Project Report - QUDEDIS Exchange Grant
- Dynamics of Ultra Cold Atoms in Disordered Potentials -

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I. DESCRIPTION OF THE WORK CARRIED OUT AND THE RESULTS OBTAINED

In this project, we have theoretically investigated ultra-cold gases in disordered potentials. The main aim was to study damped Bloch oscillations of Bose-Einstein condensates in disordered optical lattices. In particular, we have addressed the important interplay of disorder and interaction effects on the system dynamics. Our numerical results show, that interactions can either enhance or diminish the damping rate, depending on the strength of the nonlinearity. For strong nonlinearity, the damping rate is larger then in the single particle case, due to effects of the so-called dynamically instability[1, 2]. In the regime of weak nonlinearity, the damping rate may be significantly smaller then in the single particle case due to an interaction induced *dynamical screening* of the disorder potential.

In continuation of my previous work in Barcelona (QUDEDIS Grant 1365), we have expanded our numerical investigations of Bloch oscillations in the presence of disorder and interactions. As a main result, we have moreover developed two analytical approaches to this problem. They can serve to qualitatively explain the numerically observed damping rates. This work was carried out in close collaboration with the theoretical research group of Prof. Santos and the experimental research group of Profs. Ertmer/Arlt at Leibniz Universät Hannover, and serves as a direct guide for the current experiments there.

In a second line of research, we have studied the introduction of a random field in a system with continuous symmetry. A two-component Bose-Einstein condensate with a random Raman coupling between the components served as a model system for our investigations. In my previous stay in Barcelona (QUDEDIS Grant 1365), we have studied such systems in 1D and in 2D, and we have observed the emergence of order, if the random field breaks the continuous symmetry. As a result of this Exchange Grant, our studies could be extended to the 3D case, confirming our findings in lower dimensions.

The results of this work will be published in two papers. The results on random field induced order, have already been submitted to Phsical Review Letters [3]. Our results on damped Bloch oscillation will be submitted these days.

In the following we describe our work on random field induced order (section II.) and on the dynamics of Bloch oscillations in disordered lattice potentials (section III.).

II. RANDOM FIELD INDUCED ORDER IN A TWO-COMPONENT BOSE-EINSTEIN CONDENSATE

Systems with continuous symmetry do not exhibit long range order in two dimensions for any finite temperature. This is a consequence of the famous Mermin-Wagner-Hohenberg theorem [4]. As recently proposed [5], the introduction of a random field can lead to the appearance of order, if the field breaks the continuous symmetry.

We have studied this effect with a two-component Bose-Einstein condensate and random coupling between the components. This system can be experimentally realized by two internal atomic states and a random Raman coupling between them. The dynamics of the condensate wavefunctions Φ and Ψ is determined by the coupled equations

$$\begin{aligned} i\hbar\partial_t\Psi &= \left[-\frac{\hbar^2}{2m_1}\nabla^2 + V_1 + g_{11}\left|\Psi\right|^2 + g_{12}\left|\Phi\right|^2\right]\Psi + \frac{\Omega}{2}\Psi \\ i\hbar\partial_t\Phi &= \left[-\frac{\hbar^2}{2m_2}\nabla^2 + V_2 + g_{22}\left|\Phi\right|^2 + g_{12}\left|\Psi\right|^2\right]\Phi + \frac{\Omega}{2}\Phi, \end{aligned} \tag{1}$$

where V_i, m_i and g_{ij} denote the respective external trapping potentials, atomic masses and interaction coupling constants. The Raman coupling of the two components is governed by the field $\Omega(\mathbf{r})$ and is assumed to be resonant and real-valued in the equations above. In the absence of the coupling field the system obeys a continuous symmetry, which is generated by translations of the global phase difference of the two condensate wavefunctions.

We have numerically propagated 3D initial condensate wavefunctions according to the coupled equations (1) in imaginary time to determine the ground state of the system. In the abscence of disorder, the ground state is degenerated in respect to a shift of the phase difference between the two components. However, the introduction of an arbitrarily small coupling field Ω with spatial disordered profile and zero meanvalue lifts this degeneracy and causes a phase difference, that fluctuates around $\pm\pi/2$. We have investigated this effect for a quasi-random, incommensurate sinusoidal field configuration $\Omega(x,y,z) \propto \sum_{u\in\{x,y,z\}}[\sin(u/\lambda_R) + \sin(u/(1.71\lambda_R))]$ with $\lambda_R = 4.68\mu m$. This coupling field can be experimentally realized by a so-called optical super lattice configuration. In Figure 1 we show the phase difference of the two condensate wavefunctions for the case of a spherical harmonic trap with frequency $\omega = 2\pi \times 30$ Hz and $N = 10^5$ particles. The phase difference indeed fluctuates around $\pi/2$ with very small amplitudes.

