Topological String Theory, Modularity and non-perturbative Physics

Organizers: Ludmil Katzarkov (Univ. Wien), Albrecht Klemm (Univ. Bonn), Max Kreuzer (TU Wien) and Don Zagier (MPIM Bonn)

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Preprints contributed: [1],[10],[9],[8], [4],[7],[5],[3],[2], [11],[12],[6], [13],[14],[15]

Report on the Program

The activity brought together physicists working on string theory, gauge theory and matrix models with mathematicians with expertise in the fields of homological mirror symmetry, Gromov-Witten theory and automorphic forms.

Since the formulation of topological string and field theories by Schwarz and Witten in the eighties, physical methods of of solving topological field theories and string theories have lead to a multitude of highly nontrivial conjectures. Different then the ones from generic field and string theories these conjectures are mathematical relevant, because the observable as well as the path integral in topological theories either already mathematically well defined or at least such a definition is within reach. The most prominent examples for such conjectures are Mirror Symmetry, the dualities between weakly and strongly coupled field and string theories as well as large N-dualities relating open- and closed string theories. The idea of the activity was on the one hand to make progress in formulating the above mentioned conjectures mathematically precisely, which makes them often more far more general then their physically motivated origins, and eventually turning them into theorems. On the other hand topological theories are models for and in fact subsectors of realistic physical models. Therefore more physical lessons can be drawn from them and further physical applications should be developed.

Particular fruitful is the study of strong weak/coupling dualities and large N-dualities, because they turned out to be key sources of insights into the non-perturbative behavior of physical theories. Marcos Mariño focussed his lectures on this topic. Finally the symmetries discovered in the topological theories are often automorphic groups like $SL(2,\mathbb{Z})$ and correspondingly their correlations functions are automorphic forms. For this reason we were particular lucky to have with Don Zagier a world expert on modular forms among the organizers. The correlation functions of topological theories receive in a controlled way

contributions from the BPS states of the super symmetry theories and can be thought as generation functions for the multiplicities of the latter. The various techniques for the explicite calculation of these correlators were explained in the Lecture by Albrecht Klemm. In this context the stability of BPS states and the wall-crossing formulas for their degeneracies are of utmost importance. These wall-crossing formulas are related to other interesting phenomena in mathematical physics. One incarnation of walls of marginal stability are Stokes lines as was discussed in the context of matrix model by Marcos Mariño. As explained in the talks by Boris Pioline and Sergei Alexandrov they are also relevant to the calculation of the metric on the Hyper-Kähler moduli space of $N = 2$ supersymmetric theories. More recently it has been realized that wall-crossing effects and mock modularity

Mathematically mirror symmetry has lead to surprising new methods to calculate Gromov-Witten – and related symplectic invariants on Calabi-Yau manifolds. For example it relates the generating functions of the closed Gromov-Witten invariants or BPS invariants on the Calabi-Yau manifold M to automorphic forms reflecting discrete symmetries acting on the complex structure moduli of the mirror manifold W, which are captured by the variation of Hodge structures.

Combined with physical constructions, which map the gauge – or string coupling to complex moduli, such as Seiberg-Witten theory, type II/ heterotic string duality and F-theory, the study of automorphic forms has become important to capture non-perturbative physics in gauge — and string theory.

To study non-perturbative effects in physics from first principles, i.e. without using duality symmetries one has to consider the simplest examples, which for both gauge- and string theory are given by matrix models, which admits large N expansions. Since the early work of Penner and Kontsevich on the intersection numbers in the moduli space of curves and Matrix Airy functions, matrix model techniques have played an increasing role in the analysis of topological string theory.

Most of these construction are based on dualities which are particularly simply realized on the topological sector of supersymmetric theories. These topological sectors determine the leading terms in the low energy effective action of supersymmetric theories. Generally dualities and their application are best understood for theories with extended supersymmetry algebras labelled by the number N of super symmetry generators. The physically most interesting and mathematically most challenging situation is $N = 1$ super symmetry in 4d as realized by string compactification of type II string on Calabi-Yau threefolds in the presence of supersymmetric D-branes. Thomas Grimm gave in his lecture an overview over topological couplings in semi realistic effective actions.

Mirror symmetry applies indeed to open string theory with supersymmetric D-branes as boundaries. The latter version lead to the formulation of homological mirror stating the equivalence of the derived category of coherent sheaves on M called $D(M)$ — a complex structure-dependent category — to the Fukaya category $F(W)$ of W, defined by Floer homology of Lagrangian sub manifolds. A somewhat simplified version is as follows: Let $\mathcal{D}(M)$ be a differential graded model for $D(M)$, and similarly $\mathcal{F}(X)$ the A_∞-category underlying $F(X)$. Then there is a full and faithful A_∞-functor

$$\mathcal{F}(W) \longrightarrow \mathcal{D}(M)$$

which becomes an equivalence after a suitable formal completion on the left hand side. In particular, $F(W)$ is identified with a full sub category of $D(M)$.

