Scientific Report

Radio-frequency potentials on atom chips

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Purpose of the visit

The goal of this exchange visit was to develop a robust theory for describing arbitrarily polarised radio frequency fields with the aim to use them to coherently control the centre-of-mass state of single, cold atoms trapped on an atomchip. Having such a description would allow us to dramatically extend the number of 'buttons' available in experiments and make new regimes of manipulation available. My time spent in the University of Nottingham has not only allowed me to develop this theory through discussions with Dr. Thomas Fernholz and Dr. Igor Lesanovsky, but has also helped to lay the foundations for further close collaboration between the experimental group in Nottingham and theoretical group in Cork.

Description of the work carried out during the visit

Before undertaking this exchange visit we had shown that it is possible to use rf-fields to manipulate single atoms in complex potentials using linearly polarised RF fields in order to achieve coherent transport by adiabatic passage between three distinct traps [1]. One significant assumption that made the work theoretically accessible was the restriction to one-dimensional systems with linear polarization of the rf field. The model we used to describe adiabatic potentials created by many different radio frequencies [2] also was no longer applicable when the frequencies come too close. With our new model we have overcome both of these issues.

During the visit to the University of Nottingham I worked closely with Dr. Thomas Fernholz and Dr. Igor Lesanovsky. Dr Lesanovsky is an expert in Floquet theory and both he and Dr. Fernholz have significant experience on the theory of rf-trapping [3-5]. This involved meeting on a regular basis with Dr. Fernholz to discuss my day to day progress and also longer monthly meetings with both Dr. Fernholz and Dr. Lesanovksy to discuss new developments, tackle problems and decide on the next element of the model to focus on.

Description of the main results obtained

The basis of our new model is Floquet Theory [6]. It was developed to provide solutions to Schrodinger equations with a Hamiltonian periodic in time. The interactions of a quantum system with an oscillating field can be studied by replacing the time-dependent Hamiltonian with a time-independent Hamiltonian represented by an infinite matrix.

If we consider an atom confined in a static magnetic field $\vec{B_s}$ subjected to a radio frequency field of the form

$$\vec{B_{rf}}(r,t) = \vec{B}(r)\cos(\omega t)$$

the Hamiltonian for this system, taking only the spin degree of freedom, is

$$\hat{H} = \mu_B g_F \vec{S} \cdot (\vec{B_s} + \vec{B}_{rf})$$

Our objective is to write down the Floquet matrix, H_f, which will allow us to remove the time dependence from the Hamiltonian. H_f is defined as

$$H_f = \begin{pmatrix} H_0 + 2\hbar\omega & H_{-1} & H_{-2} & H_{-3} & H_{-4} & ... \\ H_1 & H_0 + \hbar\omega & H_{-1} & H_{-2} & H_{-3} & ... \\ H_2 & H_1 & H_0 & H_{-1} & H_{-2} & ... \\ H_3 & H_2 & H_1 & H_0 - \hbar\omega & H_{-1} & ... \\ H_4 & H_3 & H_2 & H_1 & H_0 - 2\hbar\omega & ... \end{pmatrix}$$

and each Floquet *block* H_m can be found using

$$H^m(r) = \frac{1}{\tau}\int_0^\tau H(r,t)\exp im\omega t\,\mathrm{d}\tau$$

Once we evaluate these blocks the Floquet matrix is given by

$$\hat{H}_F = \begin{pmatrix} k|B_s(r)|+2\hbar\omega_1 & 0 & 0 & \hbar\Omega^* & 0 & ... \\ 0 & -k|B_s(r)|+2\hbar\omega_1 & \hbar\Omega & 0 & 0 & ... \\ 0 & \hbar\Omega^* & k|B_s(r)|+\hbar\omega_1 & 0 & 0 & ... \\ \hbar\Omega & 0 & 0 & -k|B_s(r)|+\hbar\omega_1 & \hbar\Omega & ... \\ 0 & 0 & 0 & \hbar\Omega^* & k|B_s(r)| & ... \\ 0 & 0 & \hbar\Omega & 0 & 0 & ... \end{pmatrix}.$$

where k is defined as $k = \mu_B g_f m_f$, μ_B is the Bohr magneton, g_f is the atomic g-factor of the hyperfine level and m_f is spin quantum number. The Rabi frequency, Ω, is defined as $\Omega = \mu_B g_F m_f \frac{B_x(r)+iB_y(r)}{2}$, where $B_x(r)$ and $B_y(r)$ are the x and y components of the rf field.

