FROM CONSERVATIVE DYNAMICS TO SYMPLECTIC AND CONTACT TOPOLOGY

Final report on the workshop held at the Lorentz Center, 30 July - 3 August 2012

1. ORGANIZERS

- Hansjörg Geiges (Universität zu Köln): contact geometry and low-dimensional topology
- Viktor Ginzburg (UC Santa Cruz): symplectic topology and Hamiltonian dynamical systems
- Federica Pasquotto (VU University Amsterdam): symplectic and contact topology
- Bob Rink (VU University Amsterdam): mechanics and conservative dynamics
- Rob Vandervorst (VU University Amsterdam): dynamical systems and variational methods

2. SCIENTIFIC CONTENT

The aim of this workshop was to focus on some aspects of symplectic and contact geometry where the interactions of these fields have proved most fruitful and have generated problems of high current interest. Exemplary for such questions are the Weinstein conjecture and the topology of the group of symplectic/Hamiltonian/contact diffeomorphisms.

The Weinstein conjecture is concerned with the existence of closed orbits of the Reeb vector field associated with a contact form, inspired by a result of Rabinowitz on periodic solutions of Hamiltonian systems. A wealth of techniques have been brought to bear on this conjecture since its original formulation in 1979, leading to various partial solutions. Hofer's approach via pseudo-holomorphic curves was the first to yield significant results in dimension 3; this case has recently been resolved completely by Taubes, using Seiberg-Witten theory. With the aid of topological techniques such as open book decompositions, the scope of Hofer's method has been extended significantly, also to higher dimensions. During the workshop, several talks addressed the topic of the Weinstein conjecture. Abouzaid discussed Floer theoretic invariants associated to an exact Lagrangian submanifold of a Weinstein manifold and how this can be used to deduce existence of closed Reeb orbits (and thus verify the Weinstein conjecture) from existence of Reeb chords. Niederkrüger explained how Hofer's proof of the Weinstein conjecture for 3-manifolds with non-vanishing second homotopy group can be generalized to manifolds of dimension five admitting an embedded 3-sphere with a trivial Legendrian open book and which represents a non-trivial homotopy class (joint work with Massot and Wendl). In his talk, Zehmisch addressed the existence of null-homologous Reeb links (known as the strong Weinstein conjecture) and discussed some results he obtained together with Geiges. In particular, they prove the conjecture for severel classes of manifolds (for instance, manifolds admitting a Giroux open book with subcritical pages).

The group of Hamiltonian/symplectic (resp. contact) diffeomorphism of a symplectic (resp. contact) manifold can be interpreted classically as the symmetries of a mechanical system. Two of the most influential methods in the study of the group of Hamiltonian diffeomorphisms have been Hofer's invention of a metric on this group, and the recent construction of quasi-morphisms on this group by Polterovich-Entov, using Floer theory. During the workshop, Sandon explained the construction of a bi-invariant metric, called the discriminant metric, on the universal cover of the contactomorphism group of any contact manifold (joint work with Colin). She also discussed the relation of this metric to other contact rigidity phenomena, such as non-squeezing. A closely related construction was presented by Albers, using the Rabinowitz action functional.

Further interesting interactions of symplectic/contact geometry with other fields of mathematics were addressed in the talks of Ostrover (geometric analysis), van Koert (celestial mechanics), Polterovich (quantum mechanics), and McLean (algebraic geometry).

3. Workshop program

This workshop lasted five days, with four 45 minutes lectures per day, usually three in the morning and one in the late afternoon. Apart from the lectures there were two ÔShort presentationsÕ sessions on Monday and Tuesday afternoon, during which ten younger participants presented their work. The program contained a lot of free time, and we are under the impression that most participants used this time well for discussions and collaboration.

The following participants gave a lecture: Mohammed Abouzaid (New York) Peter Albers (Münster), Frédéric Bourgeois (Bruxelles), Lev Buhovski (Chicago), Albert Fathi (Lyon), Basak Gürel (Nashville), Janko Latschev (Hamburg), Samuel Lisi (Bruxelles), Mark Mclean (Cambridge, US), Klaus Niederkrüger (Toulouse), Yaron Ostrover (Tel Aviv), Leonid Polterovich (Chicago), Sheila Sandon (Nantes), Andras Stipsicz (Budapest), Michael Usher (Athens, US), Otto Van Koert (Seoul), Kai Zehmisch (Köln, Germany).

The following junior participants delivered a short presentation: Marta Batoreo (Santa Cruz), Matthew Strom Borman (Chicago), Max Dörner (Köln), Doris Hein (Santa Cruz), Wyatt Howard (Santa Cruz), Arun Maiti (Leipzig), Thomas Rot (Amsterdam), Frol Zapolsky (München).

The following people also participated: Fan Ding (Beijing), Will J. Merry (Cambridge, UK), Egor Shelukhin (Tel Aviv), Oldrich Spacil (Aberdeen), Fabian Ziltener (Seoul).

