

**Scientific report on the International School in Geometry and  
Physics  
Geometry and Quantization of Moduli Spaces  
an ITGP midterm activity**

Centre de Recerca Matemàtica, Bellaterra (Barcelona), 26–30 March, 2012

Integrated in the Research Programme  
Geometry and Quantization of Moduli Spaces  
CRM, Barcelona, March–June 2012

## **1. Summary**

This Advanced School was the first of the four main activities of the Research Program "Geometry and Quantization of Moduli Spaces", which is taking place in the CRM (Centre de Recerca Matemàtica, Bellaterra, Barcelona) from March to June 2012. Its aim has been to offer an introduction to four different subjects (which were finally reduced to three, see the explanation below) related to moduli spaces, with a target audience of PhD students and more generally researchers interested on the field of moduli spaces.

The choice of subjects has been guided both by their intrinsic mathematical relevance and by the topics to which the subsequent activities of the Research Program "Geometry and Quantization of Moduli Spaces" will be devoted. A common feature of a big portion of the contents of the courses is the role played in their origin by ideas and intuitions coming from physics. The notion of moduli space has proved in the last decades to be crucial in the understanding of high energy physics, and as such it has led to an interplay between physics and geometry which has been extraordinarily fruitful in both fields.

This advanced course was intended to prepare people for the workshops and final conference of the Research Program. The main focus was, as in the other activities of the Research Program, on the moduli space of representations of the fundamental group of a surface in Lie groups, but more general notions have also appeared (notably in the course on TQFT's). The speakers of the courses were chosen among the leading experts of the area.

The audience consisted of roughly 40 people.

At the beginning of the Advanced School informal notes prepared by the speakers were delivered to the participants.

Web page of the activity: <http://www.crm.cat/acmodulispace/>

## 2. Description of the scientific content and discussion at the event

The Advanced School consisted of the following three courses:

- Vladimir Fock (Université Louis-Pasteur, Strasbourg): *Integrable systems, dimers and local systems*. This was a course of five 1-hour sessions. The subject of the course was some constructions of Poisson structures and integrable systems on moduli spaces of local systems using dimers. These were the contents of the course.
  1. Integrable systems and r-matrices. Symplectic leaves on  $G$ ,  $G = AdG$  and the Weyl group; discrete transformation commuting with integrable flows; cluster parametrisation of  $G^u$  and  $G^u = AdH$ ; Poisson structure and coordinate change; isomorphisms between double Bruhat cells.
  2. Generalization to loop groups. Realisations of the coextended loop group; definition and realisation of the coextended affine Weyl group.
  3. Dimers. Recollection about dimers; face partition function; dual surface and Poisson brackets; equivalence of bipartite graphs; GK integrable systems.
  4. Dimer partition functions and characteristic polynomials
  5. Mutations and discrete flows.
- Constantin Teleman (University of Berkeley): *Topological Quantum Field Theories*. This was a course of six 1-hour sessions. The subject was extended TQFT's and the cobordism hypothesis. These were the contents of the course.
  1. *Old style* TQFT's: axioms of Atiyah, Segal and Witten; 1- and 2-dimensional TQFT's; 2-dimensional TQFT's associated to group rings of finite groups.
  2. Witten's construction of a 2-dimensional TQFT's using a compact Lie group (generalizing the case of finite groups).
  3. Strict and weak higher categories and their role in extended TQFT's.
  4. 2-dimensional extended TQFT's: the case of finite groups.
  5. Homotopy types and extended TQFT's.
  6. Braided tensor categories.
  7. Lurie's cobordism hypothesis.
- Richard Wentworth (University of Maryland): *Higgs bundles and local systems*. This was a course of six 1-hour sessions. The subject was local systems, Higgs bundles and opers. These were the contents of the course.
  1. The Dolbeault Moduli Space; Higgs bundles; the moduli space; the Hitchin-Kobayashi correspondence.
  2. The Betti Moduli; representation varieties; local systems and holomorphic connections; the Corlette-Donaldson theorem; Hyperkähler reduction.
  3. Differential Equations; uniformization; higher order equations; opers; the Eichler-Shimura isomorphism.

The original program included a fourth course by Prof. Marc Burger (ETHZ, Zürich) on *Bounded cohomology and local systems*. Unfortunately, due to health problems Prof. Burger had to cancel his trip to Barcelona one day before the beginning of the school. Due to the lack of time, it was not possible to look for an alternative professor, so the course was removed from the schedule. The other three lecturers very kindly agreed to expand their courses by one hour each, and this allowed us to fill in most of the time slots intended for Prof. Burger's course.

### 3. Assessment of the results and impact of the event on the future direction of the field

Given the nature of this event (an advanced school, and not a workshop), we expect its impact on future directions of the field to be evident in the mid-term, mainly through the future contributions of its participants. The combination of the three courses gave a coherent and varied description of the present state of the art in the field of moduli spaces of local systems on surfaces. The participants of the course should consequently be well equipped now to make significant contributions to the field in the following years.

Besides this direct impact, we should mention that we are at present investigating the possibility of publishing a book with written notes of the courses. The book would be published by Birkhäuser in its collection devoted to the CRM Advanced Courses. We very much hope that such a book will provide a valuable tool to mathematicians working in the field.

### 4. Final program of the meeting

#### Schedule

	Monday 26 March	Tuesday 27 March	Wednesday 28 March	Thursday 29 March	Friday 30 March
9:30-10:30	Teleman	Teleman	Teleman	Cancelled sessions*	Teleman
10:30-11:00	coffee/tea	coffee/tea	coffee/tea		coffee/tea
11:00-12:00	Wentworth	Wentworth	Fock		Wentworth
12:15-13:15	Teleman	Fock	Excursion		Teleman
13:15-14:45	lunch	lunch			lunch
14:45-15:45	Wentworth	Wentworth			Wentworth
16:00-17:00	Fock	Fock			Fock

\* On Thursday all sessions were cancelled, due to a general strike announced by trade unions. Based on recent experience, it was expected (as it was indeed the case) that the access to campus and the train service were to be blocked by people involved in the strike. The school was rescheduled accordingly, increasing the number of talks per day to leave Thursday free of activity while preserving the contents of the courses.