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Scientific Report on the research activity:

# "Research of Close Approaches between Asteroids for Determining Masses of Asteroids" 

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within a short visit grant from December 5 till December 17, 2011 (13 days)

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## PURPOSE OF THE VISIT

The space astrometric mission Gaia will result in accurate astrometry of asteroids besides idem for stars of our Galaxy. The positions of asteroids should allow to research the small effects, perturbations in the motion of Solar system objects. The best conditions for determining small gravitational perturbations caused by the large asteroids on the smaller ones consist in measuring deflection angles during their close approaches. The success of such measurements depends upon the value of the angles, especially if the approach occurs in the beginning or end of the Gaia mission. So, the work around can be in using astrometry ground-based observations of the perturbed asteroids made before the beginning of Gaia mission. The important organizational idea is to make predictions of the motion of known asteroids and to find out close approaches with the larger bodies. The earlier available calculations cover substantially smaller ensemble of asteroids, so the present calculations will make benefit in using the available data before Gaia launch.

The aim of the suggested project consists in making systematical calculations for the list of known numbered 283317 asteroids for the closest 5 years in cooperation with the colleagues from Institute of Celestial Mechanics and Calculation of Ephemerides - Paris Observatory. These calculations will also supply observers within the Gaia Follow Up Network with the necessary astrometry requirements for the deflection angle to be successfully measured.

## DESCRIPTION OF THE WORK CARRIED OUT DURING THE VISIT

Today we know on 574121 asteroids among which 310376 are "numbered" asteroids, the bodies with confidently certain orbits for which the unambiguous prediction can be made for motion for a limited period of time in both the past, and the future, i.e. their unambiguous identification is possible in the historical observations. Other asteroids represent the bodies for which yet there are no possibilities to make such a prediction of motion since these asteroids have been observed in one-two occurrences.

For getting the prediction of motion of an asteroid which satisfies contemporary requirements to the accuracy of its position, e.g. INPOP by IMCCE, DE4XX by Jet Propulsion Laboratory (JPL), it is necessary to make integration of the corresponding relativistic equations of motion considering gravitation of the Sun, big planets, including their figures, 300 large asteroids, other factors. Computing complexity of this problem can be estimated as a solution of $6 * \mathrm{~N} \approx 3.4 * 10^{6}$ ordinary differential equations, where N is the number of interacting bodies, with a relative error not worse than $\frac{\Delta r}{r}=\frac{0.01^{\prime \prime}}{206265} \approx 10^{-8}$ which represents a calculation consuming problem.

For solution of a problem of mass determination of large asteroids, it is necessary to select such small asteroids which will have close encounters with large ones. The perturbing asteroid will essentially change the orbit of perturbed asteroids within the encounter. The selection of such small asteroids should be made upon the prediction of their motion for the next years including prospective operating time of Gaia, e.g. 2011-2020. To decrease significantly computing time of solution such a problem, the preliminary search of close encounters between asteroids can be made using the well-known exact solution of two-body problems "an asteroid and the Sun" for each of 574121 asteroids. This assumption is correct since elements of an orbit of asteroids essentially change on time scale of a century. The encounter between asteroids is characterized by minimal distance occurred, which was easily formulated as a proper mathematical problem. The supremum for minimal distances between asteroids was accepted in 0.05 a.u.

The search of a minimum of distances between two asteroids was made numerically in ecliptic system with the beginning in the Sun. Two algorithms were developed in C programming language: 1) the algorithm uses search of a minimum of distance between asteroids in two variables - eccentric anomalies with equality condition on mean anomalies; 2) the algorithm uses search of a minimum of distance between asteroids in one variable - time. The comparison calculations using these two algorithms showed that the second algorithm is considerably efficient in speed than the
first one.
Presence of several minima on the investigated time interval made the basic complexity of the problem. As far as the square of relative distance between asteroids is a smooth function of time, the hypothesis was used about existence of a region of "attraction" around each minimum where it can be found by a local minimization algorithm. Systematic search within each cell of some regular mesh was developed. The only necessary parameter was the size of a cell, which can be chosen from the problem physics. The local minimization algorithm should necessarily recognize saddle points. The described above algorithm for search of several extrema was developed and checked against the test function of Goldstein and Price of two variables having one global and four local minima in a set area.

