The Quantum Vacuum and the Casimir Effect

State of the art.The Casimir force [1] is maybe the most accessible experimental consequence of vacuum fluctuations in the macroscopic world (see fig.1). It is comparably small but recently it has been measured with modern experimental techniques and several experiments reached an accuracy in the % range [2]. An accurate comparison with theory of the measured Casimir force is a key point for the experiments searching for new short range weak forces predicted in theoretical unification models. Similar experiments were also performed with Micro-Electro-Mechanical Systems (MEMS) [3, 4], promising tiny devices containing metallic elements on a micron/submicron scale. Due to the small distances between its elements, the Casimir force becomes very important for these systems.

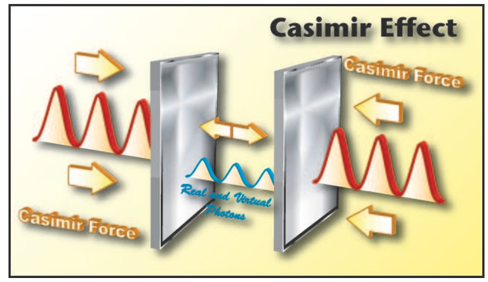

Figure 1: *Original geometrical configuration used by H.B.G. Casimir in 1948. Two planar parallel mirrors, which are facing each other in quantum vacuum, are attracted to each other. For two mirrors with a surface of 1cm^2, separated by a distance of 1μm, it equals* $0.1\mu N$.

The Casimir force could represent a useful tool to control ultracold atoms at an the same time the precision attained in manipulate ultracold atoms can be used to probe the casimir effect. For instance, very recently [5] Eric Cornell and his colleagues at the University of Colorado in Boulder and JILA, employed for this purpose a Bose-Einstein Condensate. Ultracold atoms trapped near surfaces are modern devices which exploit peculiar quantum laws to do computations (atom-chip). Casimir effect is an unavoidable effect and its knowledge could represent an essential and useful tool in projecting, controlling and transporting atoms near macroscopic surfaces.

Moreover, the connection between Casimir effect and BEC is not limited to these situations: For example the critical temperature transition in BEC can be studied with a Casimir-like formalism.

The research performed

Plasmons

In the case of metallic mirrors I previously showed [6, 7] that there are two distinct contributions to the Casimir Force: the plasmonic and the photonic ones. The former is due to the evanescent modes associated with the collective electron excitations (plasmons) propagating on the metal/vacuum interface [8]. The latter contribution comes from ordinary propagating cavity modes. Plasmonic modes turn out to have a much greater importance than usually appreciated. For distances larger than about ∼10nm and typical metals, they even give rise to a contribution having simultaneously a negative sign and a much too large magnitude with respect to the Casimir formula. This particular behavior of the plasmonic contribution has been obtained exploiting the plasma model to describe the dielectric properties of the mirrors. Of course the plasma model represents only the simplest approximation to describe the optical response of a medium and it neglects some essential features of a real material.

In the first part of my research activity I generalized indeed a part of the previous calculation to models of dielectric functions that describe more realistic properties of metals and dielectrics, like dissipation and non-locality response to an electromagnetic field. Using a dissipative model, namely the Drude model [9], I showed in detail that, for distances shorter then the plasma wavelength, the Casimir force can be interepreted as a Coulomb interaction between the surface plasmons living at the metal/vacuum interface of each mirror. The plasmon interaction gives in this limit the main contribution and the Casimir energy is equivalent to the shift of the zero point energy corresponding to generalized plasmonic modes that, differently from the non dissipative case, depend on the dissipation rate. In the limit of small dissipation rates I get a correction of the non-dissipative result with a which is linear in the dissipation rate. The introduction of dissipation leads to a reduction of the "binding" Casimir energy, the Casimir force is then less intense. This reduction of the intensity of the Casimir force is similar to the overdamping of a harmonic oscillator [10, 11, 12]

The comparison with the dissipative oscillator can be, in a very clear way, pushed further taking into account the general expression of the Casimir energy rather then its short distance approximation. I showed that, from a very general point of view, it is possible to express the Casimir energy as a sum of terms directly connected with each single mode of the electromagnetic field vibrating inside the dissipative cavity formed by the two mirrors. In presence of dissipation the concept of a mode need of course some generalization. We face indeed two distinct modifications of the non dissipative formula: first of all the mode "frequencies" are no long real quantities but they become complex: the modes turn into resonances. This behaviour was expected and it is a general feature of the dissipative system. A less expected modification is due to an extra term which is directly connected with the interaction energy between the system, the cavity, and its environment, the reservoir with which it is at the thermal equilibrium. A clear interpretation of this term can be found examinating the interaction of a single quantum harmonic oscillator with a quantum reservoir (which can be generally described by an ensemble of interacting quantum harmonic oscillators) at thermal equilibrium. Even at zero temperature there is an interaction between the system and the reservoir which has as first effect a modification of the "bare" oscillation frequency and second, as a more intriguing effect, a non-zero average interaction energy [12]. A definite amount of energy is needed to couple or uncouple the oscillator from is environment [13] and the strength of this coupling is weighted by the dissipation rate, which is basically connected with the system/bath coupling constant. All these features are recovered in the formula I derived, which expresses the Casimir energy as a sum over the "cavity modes", the modes being nothing but generalized quantum oscillators. This formula represents the first step towards an analysis of the generalized evanescent and propagating wave contributions to the Casimir force. It is worth stressing that this opens the way to find strategies that manipulate the plasmonic contribution through the variation of some "tunable" system parameter like temperature, resonances in dielectric function, surface geometry and non-equilibrium situations. It also allows to study the crossover to repulsive Casimir force. Indeed an enhancement of the plasmonic contribution could produce a change of sign in the Casimir force, which, from a technological point of view, would certainly be important for MEMS because it would allow to prevent sticking phenomena.