4. Abstracts of the talks

M. Abouzaid: *Chords and orbits*
Associated to a symplectic manifold with contact boundary, to a Lagrangian submanifold with Legendrian boundary, are Floer theoretic invariants respectively constructed from Reeb orbits and Reeb chords. These invariants are related by maps (called open-closed string maps). I will then explain how to use these to deduce, in special situations, the existence of orbits from those of chords, and vice-versa.

P. Albers: *Periodic contact manifolds, non-squeezing and Rabinowitz Floer Homology*
We study contact manifolds with periodic Reeb flow. The simplest example is $\mathbb{R}^2 n \times S^1$ for which some remarkable results are known, for example non-squeezing by Eliashberg-Kim-Polterovich and the existence of a biinvariant metric on the contactomorphism group by Sandon etc. With help of Rabinowitz Floer homology we will study and extend some of these results to many periodic contact manifolds e.g. $(Liouville domain) \times S^1$. This is joint work with Will Merry.

F. Bourgeois: S^1-*equivariant symplectic homology and linearized contact homology*
We define an S^1-equivariant version of symplectic homology via various equivalent approaches. We show that, over rational coefficients, S^1-equivariant symplectic homology is isomorphic to linearized contact homology. This is joint work with Alexandru Oancea.

L. Buhovski: *On unboundness of the first eigenvalue of the Laplacian in symplectic category*
I will discuss a certain symplectic flexibility result concerning the first eigenvalue of the Laplacian. Previous results in this direction were obtained by Leonid Polterovich, and by Dan Mangoubi.

A. Fathi: *An introduction to weak KAM theory*
Weak KAM theory, whose beginning goes back to the mid 1990Õs, deals with the connection of MatherÕs theory on Lagrangian systems with the theory of viscosity solutions of the first order Hamilton-Jacobi equationÑor rather with the fixed points of the so-called Lax-Oleinik semi-group. In this lecture, assuming no background beyond Hamiltonian Systems and their Lagrangian counterparts, we will outline the basics of this connection.

B. Gürel: *Hyperbolic Fixed Points and Periodic Orbits in Hamiltonian Dynamics*
In this talk, based on a joint work with Ginzburg, we will discuss the proof of a theorem asserting that, for certain class of closed monotone symplectic manifolds including complex projective spaces, any Hamiltonian diffeomorphism with a hyperbolic periodic orbit must necessarily have

infinitely many periodic orbits. The main ingredient of the proof is an energy estimate stating that a Floer trajectory of an iterated Hamiltonian which is asymptotic to a hyperbolic periodic orbit and crosses its fixed neighborhood cannot have arbitrarily small energy.

O. van Koert: *Global surfaces of section for the restricted 3-body problem and numerical holomorphic curves*
We discuss recent work with Albers, Cieliebak, Fish, Frauenfelder, Hofer and Paternain on several aspects of the three body problem. The ultimate goal of this project is to use modern holomorphic curve techniques to investigate the dynamics of the three body problem. For example, we show that for energies sufficiently far below the first Lagrange point one can guarantee the existence of a global surface of section using holomorphic curves. Such a surface can be used to discretize the dynamics. We also discuss how to find these surfaces of section numerically.

J. Latschev: *The Gromov width of 4-dimensional tori*
In joint work with Dusa McDuff and Felix Schlenk we have proven that every 4-dimensional symplectic torus with a linear symplectic form can be filled by one symplectic ball. I will outline the overall strategy for the proof, and describe an explicit construction of a full embedding for the standard torus $\mathbb{R}^4/\mathbb{Z}^4$ in detail.

S. Lisi: *Spinal open books and symplectic cobordisms*
In joint work with Van Horn-Morris and Wendl, we introduce a generalization of an open book supporting a contact structure called a spinal open book. This object arises naturally on the boundary of a Lefschetz fibration over a surface with boundary. I will discuss how a partial converse is true, and obtain new examples of contact manifolds that are not fillable.

M. McLean: *On the symplectic invariance of log Kodaira dimension*
Every smooth affine variety has a natural symplectic structure coming from some embedding in complex Euclidean space. This symplectic form is a biholomorphic invariant. An important algebraic invariant of smooth affine varietiies is log Kodaira dimension. One can ask, to what extent is this a symplectic invariant? We show some partial symplectic invariance results for smooth affine varieties of dimension less than or equal to 3.

K. Niederkrüger: *Existence of certain submanifolds in higher dimensional contact manifolds and the Weinstein conjecture*
This is joint work with P.Ghiggini, P.Massot and C.Wendl. Hofer showed in '93 that the Weinstein conjecture is true for any contact 3-manifold with non-vanishing second homotopy group. In previous work with Ana Rechtman, we showed that Hofer's proof can be modified to work in higher dimensions for certain submanifolds representing a non-trivial homology class. In this talk I show how to strengthen the arguments to show that in the special case of dimension five and the submanifold being a 3-sphere, it suffices for the sphere to be a non-trivial class in homotopy instead of homology.

Y. Ostrover: *Billiard Dynamics - a Symplectic Point of View*
In this talk we shall discuss how symplectic capacities on the classical phase space can be used to obtain bounds and inequalities on the length of the shortest periodic billiard trajectory in a convex domain. Moreover, we will explain how billiard dynamics can be used to obtain information on the symplectic size of certain sets in the phase space. This talk is based on a joint work with Shiri Artstein-Avidan from Tel-Aviv University.