The developed algorithm was applied to the search of a minimum distances between all the asteroids. The output contains number or temporarily designation of asteroid, minimal distance found, Julian and calendar dates for each encounter. The orbital elements of asteroids were taken from the regularly updated database of Minor Planet Center of International Astronomical Union, http://www.minorplanetcenter.net/iau/MPCORB.html. The verification of prediction of encounters was checked against the accurate integration by the HORIZONS system of JPL, $\underline{\mathrm{http}: / / s s d . j p l . n a s a . g o v / ? h o r i z o n s ~ f o r ~ f e w ~ a s t e r o i d s . ~ T h e ~ c o m p a r i s o n ~ h a s ~ s h o w n ~ a b s o l u t e ~ d i f f e r e n c e s ~}$ less than one day in time moment, and a big sensitivity of minimal distance to the osculating orbital elements of asteroids giving only the correct order of a value. The last fact advices to take the supremum of minimal distances with safety.

## DESCRIPTION OF THE MAIN RESULTS OBTAINED

1. The specialized algorithm is developed for search of the minimal distances between two bodies, moving on two heliocentric orbits arbitrary located in space. The algorithm finds all minima of distance between bodies.
2. The search for close encounters of 300 large asteroids with all the known 574120 asteroids (December 14, 2011), including 310376 numbered asteroids, is made on a time interval 2011-2020. There was found 282 thousands of encounters on distances less than 0.05 a.u. The sample of 80 closest encounters from the corresponding file is presented in the table below.

| Perturbing asteroid | Perturbed asteroid | Minimal distance, a.u. | Julian date of encounter | Calendar date of encounter |
| :---: | :---: | :---: | :---: | :---: |
| 143 | K07UB3V | 1. 4 4E-4 | 2456360.7 | 2013-03-09.2 |
| 233 | K07F27C | 1. $6 \mathrm{E}-4$ | 2458290.4 | 2018-06-20.9 |
| 95 | E4145 | 2.19E-4 | 2458511.5 | 2019-01-28.0 |
| 71 | 47353 | $2.27 \mathrm{E}-4$ | 2456971.6 | 2014-11-10.1 |
| 1 | K05QH9S | $2.43 \mathrm{E}-4$ | 2457787.4 | 2017-02-02.9 |
| 4 | B4488 | 2.51E-4 | 2457061.2 | 2015-02-07.7 |
| 29 | 79199 | 2.66E-4 | 2458582.4 | 2019-04-08.9 |
| 471 | 09673 | 2.73E-4 | 2455765.0 | 2011-07-22.5 |
| 455 | K04X03G | 2.91E-4 | 2458420.0 | 2018-10-28.5 |
| 505 | K9197 | $3.51 \mathrm{E}-4$ | 2459005.9 | 2020-06-05.4 |
| 654 | K03UF2F | $3.56 \mathrm{E}-4$ | 2456554.1 | 2013-09-18.6 |
| 16 | S8951 | 3.65E-4 | 2458247.3 | 2018-05-08.8 |
| 209 | 24343 | $3.68 \mathrm{E}-4$ | 2457322.4 | 2015-10-26.9 |
| 202 | K11BA9U | $3.93 E-4$ | 2456138.8 | 2012-07-30.3 |