In [7] Floquet theory is extended to systems that have multiple different radio frequencies. The many-mode Floquet matrix that is required is much too large to be reproduced here so we show an extract from it assuming we have only two frequencies ω_1 and ω_2

$$H_f = \begin{pmatrix} ... & ... & ... & ... & ... & ... \\ k|B_s(r)|+\hbar\omega_1+\hbar\omega_r & 0 & 0 & \Omega_1^* & 0 & ... \\ 0 & -k|B_s(r)|+\hbar\omega_1+\hbar\omega_r & \Omega_1 & 0 & 0 & ... \\ 0 & \Omega_1^* & k|B_s(r)|+\hbar\omega_r & 0 & 0 & ... \\ \Omega_1 & 0 & 0 & -k|B_s(r)|+\hbar\omega_r & \Omega_1 & ... \\ 0 & 0 & 0 & \Omega_1^* & k|B_s(r)|-\hbar\omega_1+\hbar\omega_r & ... \\ 0 & 0 & \Omega_1 & 0 & 0 & ... \\ 0 & 0 & 0 & \Omega_2^* & 0 & ... \\ 0 & 0 & \Omega_2 & 0 & 0 & ... \\ 0 & \Omega_2^* & 0 & 0 & 0 & ... \\ \Omega_2 & 0 & 0 & 0 & \Omega_2 & ... \\ 0 & 0 & 0 & \Omega_2^* & 0 & ... \\ 0 & 0 & \Omega_2 & 0 & 0 & ... \\ 0 & 0 & 0 & 0 & 0 & ... \\ ... & ... & ... & ... & ... & ... \end{pmatrix}.$$

Here $\omega_r = \omega_1 + \omega_2$ and Ω_1 and Ω_2 are the Rabi frequencies associated with the first and second rf fields, respectively.

Once we diagonalize this matrix we produce the eigenenergy structure of the atom which is shown in Figure 1.

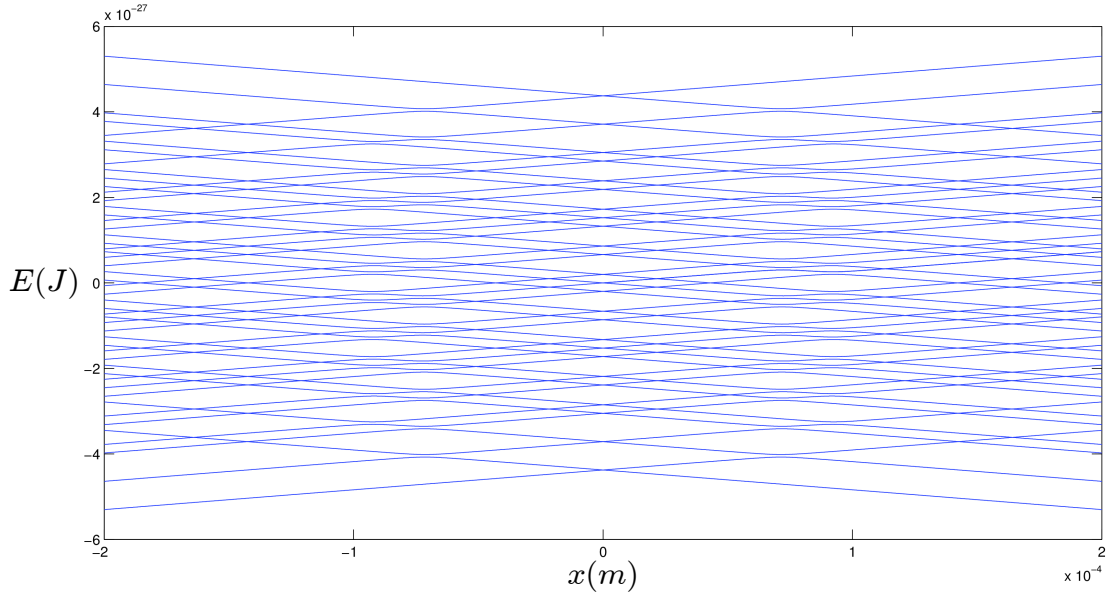

Figure 1
Eigenengeries for an atom trapped in a linear magnetic static field irradiated by two linear polarized rf fields with $\omega_1 = 1MHz$ and $\omega_2 = 1.3MHz$.

The last task we have to complete is the calculation of the adiabatic potential from these eigenenergies. This is done numerically by identifying avoided level crossings and then transferring to the corresponding eigenstate. Figure 2 shows the adiabatic potential generated from the eigenenergies in Figure 1.