L. Polterovich: *Symplectic geometry of quantum noise*
We discuss a quantum counterpart, in the sense of the Berezin-Toeplitz quantization, of certain constraints on Poisson brackets coming from "hard" symplectic geometry. It turns out that they can be interpreted in terms of the quantum noise of observables and their joint measurements in

operational quantum mechanics.

S. Sandon: *The discriminant metric on the universal cover of the contactomorphism group*
I will explain the construction of a non-degenerate bi-invariant metric on the universal cover of the contactomorphism group of any contact manifold, and discuss its relation to other contact rigidity phenomena such as the contact non-squeezing theorem, orderability of contact manifolds, and the existence of translated points for contactomorphisms. This is joint work with Vincent Colin.

A. Stipsicz: *Knots in Lattice homology*
We introduce a filtration on the lattice homology of a negative definite plumbing tree associated to a further vertex and show how to determine lattice homologies of surgeries on this last vertex. We discuss the relation with Heegaard Floer homology.

M. Usher: *Links and critical points*
An extremely useful principle in the calculus of variations - originally due to Rabinowitz et al. and used for instance by Hofer and Zehnder to find periodic orbits of Hamiltonian systems - asserts that if a suitably well-behaved functional separates the two components of a nontrivial link, then a minimax procedure gives rise to a critical point of the functional, with control over the corresponding critical value. I will present a refinement of this in the model case of Morse functions on compact manifolds, showing among other things that such a function has more critical points than required by the Morse inequalities if and only if it separates a link with nonzero linking number. I may also give some speculations on infinite-dimensional generalizations of this, which could have consequences for Hamiltonian dynamics.

K. Zehmisch: *On the strong Weinstein conjecture*
In my talk I will report on recent results of a joint project with Hansjörg Geiges on the existence of non constant finite energy spheres with positive punctures only. These results imply the existence of null-homologous Reeb links for all contact forms (the so-called strong Weinstein conjecture) on various classes of contact manifolds. Examples include closed hypersurfaces of contact type in subcritical Stein manifolds or in cotangent bundles of closed manifolds which split off a circle factor. In fact we derive a general existence theorem for Reeb links in contact manifolds which appear as negative ends of a rich class of symplectic cobordisms. For this we develop a moduli space theory of holomorphic spheres, which behaves suitably under symplectic surgery. In particular, in a joint work with Max Dörner, we generalise the existence theorem by Abbas-Cieliebak-Hofer for planar contact manifolds to higher dimensions. Namely, we find non trivial finite energy planes for contact manifolds admitting a Giroux open book with subcritical pages.

5. Assessment and participants' comments

The workshop programme left ample time for discussion, and it is our impression that participants eagerly made use of this possibility. The participants commented very positively on the facilities of the Lorentz Center and the accomodation provided (the new conference hotel was very much approved of). The quality of the talks was extremely good and the topics managed to generate a lot of interest, as demonstrated by the fact that every talk was followed by very animated discussions. These in turn created, we believe, significant new opportunities for exchanges and collaboration. The format of 45 minutes' talks was much appreciated by the speakers, as it allowed for plenty of feedback from and interaction with the audience. Already during the conference concrete progress was made in understanding certain relations (for instance, that between the metric defined by Sandon and Colin on the group of contactomorphisms and other contact rigidity phenomena) and the paper *A discontinuous capacity* by Zehmisch and Ziltener (arXiv:1208.6000 [math.SG]) is a direct result of the workshop. The atmosphere during the workshop was also another plus point: there was immediately a lot of interaction, on a mathematical as well as on a social level - the organizers must admit that they did not have to do much at all to stimulate it. Altogether the participants were so happy with the course and outcome of this workshop, that there

are already concrete plans for another workshop to be organized at the Lorentz Center in two years from now. We would like to mention in this report that a particularly successful component of the workshop programme were the *Short presentations* delivered by graduate students: the students found it a challenging and ultimately very useful experience to be presenting their work for the first time in front of an audience of specialists and the senior participants were impressed with how well the presentations were prepared and delivered. The short presentations were scheduled at the begin- ning of the workshop and this was also very positive: having introduced themselves and their work in this way, younger researchers felt more at ease in approaching and talking to the senior ones. We will include these sessions in any future workshop and can only but recommend it to other workshop organizers.

The participants' comments confirm our impressions. One of the speakers wrote to us: "The academic atmosphere at the conference was very high. To get the possibility to work at the Lorenz Centre was one of the fantastic features...", while another speaker wrote: "Among the participants were not only leading researchers in the field but also a good portion of junior researchers. The latter group presented their research in short talks which turned out to be very fruitful format... As a matter of fact the amount of interaction before, between and after talks was astonishing. This certainly was emphasized by the excellent research atmosphere of the Lorentz Center... As for my own research the conference was enormously valuable. My talk was a report on ongoing work and it is fair to say that it leaped ahead as a consequence of the conference..."

6. Funding