existence of generalized Hall-Segal-Bargmann transforms.

During the visit, we were able to successfully complete the project. In particular, we are now able to construct coherent-state-type transforms for a large class of Hamiltonians on the cotangent bundle of an arbitrary compact Lie group. Moreover, we see clearly why the quadratic case is special (this case yields the standard coherent state transform of Hall), and can relate these results to the fact that the heat kernel (which appears in the standard coherent state transform) yields a semigroup, whereas the generalized transforms do not have the semigroup property. We also understand unitarity of the

coherent-state-type transforms for our class of Hamiltonians. The project is essentially finished, and we are now editing a final draft of the accompanying article "Complex time evolution in geometric quantization and generalized coherent state transforms" which we expect to submit in the next couple of weeks.

It is expected that this line of research and my collaboration with J. Mourão and J. Nunes will continue. In particular, we have already begun to think about a semiclassical analysis of the (non)unitarity of the coherent-state-type transforms discussed above. After the semiclassical analysis is finished, we would like to study similar constructions on toric manifolds (here, we think of toric manifolds as smooth compactifications of cotangent bundles of products of circles). The motivation behind this line of research is to understand the relationship between real Lagrangian fibrations (by the cotangent fibers in the case studied here, and the singular fibration by real tori in the toric case) and complex structures (induced by Thiemann's construction) at the level of geometric quantization. For cotangent bundles of compact Lie groups, via the work of Hall, this can be rephrased as the relationship between the space of square-integrable functions on a compact Lie group and the Segal-Bargmann weighted space of holomorphic functions on the complexification of the Lie group.