| 200 | T8669 | 4.09E-4 | 2459208.5 | 2020-12-25.0 |
| :---: | :---: | :---: | :---: | :---: |
| 223 | K04P71W | $4.16 \mathrm{E}-4$ | 2457837.5 | 2017-03-25.0 |
| 147 | K04Tb0M | 4.27E-4 | 2458699.9 | 2019-08-04.4 |
| 674 | 51798 | 4.3E-4 | 2455617.5 | 2011-02-25.0 |
| 10 | K09SL7W | 4.31E-4 | 2457411.2 | 2016-01-23.7 |
| 240 | K05UD5E | 4.38E-4 | 2457927.3 | 2017-06-22.8 |
| 5 | K09W07X | 4.45E-4 | 2459038.1 | 2020-07-07.6 |
| 240 | K08SQ0E | $4.46 \mathrm{E}-4$ | 2458044.0 | 2017-10-17.5 |
| 344 | C9180 | 4.52E-4 | 2458326.9 | 2018-07-27.4 |
| 192 | E4411 | 4.6E-4 | 2457386.5 | 2015-12-30.0 |
| 680 | 34664 | 4.6E-4 | 2457790.2 | 2017-02-05.7 |
| 111 | K0 6BDOY | 4.62E-4 | 2458422.6 | 2018-10-31.1 |
| 230 | G2321 | 4.67E-4 | 2456083.4 | 2012-06-04.9 |
| 308 | K06SB4E | 4.82E-4 | 2455909.1 | 2011-12-13.6 |
| 488 | K02TZ6R | $4.92 \mathrm{E}-4$ | 2458238.3 | 2018-04-29.8 |
| 63 | 68784 | 4.96E-4 | 2456561.6 | 2013-09-26.1 |
| 72 | B8220 | $4.97 \mathrm{E}-4$ | 2455796.8 | 2011-08-23.3 |
| 9 | K02RI2G | 5.03E-4 | 2458817.1 | 2019-11-29.6 |
| 356 | K06SE5P | 5.03E-4 | 2457157.0 | 2015-05-14.5 |
| 690 | 15980 | $5.1 \mathrm{E}-4$ | 2456852.9 | 2014-07-14.4 |
| 286 | U8934 | $5.13 \mathrm{E}-4$ | 2456958.7 | 2014-10-28.2 |
| 89 | C3255 | 5.16E-4 | 2459176.1 | 2020-11-22.6 |
| 419 | B8071 | 5.41E-4 | 2456419.6 | 2013-05-07.1 |
| 105 | C2934 | $5.55 \mathrm{E}-4$ | 2458410.7 | 2018-10-19.2 |
| 12 | L1859 | 5.61E-4 | 2457912.5 | 2017-06-08.0 |
| 980 | K07VL6V | 5.65E-4 | 2457596.4 | 2016-07-26.9 |
| 49 | 32874 | $5.78 \mathrm{E}-4$ | 2458828.4 | 2019-12-10.9 |
| 345 | S2285 | $5.86 \mathrm{E}-4$ | 2457474.5 | 2016-03-27.0 |
| 514 | L3751 | 5.96E-4 | 2456212.3 | 2012-10-11.8 |
| 91 | I4791 | 6.13E-4 | 2458881.1 | 2020-02-01.6 |
| 407 | P4875 | $6.24 \mathrm{E}-4$ | 2455839.6 | 2011-10-05.1 |
| 356 | L1813 | $6.25 \mathrm{E}-4$ | 2457088.4 | 2015-03-06.9 |
| 71 | K08X21N | 6.36E-4 | 2456928.1 | 2014-09-27.6 |
| 141 | K05QH9F | $6.42 \mathrm{E}-4$ | 2456767.1 | 2014-04-19.6 |
| 22 | 86598 | 6.52E-4 | 2457782.3 | 2017-01-28.8 |
| 22 | L6451 | 6.69E-4 | 2455695.8 | 2011-05-14.3 |
| 94 | 96962 | $6.76 \mathrm{E}-4$ | 2456748.6 | 2014-04-01.1 |
| 58 | G4141 | $6.91 \mathrm{E}-4$ | 2457010.2 | 2014-12-18.7 |
| 469 | D6305 | $7.14 \mathrm{E}-4$ | 2456515.9 | 2013-08-11.4 |
| 674 | 12619 | 7.2E-4 | 2458477.4 | 2018-12-24.9 |
| 230 | 09215 | $7.24 \mathrm{E}-4$ | 2457955.4 | 2017-07-20.9 |
| 88 | K09F20E | 7.29E-4 | 2458254.6 | 2018-05-16.1 |
| 476 | K10U57Q | 7.31E-4 | 2456293.9 | 2013-01-01.4 |


