

Scientific report on the conference **New perspectives in topological field theories** within the ESF activity ‘Interactions of Low-Dimensional Topology and Geometry with Mathematical Physics (ITGP)’

**Date:** 1 week, Mon.–Fri., 27th to 31st of August 2012.  
**Place:** Hamburg, Germany  
**Participants:** 50  
**Organisers:** Catherine Meusburger (Univ. Erlangen-Nürnberg),  
Ingo Runkel (Univ. Hamburg) – ESF contact person,  
Christoph Schweigert (Univ. Hamburg),  
Jörg Teschner (DESY Hamburg),  
Christoph Wockel (Univ. Hamburg)

## 1 Summary

The focus of this meeting lay on recent new developments in mathematics and theoretical physics which are related to topological quantum field theory. Specifically, we had in depth reports of progress in the areas

1. quantisation and higher structures in topological field theories,
2. non-compact three-dimensional Chern-Simons theories,
3. categorified link invariants.

There were 21 talks altogether, of which 3 were 90 minutes introductory talks in each of the above areas and 14 were research talks by the invited speakers of one hour each. The remaining 4 talks formed an open session of half-hour talks for which we had invited applications of the doctoral and postdoctoral participants registered for the meeting. The meeting provided ample time and opportunity for scientific exchange between the participants: we had long coffee and lunch breaks, and apart from the lecture hall itself there was a discussion room constantly at the disposal of the participants. Finally the free afternoon on Wednesday was used by many participants for discussions, both at the conference site and during the organised excursion.

We were very pleased to win many internationally leading experts as speakers and participants of this conference. The mix of speakers from mathematics and theoretical physics has worked well and the talks were accessible to members of both communities.

## 2 Scientific content

Three-dimensional topological field theory is an underlying theme for many recent advances in mathematics and theoretical physics. For this conference we selected the three topics listed above since they exhibited a particularly large potential for scientific interaction. In each of these topics, different questions are in the foreground and different tools from

mathematics and physics are used. This conference has enabled experts to understand the main questions and methods in the other areas, and to assess the current state of the art in these fields. Most importantly, of course, the conference allowed experts from different topics to engage in direct discussions.

The format of the conference was chosen precisely with the aim to make all three different research directions accessible to all participants. A key component were therefore three introductory lectures, for which we could find three high profile speakers:

1. Peter Teichner gave a “Survey of functorial field theories”, in which he explained the description of extended field theories via higher categories,
2. Sergei Gukov gave an “Introduction to the volume conjecture”, in which he outlined the origin of the conjecture and its field theory interpretation,
3. Dror Bar-Natan gave “A quick introduction to Khovanov homology”, where he described the transition from the Jones polynomial to the finer link invariant given by Khovanov homology.

To describe the main themes discussed in the research talks, in the following we roughly group the individual talks under headings 1.–3. as mentioned above.

## 1. Quantisation and higher structures in topological field theories

Several talks addressed aspects of the construction of field theories:

- Topological terms are additions to the action of a classical sigma model which do not depend on the metric on the domain of the sigma model. Peter Teichner explained how such topological terms can be understood as invertible field theories.
- The passage from the classical to the quantum description is notoriously difficult in topological field theory. Jørgen Andersen discussed how Reshetikhin-Turaev TFTs can be obtained via the quantisation of a moduli space of flat connections.
- Alberto Cattaneo described classical and quantum field theories on manifolds with boundaries. This gave a uniform view on classical and quantum theory via extended field theories and allows a new approach to quantisation.

On a more general note, Theo Johnson-Freyd discussed a technique of evaluating Feynman integrals based on localisation to a critical locus which may be useful in computing amplitudes of TFTs.

Apart from extended field theories, where data of an  $n$ -category is associated to submanifolds of various codimensions, higher categories also appear when one includes so-called defects into the model. This aspect was addressed in the following contributions:

- John Barrett gave a detailed account of a diagrammatic language for dualities in Gray categories and explained its relation to 3d TFTs with defects. The concrete description of such a defect tricategory was addressed in the contribution by Gregor Schaumann.

- In one dimension less, for 2d TFTs, Nils Carqueville explained how Landau-Ginsburg models with defects can be described by matrix factorisations, and how the latter form a bicategory with adjoints. This allows for a new and shorter proof of the so-called Cardy-condition in open/closed two-dimensional TFTs. An A-infinity version of these open/closed TFTs associated to Landau-Ginsburg models was discussed by Ed Segal.

In a slightly different direction, Hendryk Pfeiffer presented results on canonical bases for  $\mathfrak{sl}(2)$ -type fusion categories. Such categories arise for example as certain subquotients of the representation category of quantum  $\mathfrak{sl}(2)$  and can be used as input in the Reshetikhin-Turaev construction of three-manifold invariants.