The first greater activity of the program was the workshop "D-branes, Effective Actions and Homological Mirror Symmetry." It run from the 21st -30st June of 2010 and was devoted to homological mirror symmetry and their physical applications. We aimed for longer, as far as possible, self consistent presentations on key topics of HMS.

- Mohammed Abouzaid, Ludmil Katzarkov, Maxim Kontsevich and Toni Pantev presented new results related to Fukaya categories
- Matrix factorization, effective actions, and D-brane mirror symmetry and superpotentials were summarized by the physicists Richard Garavuso, Manfred Herbst, Johanna Knapp, Emanuel Scheidegger. Nils Carqueville talked about quiver gauge theory and topological strings.
- Serguei Barannikov and Yoannis Vlassopoulos presentations centered around A_∞ algebras topological quantum field theory and matrix models.
- Albrecht Klemm and Jan Soibelmann talked about the relation of Donaldson-Thomas invariants to modular forms and integrable systems.
- Grigory Mikhalkin gave an introduction to tropical geometry and Mark Gross gave applications of the latter to mirror symmetry.
- Mixed Hodge structures were discussed in the talks by Gregory Pearlstein and related to Fukaya categories by Ludmil Katzarkov and Maxim Kontsevich
- Matt Ballard, David Favero and Ludmil Katzarkov lectured on spectra of categories.
- Denis Auroux talked about "Mirror symmetry for blowups and hypersurfaces in toric varieties."

- A. Efimov talked about "Formal completion of a category along a subcategory" and "HMS for $P^1 - \geq 3 points$." Charles Doran talked about "Modular Invariants for Lattice Polarized K3 Surfaces" and "Toric Hypersurface Normal Forms and the Kuga-Satake Hodge Conjecture."

For the conference which took place from July 19 - 28, 2010 we had planned four overview talks lasting up to five hours each on a subject, which is as explained above is central for the understanding of non-perturbative physics.

- Thomas Grimm explained the occurance of "Holomorphic Couplings in effective 4d Supergravity Action from higher Dimensions." The lecture started with an overview of the geometrical properties of supersymmetric string, M- and F-theory compactification manifolds. The general constraints from super symmetry on the effective action was discussed as well as the expected duality symmetries and their constraints on moduli dependent couplings.

- Albrecht Klemm reported on "Integrability in Topological String Theory." This lecture reviewed supersymmetric localization, the A- and B- model twisting, the observables in the so defined topological theories, their relation to BPS states and stability and wall-crossing formulas for the latter. It explained the mirror principle and how the variation of Hodge structure, the relation between holomorphicity and modularity, direct integration and the gap condition leads to a solution of this theory and contrasted that to other approaches.

- Marcos Mariño described recent developments in the understanding of "Non-Perturbative effects in Matrix model, Chern-Simons Theory and Topological Strings." He started with the definitions of classical asymptotics, Stokes phenonema and Borel summability. Non-perturbative effects were then discussed in the an-harmonic oscillator, in Chern Simons theory, matrix models and topological string theory. In particular large N expansion techniques were discussed. Very useful lecture notes are available at "http://www.th.physik.uni-bonn.de/People/rauch/viennamarino.pdf"

- Don Zagier gave a beautiful lecture "Properties of Modular Form and their Asymptotics" on modular forms with special emphasis on the newly discussed phenomenon of mock modularity. After developments pushed by Bringman and Zwegers large classes of the latter can be systematically completed to non holomorphic but modular forms and many properties, e.g. their asymptotics can be studied using similar methods then for ordinary modular forms.

Beside the longer lectures we had talks on recent trends in non-perturbative gauge theory and string theory. Serguei Barannikov, Andrea Brini, Nadav Drukker, Kashani-Poor, Ricardo Schiappa, Piotr Sulkowski and Pavel Putrov talked on the correspondence between matrix model, gauge theory and string theory. Alexander Belavin, Semyon Klevtsov, Alexei Morozov, Alexander Popolitov and Alexei Morozov talked on the recently by Alday, Gaiotto and Tachikawa discovered relation between 4d N=2 sypersymmetric gauge theory and Liouville Theory. Jan Manschot and Don Zagier talked on the relation between wall crossing and mock modularity. Christoph Keller on "Siegel modular forms and CFT partition functions at genus two." Sergei Alexandrov and Boris Pioline on the hypermultiplet moduli space. Bengt Nilsson on the action of the M2 brane. Xenia della Ossa reported on the observations by Candelas and della Ossa relating mirror symmetry calculations in a very intriguing and not completely understood way to number theory. More detailed information and abstracts are of the talks are available at http://hep.itp.tuwien.ac.at/ kreuzer/TSTMP.html.

Contributing Participants

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