The thermal problem

The plasmonic mode analysis could also give a clearer input to the "thermal" problem on which I worked in the second part of my stay. This is an outstanding problem in the theory of the Casimir effect, which deal with the foundations of the theory itself [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. The essence of the problem lies in the classical contribution to the Casimir force, which dominates at large distances between plates or at high temperature. Calculations made for ideal metals at finite temperature show that both polarizations of the electromagnetic field give equal contributions to the force. At the same time the Lifshitz theory of fluctuating fields predicts zero contribution for one polarization when the Drude model was used. Strictly speaking, the result is that the classical contribution to the Casimir force with the Drude model is half the value of the ideal metal. At the same time one can show that there is no continuous transition from the "real" to the ideal metal, a thing that seems at least intriguing and that gives rise to some doubts on the validity of the previous results. A reliable expression for the temperature dependence is important because although the classical contribution to the Casimir force vanishes at zero temperature, it is relevant at room temperature or at distances of a few μm where experiements are reaching a good accuracy.

A new round of discussion has started when a thermodynamical problem connected with the Casimir free energy has been revealed. The idea was to use the Nernst heat theorem as a guiding principle to choose between different approaches to the temperature correction [18]. According to this theorem the entropy must go to zero in the limit of zero temperature. At the beginning it seemed that this analysis confirmed the approach leading to a contribution of both the polarizations. However, the following analysis revealed that the situation is not as simple and, therefore, we have a confusing situation where each approach has its own reasoning.

The work I performed aimed from one point of view to provide a general framework for all the previous trials to understand the problem; from the other I tried to get some insight on the physicality of the problem.

From the physical point of view the utilization of the Drude model is well motivated by the fact that it seems to work well especially at low frequencies. Bordag et al. [25] used as dielectric function the non-dissipative plasma model [9] rather of the Drude one motivated by the fact that the dissipation rate should vanish in the zero temperature limit. They showed that in this way it was possible to avoid this discontinuity and found a contribution from both the polarizations. It was argued then that, at low temperatures, it is the plasma model which should be used. Therefore it seems that one of possible explications for the particular behavior of the thermal correction of the Casimir effect relies on the fact that the Drude model is not mathematically accurate to describe the behavior of metal optical response.

There are several reasons that support this hypothesis. First of all the Drude model does not take into account the non locality of the interaction between the electromagnetic field a the conduction electrons. This means that when the radiation wavelength becomes smaller then the electron free-path length, the real optical response of the metal remarkably differs from the one predicted. In this case we are in fact in the region of the anomalous skin effect. Another argument claims that the description given by the Drude model in the very small frequency domain is probably a rough approximation of the real one. In this region a better description should be given by the surface impedance approach. Also in this case there are several models for the surface impedence, differing for example the ones from the others for the fact to include or not a non local description [26, 21, 27]. This variety of model comes also from the difficulties of a accurate experimental measurement of the surface impedance

Now, almost all the tentatives to understand and perhaps solve the weird behavior of the thermal Casimir energy implied a recalculation of the thermal correction using different models for the dielectric function. In every case, however, the results are not conclusive. The encountered difficulties are always the same and they can be roughly resumed in the following list

1. At long distances the thermal behavior is dominant but in the case of real metals the correction is half of the one for ideal metal.
2. The value of the Casimir entropy at zero temperature is not zero, at least when dissipation does not go to zero
3. In a well defined domain of distance the Casimir entropy is negative

The second point is in direct contrast with the Nernst theorem while for the other two we can put forward some physical explanations.

What I did is to show from a general point of view the reasons for a violation of the Nernst theorem and I proposed a easy way to check if a particular model can produce such a violation. Concerning the point 2. and 3. I put into evidence that the these “difficulties” are a characteristic feature of the Lifshitz formula for the Casimir force and are quite model-independent once we assume that the TE polarization does not contribute to the force in the low frequency limit. This means that, within this assumption, something in the Lifshitz formula itself remans to be understood that is independent of the models we use to describe the optical response for a medium.

Now, again, although there are some simple physical explications and independent calculations that seem to support the previous assumption, we have the strange discontinuity with the ideal case that remains unclear. In particular it seems that in the high temperature limit this is due to the fact that in the ideal case one sets the tangential component of the electromagnetic filed to zero and does not consider the thermal fluctuations which are in general present because of the microscopic and fluctuating structure of the medium.

Unlikely for the present time “high temperature” region is not accessible to experiment with a sufficient accuracy and, at first sight, the low temperature experimental results seem to be in agreement more with the ideal (contribution of both polarizations) case behavior than with the more realistic one (just one polarization). It worth stressing however that recently some author expressed some doubts [28, 22] on the claimed experimental accuracy of those experiment and the more realistic one could be also compatible within the error bar.

This last point shows clearly the confusion over the thermal correction of the Casimir effect and that there is still a lot of work to do.

The previous results will be submitted for publication.

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