## 2. Non-compact three-dimensional Chern-Simons theories

Topological field theories obtained from Chern-Simons theories with non-compact gauge group behave in many ways very differently from those with compact gauge group and from the theories discussed in 1. They allow for continuous parameters which result in interesting analytic behaviour, they may have infinite dimensional state spaces, and their partition functions may be singular for certain three-manifolds.

The topics addressed in the individual talks were as follows.

- Hitoshi Murakami explained the relation between hyperbolic volumes of knot complements and the coloured Jones polynomial.
- Rinat Kashaev discussed a generalised TQFT based on the quantum Teichmüller theory. Under certain conditions such generalised TQFTs can be shown to have well-defined partition functions. Tudor Dimofte discussed such partition functions for three-manifolds with boundary and gave a relation to 4d super-conformal field theories and to defects therein.
- Masahito Yamazaki explained how quantum dilogarithms and elliptic gamma functions enter the study of hyperbolic volumes and of invariants obtained from non-compact Chern-Simons theories. He also explained the relation to certain 3d and 4d gauge theories.

In a related direction, Miguel Tierz showed how to use matrix models techniques to compute partition functions of Chern-Simons theories on certain types of three-manifolds.

## 3. Categorized link invariants

The knot invariants obtained from the Jones-polynomial and its variants have found a highly nontrivial generalisation in Khovanov homology. This generalisation can be understood as a categorification of the Jones polynomial, suggesting a four-dimensional interpretation. New developments in knot homology and four-dimensional ‘knot’ invariants were presented in the following talks:

- Sergei Gukov related topics 2 and 3 by considering a categorification of the quantum volume conjecture. One effect of this is to replace the doubly-graded so-called A-polynomial of the quantum volume conjecture by a triply-graded version, the super-A-polynomial. These polynomials were further discussed in the contribution by Satishi Natawa, who also gave examples for special types of knots.
- Scott Morisson used Khovanov homology to define vector-space valued invariants of four-manifolds and discussed the ‘blob-complex’, which realises these invariants as the zeroth cohomology of a complex.
- Dror Bar-Natan described a generalisation of three-dimensional link-invariants to four dimensions where he presented an invariant of knotted 2- and 1-spheres. These invariants were related to the Alexander polynomial and to 4d BF topological field theory.

### 3 Results and Impact

The conference has achieved its main aim of bringing together experts of different directions related to three-dimensional topological field theories, both from mathematics and physics, in order to start new interactions and enable the propagation of new questions, methods and ideas between communities. Such discussions could clearly be observed during the lunch and coffee breaks, and during the question sessions after the talks. Some examples of such points of discussion were that non-compact Chern-Simons theory does not fit into Lurie’s classification since it can have infinite-dimensional state spaces and is hence not fully dualisable, the mathematical implications of the dual gauge theory descriptions suggested by string theory that featured in several talks, the relation between the various appearances of defects and domain walls, and many more.

The conference also fostered interaction between European research institutions, as is testified by the participation of experts from Denmark, Great Britain, Poland, Spain, Switzerland, and Germany. In addition, the conference certainly raised the awareness about this field within Germany and helped to establish Hamburg as one of the centres where the interaction between mathematics and physics is stressed, and where experts from both directions can gather for fruitful exchanges. This is confirmed by the fact that a significant number of participants came from abroad and even from overseas at their own expense.

We are convinced that there is still much potential in the interaction between mathematics and theoretical physics in the area of topological field theory, but also more broadly in quantum field theory and string theory. The success of this event is a strong motivation for us to organise further activities in this direction.

## Annex: Conference program

### Monday, August 27

10:00 - 11:00	Andersen, Jørgen	The geometric construction of the Reshetikhin-Turaev Topological Quantum Field Theory
11:00 - 11:30	<b>Coffee break</b>	
11:30 - 12:30	Carqueville, Nils	Defects and adjunctions in Landau-Ginzburg models
12:30 - 14:00	<b>Lunch break</b>	
14:00 - 15:30	Teichner, Peter	Survey of functorial field theories (overview talk)
15:30 - 16:00	<b>Coffee break</b>	
16:00 - 17:00	Teichner, Peter	Invertible field theories and differential cohomology

### Tuesday, August 28

09:30 - 11:00	Bar-Natan, Dror	A quick introduction to Khovanov homology (overview talk)
11:00 - 11:30	<b>Coffee break</b>	
11:30 - 12:30	Pfeiffer, Hendryk	Canonical Bases for objects of $SL(2)$ Fusion Categories
12:30 - 14:00	<b>Lunch break</b>	
14:00 - 15:00	Morrison, Scott	Khovanov homology for 4-manifolds
15:00 - 15:30	<b>Coffee break</b>	
15:30 - 16:00	Nawata, Satoshi	Super-A-polynomials of twist knots
16:00 - 16:30	Tierz, Miguel	Matrix models in Chern-Simons theory
16:30 - 17:00	Johnson-Freyd, Theo	Nonperturbative integrals, imaginary critical points, and homological perturbation theory
17:00 - 17:30	Schaumann, Gregor	Towards a tricategory of defects for the Turaev-Viro model

