

# **Advances in Teichmüller theory**

**February 4-8, 2013**

**Erwin Schrödinger Institute, Vienna**

**Organized by: Louis Funar, Athanase Papadopoulos  
and Bob Penner**

## **1 Summary**

The conference took place at the Erwin Schrödinger International Institute for Mathematical Physics in Vienna, from February 4 to 8, 2013. It was part of a 3-month activity at the institute, whose subject is Teichmüller theory and its interactions with physics. The subject of the conference was “Advances in Teichmüller theory”. There were 20 talks that covered the recent advances in the theory. The talks were given by the main specialists in the field. The conference was attended by more than 60 persons, and among them about 20 people were funded in part from the ESF program ITGP. The participants came from several countries from Europe (Austria, France, Italy, Greece, Finland, Germany, Denmark, UK, Switzerland, Romania, Turkey, etc.), from the US, Canada, Japan, Korea, China, Singapore, India and other countries.

## **2 Description of the scientific content of and discussion at the event**

The conference covered the major recent advances in the theory, including the analytic and the metric aspects, quasiconformal theory, the mapping class group actions, the geometry of curves and curve complexes and the relations with moduli spaces, the algebro-geometric aspects, surface group representations, and higher Teichmüller theory. The following is a list of the major subjects presented and discussed at the conference:

1. Ideal triangulations of the Milnor fibers of plane curve singularities and the variation of the corresponding complex structure.
2. The study of the shape of the subset of moduli space consisting of surfaces whose systoles fill by using combinatorial methods and an appropriate graph.

3. Rigidity and local rigidity of curve complexes equipped with mapping class group actions.
4. The geodesic flow on Teichmüller space: the flow near the boundary strata and applications to the calculation of the exponent of the number of periodic orbits near the boundary.
5. The stratification of the moduli space of curves endowed with a nonzero Abelian differential and the problem of whether such strata are  $K(G, 1)$ .
6. A tensorial description of the Goldman Lie algebra of a surface and the embedding of the largest Torelli group (in the sense of Putman) into a completion of the Goldman-Turaev Lie bialgebra. The relation with the Morita traces
7. The canonical real-valued function on moduli space determined by the spectral theory of the Bergman metric, introduced by Kawazumi and Zhang and the relation with the Arakelov-Green function, the pointed harmonic volume, and the height of Ceresa's cycle.
8. The generalization of Teichmüller's classical mapping problem for plane domains to three-space.
9. The geometry of representations of the fundamental group of a surface into  $SU(2, 1)$  and a dictionary between the theories of minimal surfaces in  $RH^3$  and minimal Lagrangian surfaces in  $CH^2$ .
10. The study of shear coordinates of hyperbolic surfaces including an analytic formulation without divergences for shear deformations and weighted ideal length sums, The symplectic geometry of shears and weighted ideal length sums, the definition of an elementary 2-form for shear weights, the relation to the Symplectic structure on decorated measured foliation space, a distances-sum formula for the Riemannian pairing of shears and an exact relation for distances between lines in the classical modular tessellation.
11. The relation between the modular group, indefinite binary quadratic form, dessins d'enfants and the universal Teichmüller space.
12. Deformations and rigidity properties of circle diffeomorphisms in relation with the universal Teichmüller space.
13. The relation between the space of convex polygons in the real projective plane (up to projective transformations) and the space of polynomial

holomorphic cubic differentials on the complex plane (up to complex affine transformations). Connections with the study of meromorphic Higgs bundles over the projective line and with the Stokes phenomenon for meromorphic connections.

14. The new pressure metric on the Hitchin component in the space of (conjugacy classes of) representations of a closed surface group into  $PSL(n, R)$  and on more general deformation spaces. application to the fact that the Hausdorffdimension of the limit set varies analytically over deformation spaces of convex cocompact Kleinian groups.
15. For a finite volume hyperbolic manifold with non-empty totally geodesic boundary, the study of the distribution of the time for the geodesic flow to hit the boundary and of the moments of the associated random variable in terms of the orthospectrum.
16. Compactifications of real projective structures via marked length spectrums and Patterson-Sullivan geodesic currents.
17. Operads and their actions on moduli spaces and the cellulations of these spaces.
18. Dilogarithm identities on moduli spaces and relations with identities of Basmaïan, McShane and Bridgeman.
19. Relations between affine 3-manifolds, Lorentzian and hyperbolic geometry, and the classification problem for quotients of  $P^3$  by groups of affine transformations.

Each lecture was followed by a discussion, and each afternoon several discussions were held at the ESI about the days' lectures and about the whole subject.

### **3 Assessment of the results and impact of the event on the future direction of the field**

This encounter between mathematicians working in different aspects on Teichmüller theory (geometric, algebro-geometric and analytic) was very important in terms of exchanging ideas and establishing new contacts between these mathematicians. The speakers at the conference were first-rate mathematicians, and they presented in a comprehensive way important new contributions to the subject. Many young mathematicians were present, and they all learned a lot by participating to that conference.