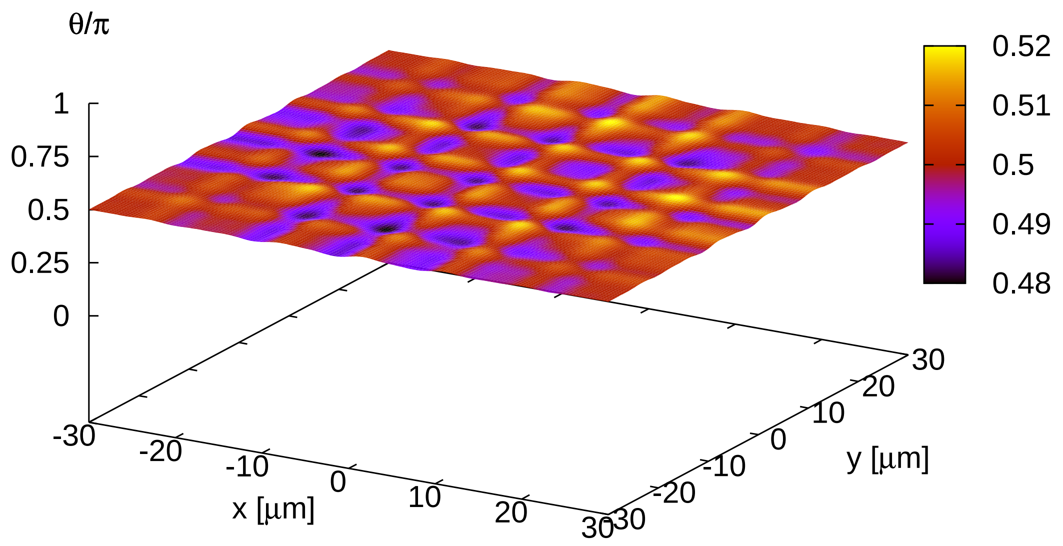

FIG. 1: Phase difference $\theta = \phi - \psi$ between the condensate wavefunctions in the presence of a random field of strength $\hbar\Omega_R \approx 5 \times 10^{-3}\mu$ in the plane $z = 0\,\mu m$.

This effect can be understood by considering the energy functional of the coupled system:

$$E = \int d^3r \left[\frac{\hbar^2}{2m_1} |\nabla\Psi| + \frac{\hbar^2}{2m_2} |\nabla\Phi| + V_1 |\Psi|^2 + V_2 |\Phi|^2 \right.$$
$$\left. + g_{11} |\Psi|^4 + g_{22} |\Phi|^4 + g_{12} |\Psi|^2 |\Phi|^2 + \Omega\Phi\Psi^* + \Omega\Phi^*\Psi \right]. \quad (2)$$

Let us denote the maximal amplitude of the Raman field by Ω_R. The density profile is only barely effected by the Raman coupling, if the chemical potential μ is much larger then the coupling field $\mu >> \hbar\Omega_R$. In this case, the effects of the random coupling on the ground state can be understood by considering the interplay of the kinetic terms and the terms containing the random field. The latter can be rewritten by inserting $\Psi = |\Psi| e^{i\psi}$ and $\Psi = |\Phi| e^{i\phi}$:

$$E_{rf} = \int d^3r \, 2\,\Omega(\mathbf{r}) \; |\mathbf{\Phi}| \, |\mathbf{\Psi}| \cos(\phi - \psi). \quad (3)$$

This term is minimized, if the sign of $\cos(\phi - \psi)$ is opposite to the sign of $\Omega(\mathbf{r})$. Since $< \Omega >= 0$ and $\Omega(\mathbf{r})$ is spatially rapidly varying, this can be achieved only at the cost of kinetic energy. The total energy is in this case minimized by $< \cos(\phi - \psi) >= 0$ and very small fluctuations around this value. This behaviour can be well observed in Figure 1.