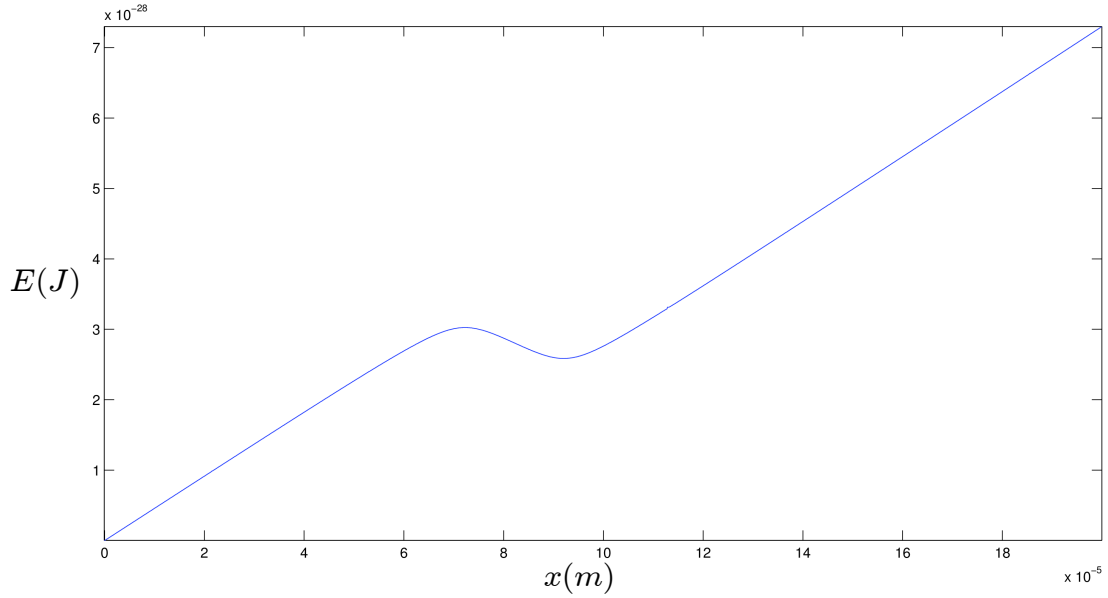

Figure 2
Adiabatic potential generated from eigenenergies in Figure 1.

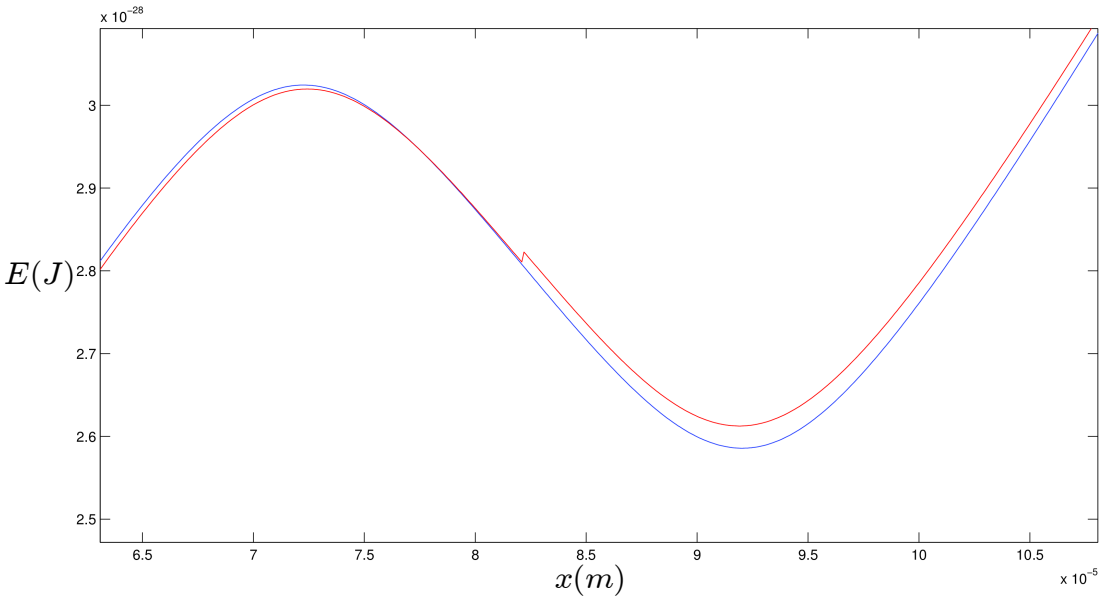

Figure 3

Comparison between old potential (red) and Floquet potential (Blue). Notice the discontinuity in the old potential at $x = 8.2 \times 10^{-5} m$

Figure 3 shows the potential that the theory developed in [2] predicts (red) plotted over the new Floquet potential (blue). As one can see from the non-continuous jump, the old model is unable to accurately generate the potential when frequencies are close. The new Floquet approach clearly offers significant corrections to the previous method and also allows to investigate radio frequencies which are much more closely spaced than were previously possible.