The atmosphere during the conference was very good in terms of willingness to learn and to exchange new ideas. Several new collaborations were born during this conference. The conference took place at the beginning of the three-month special period on Teichmüller theory at ESI Vienna and several people, at the end of the conference, decided to come back to ESI a few weeks later during the program, in order to continue collaborations which started during the conference. In the attendance there were also several PhD students, from Europe, Asia and America. Although some of the advanced lectures were difficult for them to follow, the students have certainly gained some experience and new ideas and they learned what are the most active problems in the area. It was also the occasion for them to encounter several mathematicians from all over the world and to discuss with them about their work. The contacts they made will be very helpful for them when they will be looking for a job.

## 4 Final program

### Advances in Teichmüller theory

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All lectures take place in the ESI Boltzmann Lecture Hall

- **Monday, February 4, 2013**

**08:45 – 9:30** Registration, coffee and opening

**9:30 – 10:25** Norbert A'Campo

*Ideal triangulations and variation of the complex structure by monodromy*

**10:25 – 11:20** Hugo Parlier

*Relative shapes of thick subsets of moduli space*

**10:20 – 10:35** Coffee break

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<sup>1</sup>The conference is partially funded by the ESI (University of Vienna), ITGP (ESF), the GEAR network (NSF) and JSPS (Japan).

**11:35 – 12:30** Javier Aramayona  
*Finite rigid sets in curve complexes*

**12:30 – 14:00** lunch break

**14:00 – 15:00** Ursula Hamenstädt  
*The Teichmüller flow near the boundary of strata*

• **Tuesday, February 5, 2013**

**09:10 – 10:10** Gabriele Mondello  
*Abelian differentials on curves of genus 3*

**10:10 – 10:30** Coffee break

**10:30 – 11:30** Nariya Kawazumi  
*The Goldman-Turaev Lie bialgebra and the largest Torelli group*

**11:30 – 12:30** Robin de Jong  
*On the Kawazumi-Zhang invariant of a compact Riemann surface*

**12:30 – 14:00** lunch break

**14:00 – 15:00** Matti Vuorinen  
*Teichmüller's problem in space*

• **Wednesday, February 6, 2013**

**09:10 – 10:10** Scott Wolpert  
*Products of twists, geodesic-lengths and Thurston shears*

**10:10 – 10:30** Coffee break

**10:30 – 11:30** John Loftin  
*Minimal Lagrangian surfaces in  $CH^2$*

**11:30 – 12:30** Muhammed Uludag  
*Binary quadratic forms as dessins*

**12:30 – 14:00** lunch break

**14:00 – 15:00** Katsuhiko Matsuzaki  
*Rigidity of groups of circle diffeomorphisms and Teichmüller spaces*

• **Thursday, February 7, 2013**

**09:10 – 10:10** David Dumas  
*Convex polygons, complex polynomials, and hyperbolic affine spheres*

**10:10 – 10:30** Coffee break

**10:30 – 11:30** Dick Canary

*A pressure metric for the Hitchin component*

**11:30 – 12:30** Martin Bridgeman

*Moments of the boundary hitting function for geodesic flow*

**12:30 – 14:00** lunch break

**14:00 – 15:00** Inkang Kim

*Compactification of real projective and  $AdS^3$  geometry*

• **Friday, February 8, 2013**

**09:10 – 10:10** Carl Friedrich Bödigheimer

*Homology Operations for Moduli Spaces*

**10:10 – 10:30** Coffee break

**10:30 – 11:30** Kasra Rafi

*Large scale rank in Geometric Topology*

**11:30 – 12:30** Tan Ser Peow

*A dilogarithm identity on moduli spaces of curves*

**12:30 – 14:00** lunch break

**14:00 – 15:00** William Goldman

*Complete affine 3-manifolds, Lorentzian and hyperbolic geometry*

## 5 List of registered participants

Norbert A'Campo, Jorge Acosta, Toshiyuki Akita, Vincent Alberge, Daniele Alessandrini, Javier Aramayona, Andrzej Bis', Carl-Friedrich Bödigheimer, Martin Bridgeman, Richard Canary, Charalampos Charitos, Robin de Jong, David Dumas, Federica Fanoni, Ege Fujikawa, Louis Funar, William Goldman, Subhojoy Gupta, Ursula Hamenstaedt, Chris Judge, Anders Karlsson, Yasushi Kasahara, Nariya Kawazumi, Inkang Kim, Erina Kinjo, Hiroki Kodama, Takayuki Koike, Mustafa Korkmaz, Yusuke Kuno, Ana Lenzhen, Qionglin Li, Lixin Liu, John Loftin, Bruno Martelli, Gregor Masbaum, Katsuhiko Matsuzaki, Masato Mimura, Gabriele Mondello, Yurii Neretin, Frederic Palesi, Athanase Papadopoulos, Ioannis Papadoperakis, Hugo Parlier, Bram Petri, Kasra Rafi, Masatoshi Sato, Hiroshige Shiga, Weixu Su, Zhe Sun, Yuuki Tadokoro, Kazuto Takao, Jing Tao, Ser Peow Tan, Georgios Tsapogas, Muhammed Uludag, Matti Vuorinen, Scott A. Wolpert, Ayberk Zeytin