Note, that this phase-ordering effect is an analogue to the *random-field-induced-order* which has been recently discussed for lattice spin models [5]. However, in the case of quasi-randomly coupled Bose-Einstein condensates, the effect turns out to be more pronounced. We have also perfomed numerical simulations for purely random coupling fields. In these cases, the phase difference fluctuates with much larger amplitude around $\pm\pi/2$. However, the average over many realizations provides an average phase difference of strictly $\pm\pi/2$, clearly showing the robustness of the effect. A detailed discussion of the differences between quasi-random and purely random coupling fields or of the analogy to lattice spin models exceeds the scope of this report. More details can be found in [3].

III. BLOCH OSCILLATIONS IN DISORDERED LATTICES POTENTIALS

We have investigated the use of ultra-cold gases to study the dynamics of Bloch oscillations in disordered periodic potentials. Bloch oscillations constitute one of the most fundamental quantum phenomena for particles in periodic potentials. Under the influence of a constant force the particle undergoes a coherent oscillatory motion instead of being linearly accelerated [6]. The experimental observation of Bloch oscillations has already been achieved in ultra-cold gases [7] by the help of accelerated optical lattices. Optical lattices form perfect periodic potentials which are almost free from any kind of imperfections. Hence, the life-time of Bloch oscillations in these structures is by orders of magnitudes larger then for electrons in solid state samples. This makes ultra-cold gases very promising candidates to experimentally study damping mechanisms and dynamics by introducing controllable disorder in the system. Our goal was to set the theoretical background for those investigations. In particular, we have analyzed the interesting interplay between disorder and nonlinear interactions that is of essential relevance for Bloch oscillations of Bose-Einstein condensates in disordered lattice potentials. The work has been carried out in close collaboration with Prof. Santos at the Institute of Theoretical Physics at Leibniz Universität Hannover and serves as a direct guide to the experiments, which are currently performed in the group of Profs. Ertmer/Arlt at the Institute of Quantum Optics at Leibniz Universität Hannover.

Typical experimental investigation deal with condensates in 3D trap configurations, and we have previously shown (see Grant Report 1365) that radial excitation play an important role for the dynamics of Bloch oscillations in this regime. However, a good understanding of the intriguing interplay between disorder and interactions may be obtained by considering a quasi-1D situation. Let us assume that a condensate is so strongly confined by an harmonic trap of frequency $\omega_\perp$ in the xy plane, that the chemical potential μ is much smaller then the transverse level spacing $\mu << \hbar\omega_\perp$. Under these conditions, the $1D$ dynamics of the condensate wave function Φ along the z-axis, is given by the Gross-Pitaevskii-equation

$$i\hbar\partial_t\Phi = \left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V(z) + g|\Phi|^2\right]\Phi, \tag{4}$$

where m denotes the atomic mass, and $g = 4\pi a_s\hbar\omega_\perp$ is the 1D coupling constant, with a_s the s-wave scattering length. $V(z)$ denotes an external potential of the form

$$V(z) = m\omega^2 z^2/2 + s\, E_r \sin^2(\bar{k}z) - Fz + V_{dis}(z). \tag{5}$$

i.e. a superposition of an axial harmonic trap with frequency ω, a tilted potential with slope F, a disorder potential $V_{dis}(z)$, and a lattice potential of periodicity $d = \pi/\bar{k}$, and depth s in units of the recoil energy $E_r = \hbar^2\bar{k}^2/2m$. The condensate ground state in the superimposed harmonic and lattice potential serves as the initial state for our simulations of the dynamics.

Figure 2 shows the averaged position $\langle z(t)\rangle = \int dz\, \Phi^* z\Phi$ for a Rubidium condensate, with $\omega_\perp = 2\pi \times 200$ Hz, $d = 412.5$ nm, $s = 5$ and $Fd/E_r = 0.05$, for different particle numbers N, axial trap frequencies ω, and disorder depths. As disorder potential, we consider Gaussian noise with correlation length $L \simeq 3.3\,\mu$m. We define the disorder depth, V_Δ as twice the standard deviation from its mean value.

