

HIGHER STRUCTURES IN MATHEMATICS AND PHYSICS 2010

Summary

The event consisted of two parts: the "Higher Structures" School (October 19-22, 2010) addressed to graduate students and young postdocs working in the field (around 50 participants), and the Conference "Higher Structures in Mathematics and Physics 2010" (October 25-29, 2010) addressed to the experts in the field (around 70 participants).

The school consisted of four mini-courses entitled

- "Conformal nets" by Andre Henriques,
- "Cluster algebras and triangulated categories" by Bernhard Keller,
- "Categorifying quantum groups" by Aaron Lauda, and
- "M. Kontsevich's graph complex and the Grothendieck–Teichmüller Lie algebra" by Thomas Willwacher.

The conference program consisted of 18 talks by the world leading experts in the field of Higher Structures. The scientific committee made a special effort in attracting young talent and giving an opportunity to speak to several brilliant young researchers both at the School and at the Conference. The conference speakers were

Mohammed Abouzaid (MIT), Serguei Barannikov (CNRS), Damien Calaque (Lyon), Alberto Cattaneo (Zürich), Gil Cavalcanti (Utrecht), Cheol-Hyun Cho (Seoul), Marius Crainic (Utrecht), Christopher Douglas (Berkeley), Giovanni Felder (ETH Zürich), Rui Loja Fernandes (IST, Lisbon), John Francis (Northwestern), Kenji Fukaya (Kyoto), Wendy Lowen (Antwerpen), Dmitry Roytenberg (Utrecht) Pavol Severa (Geneva), Boris Tsygan (Northwestern), Maxim Zabzine (Uppsala), and Xinwen Zhu (Harvard).

Both events, the School and the Conference, were very successful, with many participants coming with the outside funding, intense discussions during and outside lecture hours, and hopefully many new collaborations established during the event.

Scientific content

”Higher Structures” School

The School consisted of four mini-courses, each mini-course being four hours of lectures and a number of informal discussions between the lecturer and the participants. Here is the brief description of the mini-courses:

André Henriques (Utrecht) gave a mini-course on Conformal Nets. This is a traditional subject taking its roots in axiomatic approach to Quantum Field Theory. Henriques gave a comprehensive introduction into this topic, and explained his recent work on the relation between Conformal Nets and the higher structures arising in the context of Conformal Field Theory. The important role in this construction is played by the μ -index.

Bernhard Keller (Paris) gave an introduction into the theory of cluster algebras and cluster categories. This is a fast growing field in Algebra with applications in Geometry and Mathematical Physics. In particular, one of the recent applications of this technique is a complete proof of the Zamolodchikov’s Periodicity Conjecture for Y -systems. The mini-course was illustrated by numerous examples produced using the software created by the mini-course lecturer.

Aaron Lauda (Columbia) explained the concept of categorification for Quantum Groups and Knot Invariants. This is a fashionable topic based on the original observations of M. Khovanov on categorification of the Jones polynomial. It is known that categorified invariants of knots are strictly better (distinguish more knots) than their quantum predecessors. The categorification technique allows a presentation in terms of easily manageable graphical calculus presented in the mini-course.

Thomas Willwacher (Harvard) lectured on the relation between two mysterious objects in modern Mathematical Physics: the Kontsevich graph complex and the Grothendieck–Teichmüller (GRT) group. In particular, he explained his recent result which states that the cohomology of the graph complex (in a certain degree) is isomorphic to the Lie algebra of the GRT group.

In summary, School mini-courses gave an introduction into a variety of topics at the front line of modern research. The mini-courses were accompanied by numerous informal discussions between the participants and the

lecturers. In our view, the School was very successful. We believe, it gave a strong motivation and a lot of help for young researchers to enter and develop their tastes in this field.

Conference "Higher Structures in Mathematics and Physics 2010"

The conference covered a number of advanced research topics in Higher Structures presented by the world leading experts in the field. This includes:

Higher Structures in Symplectic Topology covered in the talks by K. Fukaya and M. Abouzaid.

A_∞ -algebras and their properties discussed by S. Barannikov and C.-H. Cho.