| 694 | T9121 | 7. $4 \mathrm{E}-4$ | 2456855.0 | 2014-07-16.5 |
| :---: | :---: | :---: | :---: | :---: |
| 372 | U8459 | 7.41E-4 | 2456474.5 | 2013-06-31.0 |
| 40 | P1039 | $7.44 \mathrm{E}-4$ | 2457054.6 | 2015-02-01.1 |
| 90 | K06QA0W | $7.45 \mathrm{E}-4$ | 2456108.5 | 2012-06-30.0 |
| 335 | K09S72D | 7.47E-4 | 2455588.8 | 2011-01-27.3 |
| 59 | K08CF6T | 7.48E-4 | 2457862.6 | 2017-04-19.1 |
| 709 | K06H00P | $7.48 \mathrm{E}-4$ | 2458197.6 | 2018-03-20.1 |
| 90 | 01221 | 7.56E-4 | 2457792.9 | 2017-02-08.4 |
| 283 | K09Q11L | 7.56E-4 | 2458900.9 | 2020-02-21.4 |
| 674 | H0108 | $7.56 \mathrm{E}-4$ | 2456643.6 | 2013-12-17.1 |
| 21 | U8928 | 7.58E-4 | 2458023.2 | 2017-09-26.7 |
| 886 | K08W17F | 7.76E-4 | 2455705.0 | 2011-05-23.5 |
| 6 | 29052 | 7.8E-4 | 2457332.8 | 2015-11-06.3 |
| 159 | 90807 | $7.82 \mathrm{E}-4$ | 2456488.6 | 2013-07-15.1 |
| 144 | A 4002 | 8.03E-4 | 2458502.8 | 2019-01-19.3 |
| 164 | H9668 | 8.11E-4 | 2456931.7 | 2014-10-01.2 |
| 3 | 67684 | 8.22E-4 | 2456568.1 | 2013-10-02.6 |
| 912 | 39790 | $8.24 \mathrm{E}-4$ | 2456498.8 | 2013-07-25.3 |
| 602 | K03UD0T | 8.25E-4 | 2456316.3 | 2013-01-23.8 |
| 3 | K08Q03E | 8.29E-4 | 2458904.7 | 2020-02-25.2 |
| 532 | K10R20U | 8.38E-4 | 2459188.0 | 2020-12-04.5 |
| 1021 | I7781 | $8.44 \mathrm{E}-4$ | 2456682.9 | 2014-01-25.4 |
| 27 | 61200 | $8.46 \mathrm{E}-4$ | 2458387.3 | 2018-09-25.8 |

## FUTURE COLLABORATION WITH HOST INSTITUTION

The developed algorithm is planned to use as a preliminary filter, allowing to choose close encounters between asteroids for making accurate integration of their motion and finding gravitational perturbations in positions, determining time moments for observations. A special observational programme for observing perturbed asteroids will be elaborated in cooperation with the Institut de Mécanique Céleste et de Calcul des Éphémérides (IMCCE) to be used in the corresponding International Joint Project (the network of ground-based observatories).

## PROJECTED PUBLICATIONS/ARTICLES RESULTING OR TO RESULT FROM THE GRANT

The preparation of a corresponding article "Astrometry Observations of Asteroids before Gaia" containing circumstances of encounters between asteroids is planned. It will contain time moments for making observations of the perturbed asteroids for the purpose of mass determination of large asteroids. The ESF activity "Gaia Research for European Astronomy Training" will be acknowledged in this publication.

