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Research project: "Anomalous transport in symmetric systems induced by non-thermal noise"

The phenomenon of absolute negative mobility (ANM) of Brownian particles moving in a spatially periodic potential and driven by both zero-mean time-periodic forces and *deterministic constant force* has been theoretically predicted [1] and experimentally confirmed in a single Josephson junction device [2]. Such a phenomenon can be described in terms of the Langevin equation for the Brownian particle:

$$M\ddot{x} + \Gamma\dot{x} = -V'(x) + A\cos(\Omega t) + F + \sqrt{2\Gamma k_B T}\,\xi(t), \tag{1}$$

where thermal equilibrium fluctuations are modelled by δ-correlated Gaussian white noise $\xi(t)$ of zero mean and unit intensity.

My main problem is formulated as follows: can the force F constant in space and time be replaced by random force (noise) $\eta(t)$. It means that we consider the equation of the form

$$M\ddot{x} + \Gamma\dot{x} = -V'(x) + A\cos(\Omega t) + \eta(t) + \sqrt{2\Gamma k_B T}\,\xi(t). \tag{2}$$

It is not obvious that such a replacement $F \to \eta(t)$ can be equivalent. We have worked on transport properties of an inertial Brownian particle which moves in a *symmetric* periodic potential $V(x) = \Delta V \sin(2\pi x/L)$.

In order to make the comparison with the case of the deterministic load $F \neq 0$, we have to assume that the mean value of the random force $\eta(t)$ is the same as F, i.e. $\langle \eta(t) \rangle = F$. It means that the random force $\eta(t)$ exerts a net force on the particle. As a model for such a force we propose a sequence of delta-shaped pulses with random amplitudes defined in terms of Poissonian white shot noise, i.e.,

$$\eta(t) = \sum_{i=1}^{n(t)} z_i \delta(t - t_i), \tag{3}$$

where t_i are the arrival times of a Poissonian counting process $n(t)$ with parameter λ. The parameter λ determines the mean number of the δ-pulses per unit time or equivalently the mean frequency of the δ-pulses. The amplitudes $\{z_i\}$ of the δ-pulses are random variables, independent of each other and of the counting Poissonian process $n(t)$. They are distributed according to the common probability density $\rho(z)$. The process $\eta(t)$ is white noise of the mean and the covariance given by

$$\langle \eta(t) \rangle = \lambda \langle z_i \rangle, \quad \langle \eta(t)\eta(s) \rangle - \langle \eta(t) \rangle \langle \eta(s) \rangle = 2D_P \delta(t-s), \tag{4}$$

where $\langle z_i \rangle$ is an average over the amplitude distribution $\rho(z)$. Its intensity D_P reads

$$D_P = \frac{\lambda \langle z_i^2 \rangle}{2}. \tag{5}$$

The Fokker-Planck-Kolmogorov-Feller type equation corresponding to the Langevin equation (2) cannot be handled by any analytical methods. Therefore we have carried out comprehensive numerical simulations of the Langevin equation (2). Details of the employed numerical scheme can be found in Ref. [3]. We have chosen time step equal to $0.002 \cdot 2\pi/\omega$ and set the initial conditions $x(0)$, $\dot{x}(0)$ equally distributed over the interval $[-\pi, \pi]$ and $[-2, 2]$, respectively. Averaging has been performed over $10^3 - 10^6$ different realizations and over one period of the external driving $2\pi/\omega$. All numerical calculations have been done using CUDA environment on modern desktop GPU which gave us possibility to speed them up to few hundreds times more than on typical present-day CPU.

By variation of shot-noise parameters, one can conveniently manipulate a pace and direction of transport. Within tailored parameter regimes, inherent stochastic nature of shot-noise induces anomalous transport analogical to the absolute negative mobility effect. We show that in the regime of absolute negative mobility, properly prepared shot-noise is equivalent to a deterministic constant force. Moreover, it is robust with respect to statistics of random amplitudes of δ-pulses.