Deformation Quantization. Recent progress in the field was presented in the talks by D. Calaque and B. Tsygan.

Generalized Geometry including Courant algebroids and generalized complex structures was a topic of the talks by G. Cavalcanti, R. Fernandes, D. Roytenberg and P. Severa.

Loop groups and integrability was the topic covered by G. Felder and X. Zhu.

Topological Quantum Field Theory and its applications were discussed by A. Cattaneo and M. Zabzine.

Deformation Theory and Category Theory were discussed in the talks by M. Crainic, C. Douglas, J. Francis and W. Lowen

In our view, the conference was successful, with a number of important new results presented, and a great number of questions posed by the audience during and outside the talks. We hope that it was stimulating for new collaborations between the participants of the meeting.

Results and impact on the future of the field

In this Section we mention the main results presented at the School and the Conference, and give our view of the impact on the future of the field.

The main results presented at the School include:

- The new relations between Conformal Nets and Representation theory of loop groups presented by A. Henriques in his mini-course.
- The periodicity theorem for cluster categories proved by B. Keller and presented in his mini-course.
- The isomorphism between the cohomology of the Kontsevich graph complex and the Lie algebra of the group GRT proved and explained by T. Willwacher.

These results will certainly have a long term impact on the development of this field of Mathematics. It is very important for young researchers to be aware of these developments. This gives them the starting point in their research, and a high hope for the rapid future progress in the field.

A variety of very interesting results was presented at the conference. Among the deepest concepts is the use of higher structures in applications to Mirror Symmetry (presented by K. Fukaya). An unexpected link between A_∞ algebras and matrix integrals was explained in the talk by S. Barannikov. A new approach to Lagrangian Field Theory was advertised by A. Cattaneo. A spectacular progress (due to D. Tamarkin) in applying higher structures in Symplectic Topology was presented by B. Tsygan. These are top class results which will have a very high impact on the future of the fields. Other talks were also very interesting, presenting possibly less important but still very beautiful new results.

School program

- André Henriques (Utrecht), "Conformal nets"
- Bernhard Keller (Paris 7), "Cluster algebras and triangulated categories"
- Aaron Lauda (Columbia), "Categorifying quantum groups"
- Thomas Willwacher (Harvard), "M. Kontsevich's graph complex and the Grothendieck-Teichmüller Lie algebra"

Conference program

- Mohammed Abouzaid (MIT), "Categorical smoothness in symplectic topology"
- Serguei Barannikov (CNRS), "Equivariant A_∞ matrix integrals and non-commutative BV formalism"
- Damien Calaque (Lyon), "Deformation quantization via generators and relations"
- Alberto Cattaneo (Zrich), "Classical and perturbative topological field theories with boundary"
- Gil Cavalcanti (Utrecht), "Normal forms and surgeries for generalized complex 4-manifolds"
- Cheol-Hyun Cho (Seoul), "Homotopy cyclic A_∞ -algebras"
- Marius Crainic (Utrecht), "Adjoint representations"
- Christopher Douglas (Berkeley), "Fusion categories are dualizable"
- Giovanni Felder (ETH Zrich), "Elliptic Dunkl operators for complex reflection groups and integrable systems"

- Rui Loja Fernandes (IST, Lisbon), "Equivariant Picard groups of Poisson manifolds"
- John Francis (Northwestern), "The E_n tangent complex, Hochschild cohomology, and higher deformations"
- Kenji Fukaya (Kyoto), "Hochschild and cyclic cohomology in Lagrangian Floer theory and Mirror symmetry"
- Wendy Lowen (Antwerpen), "Deformations of linear sites and projective schemes"
- Dmitry Roytenberg (Utrecht), "Courant algebroids and classical (1+1)-dimensional field theory"
- Pavol Severa (Geneva), "Quasi-Poisson geometry and symplectic 2-groupoids"
- Boris Tsygan (Northwestern), "Open questions in deformation quantization"
- Maxim Zabzine (Uppsala), "3D TFTs, characteristic classes and knots invariants"
- Xinwen Zhu (Harvard), "Double loop groups and reciprocity laws on algebraic surfaces"