This model can be generalized to higher dimensional potentials and rf-fields with arbitrary polarization. Figure 4 shows the potential generated when two rf-fields (σ_+ polarized) are irradiated on an atom trapped in a Ioffe-Pritchard trap. By controlling the polarization one can now create many non standard trapping geometries such as two potential minima on the ring. The discontinuities seen in the potential in Figure 4 are due to the numerical calculation of the adiabatic surface from the eigenenergies. This problem will be fixed in a future iteration of our code.

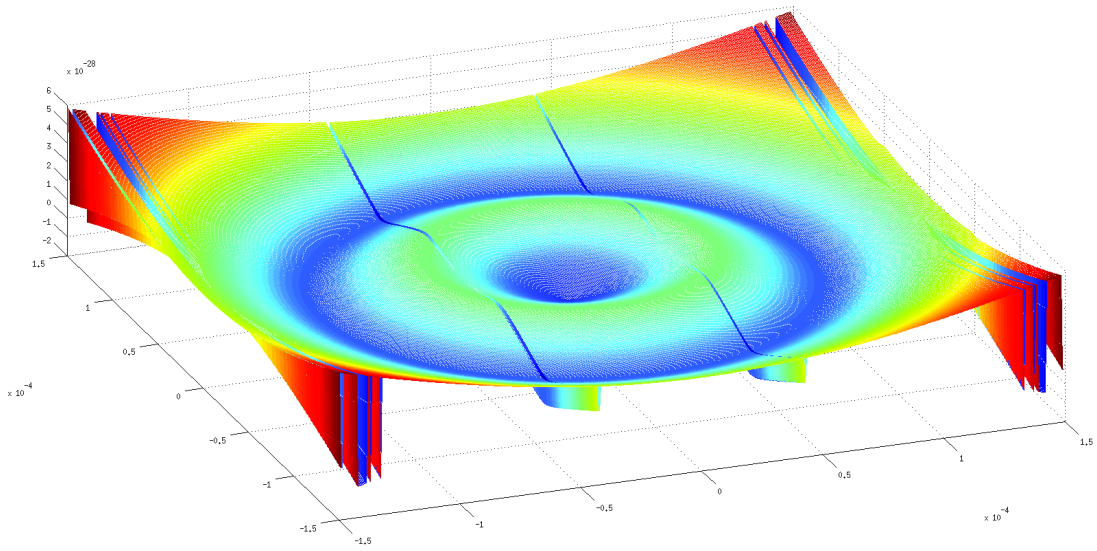

Figure 4

Potential generated when two rf fields (σ_+ polarized) are irradiated on an atom trapped in a Ioffe-Pritchard trap.

Future collaboration with host institution (if applicable)
With this these new tools we plan to work closely with our experimental partners in Nottingham to study a system which consists of two concentric rings. These rings can be created by applying three rf-fields to an atom trapped in an Ioffe-Pitchard trap on an atomchip. By then changing the polarisation of the rf-fields by π the ring potential can be transformed into that of a double well. In this way 4 distinct traps can be created, whose positions are controlled by the polarisation, relative field strength and frequency. As an example that makes use of the high configurability of such a set of potentials we suggest to study an adiabatic technique related to STIRAP (CTAP), but where the central state (trap) is now replaced by two traps. Such a system is known to not only allow the same coherent control as in [1], but by controlling the tunnel coupling between the two middle traps it also allows to take advantage of the non-abelian character of such a system to create arbitrary super-positions between the left and right traps.

Dr. Thomas Fernholz has an extensive background in such experimental setups [5] and his group is currently setting up experiments in the direction of the suggested work. Working with him and other members of the cold atom group has provided the project with direct information to ensure that our model will be as close to experimental reality as possible.

In the immediate future we will further investigate the multidimensional CTAP process suggested above. For this we will first take advantage of existing, well developed code for Matlab to simulate such adiabatic processes locally and incorporate the Floquet potentials. It is also apparent from the early numerical simulations that we will require large computational resources in order to study the time evolution of this system. For this we will draw on our experience in dealing with more numerically taxing simulation, such as the CTAP process in 3-dimensions, for which we have built a C/C++ code library running on supercomputers at ICHEC (the Irish Centre of High End Computing) or BCRI (the Boole Centre for Research in Informatics).

Projected publications / articles resulting or to result from the grant

We are currently working on a publication detailing the new Floquet model which we have developed. We will focus on making it as robust and flexible as possible and studying the interesting effects that are predicted by the theory when the frequencies are very close to resonance. We will also demonstrate applications of this method by designing systems where we can observe CTAP like effects in experimentally realistic systems which we hope will be carried out by the Nottingham group.

Other comments
I would like to take this opportunity to thank both the ESF and Polatom committees for kindly providing the financial support for this project.

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