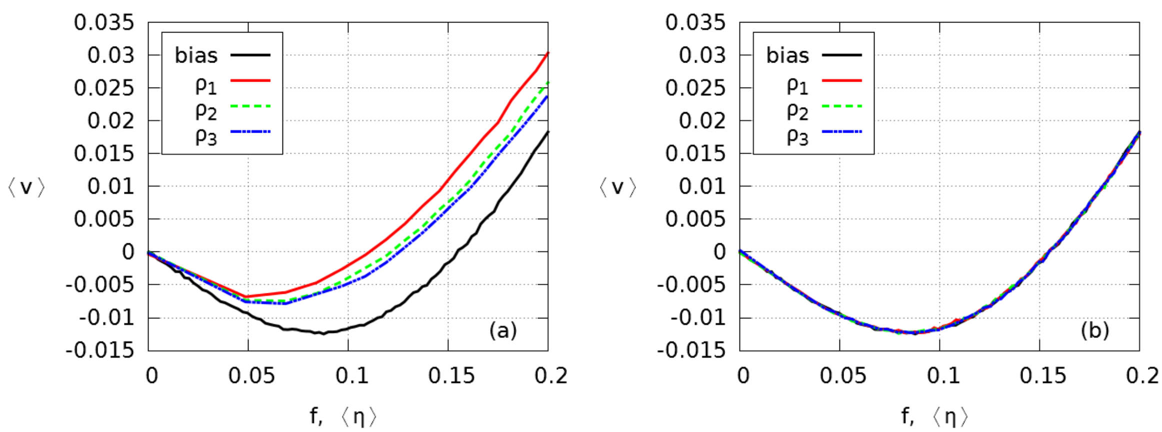

Figure 1. The dependence of the asymptotic average velocity $\langle v \rangle$ on the constant force f and the mean value of shot-noise $\langle \eta(t) \rangle$ for $\lambda = 4$ in panel (a) and $\lambda = 512$ in panel (b). Perfect equivalence of deterministic and random forces is observed for high frequency of δ-pulses of Poissonian noise. The main conclusion is: the regime of absolute negative mobility can occur in systems driven by random forces.

The dependence of the asymptotic average velocity $\langle v \rangle$ on the constant bias and mean value of shot-noise $\langle \eta(t) \rangle$ is presented in figures 1 for the case of: (i) slow frequency

$\lambda = 4$ and (ii) high frequency $\lambda = 512$ of δ-pulses. For slow frequency, one can observe smaller windows of ANM for the random forces and the deterministic force is the most effective for ANM and occurs for a wider force interval. For high frequency of δ-pulses, one can detect equivalence of the constant bias and the random force for generation of similar transport properties in the regime of ANM, Furthermore, because our previous results are insensitive the statistical distributions of amplitudes $\{z_i\}$ of the δ-kicks this is also uninfluenced by the change of their statistical properties. One can obtain the same anomalous transport effects by using either the constant bias or properly prepared shot-noise. We have to stress that the presented effect is not a ratchet effect.

These findings can readily be experimentally tested with an accessible setup consisting of a single resistively and capacitively shunted Josephson junction device.

The above results and other results on transport induced by Poissonin noise are contained in the paper:
"Anomalous transport of inertial Brownian particles induced by Poissonian noise" (authors J. Spiechowicz, J. Łuczka and P. Hänggi) which I just finished to write in Augsburg. Now I wait for response of the co-authors. We intend to publish this paper in a renowned physical journal.

[1] L. Machura, M. Kostur, P. Talkner, J. Łuczka and P. Hänggi, Phys. Rev. Lett. **98**, 040601 (2007).
[2] Nagel J, Speer D, Gaber T, Sterck A, Eichhorn R, Reimann P, Ilin K, Siegel M, Koelle D and Kleiner R 2008 *Phys. Rev. Lett.* **100** 217001
[3] Kim C, Lee E, Hänggi P and Talkner P 2007 *Phys. Rev. E* **76** 011109
Grigoriu M 2009 *Phys. Rev. E* **80** 026704 **181** 183