### Wednesday, August 29

09:30 - 11:00	Gukov, Sergei	Introduction to the volume conjecture (overview talk)
11:00 - 11:30	<b>Coffee break</b>	
11:30 - 12:30	Kashaev, Rinat	On some calculations within Teichmüller TQFT
<b>Free afternoon with excursion</b>		

### Thursday, August 30

09:30 - 10:30	Bar-Natan, Dror	Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant (overview talk)
10:30 - 11:00	<b>Coffee break</b>	
11:00 - 12:00	Barrett, John	Gray categories and their diagrams
12:00 - 13:00	Gukov, Sergei	Knot Homologies from Quantization of Moduli Space
13:00 - 14:30	<b>Lunch break</b>	
14:30 - 15:30	Yamazaki, Masahito	Quantum Dilogarithms and Elliptic Gammas in Gauge Theories
15:30 - 16:00	<b>Coffee break</b>	
16:00 - 17:00	Segal, Ed	Chern characters for matrix factorizations via non-commutative translation

### Friday, August 31

09:30 - 10:30	Murakami, Hitoshi	The colored Jones polynomial, the Chern-Simons invariant, and the Reidemeister torsion of a knot
10:30 - 11:00	<b>Coffee break</b>	
11:00 - 12:00	Dimofte, Tudor	Spin Networks, Teichmüller Theory, and RG Domain Walls
12:00 - 13:00	Cattaneo, Alberto	Classical and quantum Lagrangian field theories on manifolds with boundaries

## Annex : List of speakers

Name, Surname	University	Country
Andersen, Jørgen	University of Aarhus	Denmark
Bar-Natan, Dror	University of Toronto	Canada
Barrett, John	University of Nottingham	UK
Carqueville, Nils	University of Munich	Germany
Cattaneo, Alberto	University of Zurich	Switzerland
Dimofte, Tudor Dan	IAS, Princeton	USA
Gukov, Sergei	CALTECH, California	USA
Johnson-Freyd, Theo	UC Berkeley	USA
Kashaev, Rinat	University of Geneva	Switzerland
Morrison, Scott	UC Berkeley	USA
Murakami, Hitoshi	Tokyo Institute of Technology	Japan
Nawata, Satoshi	Perimeter Institute Waterloo	Canada
Pfeiffer, Hendryk	The University of British Columbia	Canada
Schaumann, Gregor	University of Erlangen	Germany
Segal, Ed	Imperial College London	UK
Teichner, Peter	MPI Bonn	Germany
Tierz, Miguel	The Complutense University of Madrid	Spain
Yamazaki, Masahito	Princeton University	USA

## Annex: List of participants (without speakers)

Name, Surname	University	Country
Bärenz, Manuel	University of Nottingham	UK
Barvels, Alexander	University of Hamburg	Germany
Bücher, David	University of Hamburg	Germany
Chatziyiannis, Georgios	University of Hamburg	Germany
Gray, Oliver	University of Bristol	UK
Holtkamp, Ralf	University of Hamburg	Germany
Maier, Jennifer	University of Hamburg	Germany
Meneghelli, Carlo	University of Hamburg	Germany
Meusburger, Catherine	University of Erlangen	Germany
Mombelli, Juan Martín	University of Hamburg	Germany
Morton, Jeffrey	University of Hamburg	Germany
Novak, Sebastian	University of Hamburg	Germany
Oeckl, Robert	University of Mexico	Mexico
Parzygnat, Arthur	CUNY Graduate Center	USA
Pavlov, Dmitri	University of Münster	Germany
Pawelkiewicz, Michal	DESY Hamburg	Germany
Pennig, Ulrich	University of Münster	Germany
Priel, Jan	University of Munich	Germany
Ros Camacho, Ana	University of Hamburg	Germany
Ruiz, Hans-Christian	University of Munich	Germany
Runkel, Ingo	University of Hamburg	Germany
Schäfer, Gisa	University of Bonn	Germany
Schiavina, Michele	University of Zurich	Switzerland



## Annex: List of participants (without speakers)

Name, Surname	University	Country
Schweigert, Christoph	University of Hamburg	Germany
Selmani, Sam	McGill University Montreal	Canada
Szczachor, Michal	University of Wraclaw	Poland
Teschner, Jörg	DESY Hamburg	Germany
Vartanov, Grigory	DESY Hamburg	Germany
Waldorf, Konrad	University of Hamburg	Germany
Wockel, Christoph	University of Hamburg	Germany
Yagi, Junya	University of Hamburg	Germany
Yinghua, Ai	Tshinghua University Beijing	P.R.China