FINAL REPORT

To tackle the problem of Abelianization, we have decided to investigate the case for a compact Riemann Surface of genus 2. By the main theorem in [3] we get that $\mathcal{M}^B$ the moduli space of semi-stable bundles with trivial determinant on X is isomorphic to $\mathbb{CP}^3$ when the genus of X is 2. This then implies that the determinant line bundle $\mathcal{L}^k \to \mathcal{M}^B$ is isomorphic to $\mathcal{O}_{\mathbb{CP}^3}(k)$. Thus non-abelian theta functions of level k are nothing but degree k homogenous polynomials on $\mathbb{CP}^3$.

To analyze the $sl(2)$Hitchin System for genus 2 curves, we notice that once we fix a representation of X as a hyperelliptic curve

$$\pi\colon X \to \mathbb{CP}^1$$

with 6 branch points $b.p \subset \mathbb{CP}^1$, then any quadratic differential $q \in H^0(X, K^2)$ with simple zeros is uniquely determined by a choice of two points $x_1, x_2 \in \mathbb{CP}^3 - b.p.$ In this case Hitchin's spectral curve [2] over q, S_q, is again a hyperelliptic of genus 5, but more importantly the Prym corresponding to

$$\pi\colon S_q \to X$$

call it P_q, which is the fiber of the Hitchin Map becomes more tractable. Namely, consider the genus 3 hyperelliptic curve

$$\pi\colon C_3 \to \mathbb{CP}^1$$

with 8 branch points namely $b.p \cup \{x_1, x_2\}$. Then $P_q \cong J_{C_3}/\mathbb{Z}_2$ as polarized abelian varieties, where J_{c_3} is the jacobian of C_3. Thus the smooth locus of the base of the Hitchin System, which we call QD^0, can be identified with the configuration space of two points on $\mathbb{CP}^1 - b.p.$ It then follows that $\pi_1(QD^0, q) \cong B_2(D_5)$, where $B_2(D_5)$ is the braid group of two points on a 5 punctured disk. We have calculated the hamiltonian monodromy in this situation, that is we have calculated a group homomorphism

$$M\colon \pi_1(QD^0, q) \to H_1(P_q, \mathbb{Z})$$

In the future we want to extend the above topological monodromy to monodromy in theta functions. That is, we wish to obtain the following homomorphism

$$M\colon \pi_1(QD^0, q) \to H^0(P_q, L^k)$$

where $L^k \to P_q$ is some ample line bundle on the Prym.

For the situation of genus 2 curve, a dominant map in [8] is presented

$$\Phi\colon P_q \to \mathcal{M}^B$$

We wish to study the pull back line bundle $\Phi^*(\mathcal{O}_{\mathbb{CP}^3}(k))$ on the Prym variety under this map. In particular we wish to construct the following vector bundle

$$H^0(P_q, \Phi^*(\mathcal{O}_{\mathbb{CP}^3}(k))) \to QD^0$$

Notice that once we have this vector bundle we will automatically get the monodromy we are after

$$\rho\colon \pi_1(QD^0, q) \to H^0(P_q, \Phi^*(\mathcal{O}_{\mathbb{CP}^3}(k)))$$

by the above.

It will be interesting to analyze the above monodromy representation, and especially study its irreducible subspaces. We hope this study will shed some light on Atiyah's conjecture which states that the invariant subspace of $H^0(P_q, \Phi^*(\mathcal{O}_{\mathbb{CP}^3}(k)))$ under the monodromy should be identified with the non-abelian theta functions.

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