The upper panel of Figure 1 shows the averaged BEC position for fixed nonlinearity but varrying disorder depth. It is evident, that the disorder introduces a clear damping of the oscillation amplitudes, and we have previously discussed methods for the experimental observation of this phenomenon (see Report 1365). Let us focus here on the interesting effects of interactions on the damping dynamics, shown in the lower panel of Figure 1. In this plot, the disorder depth is kept constant, and the nonlinearity is varried. For large nonlinearity (see the curve for $N = 700$), a stronger damping then in the single particle case is observed. This interaction-induced damping is related to the so-called dynamical instability [1, 2]. This instability occurs when the quasi-momentum reaches the outer parts of the Brillouin zone and small perturbations of the condensate wave function grow exponentially in time [2]. This mechanism becomes predominant with growing nonlinearity, strongly damping the Bloch oscillations.

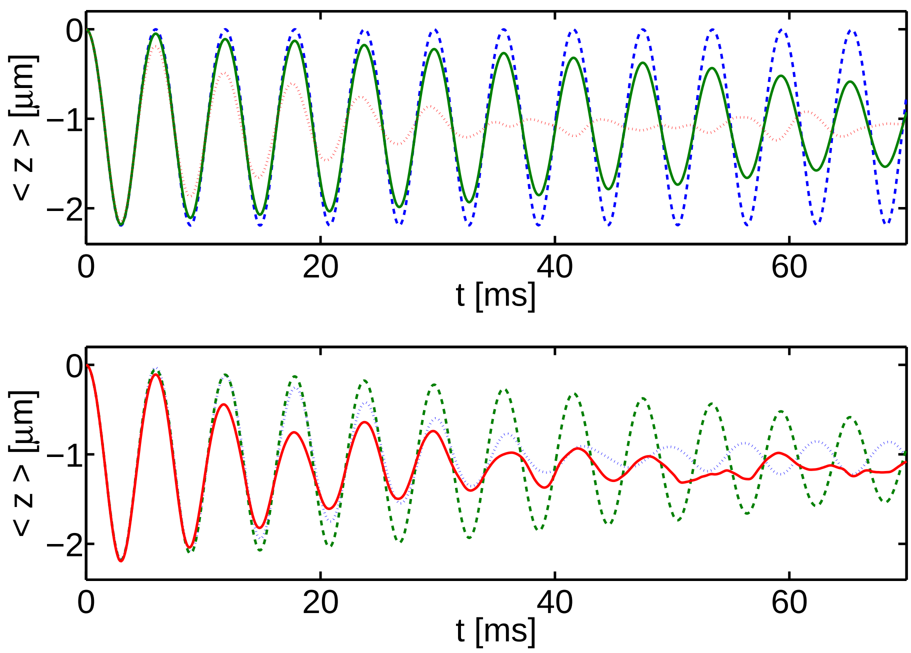

FIG. 2: Averaged position of a BEC undergoing BOs. Top frame: $N = 350$ particles, $\omega = 7\,Hz$, and disorder depths $V_\Delta/E_r = 0$ (dashed), 0.02 (solid), and 0.04 (dotted). Bottom frame: $V_\Delta/E_r = 0.02$ for $N = 1$, $\omega = 3.5\,Hz$ (dotted), $N = 350$, $\omega = 7\,Hz$ (dashed) and $N = 700$, $\omega = 10\,Hz$ (solid). The trap frequencies were adjusted to match the initial wavepacket widths.

On the contrary, the Bloch oscillation damping may be significantly reduced compared to the single particle case for weak nonlinearity (see the curve for $N = 350$). This effect is caused by an interaction-induced *dynamical screening of the disorder* and can be qualitatively understood with a semi-classical description of the Bloch oscillations. In the regime of weak nonlinearity, we can assume that the dynamics occurs within the lowest Bloch band, and that the dynamical instability is irrelevant on the time-scales considered. Let us denote the exact mean field potential obtained by solving the Gross-Pitaevskii equation by $V_{mf}(z,t) = g|\Phi(z,t)|^2$, and consider the effective single particle problem, for a particle in the lowest Bloch band under the influence of $V_{mf}(z,t)$, the tilting force F and the disorder $V_{dis}(z)$. The single particle Hamiltonian is given by $H_{eff} = -2J\cos(\hat{k}d) - F\hat{z} + V_{dis}(\hat{z}) + V_{mf}(\hat{z},t)$. In the quasi-momentum picture, $\hat{k} \to k$, and $\hat{z} = i\partial/\partial k$. We assume that the bare amplitude of the Bloch oscillations, z_{BO} is much smaller then the spatial spread of the initial wave function. This in turn implies a very narrow initial momentum distribution, centered at k_0, so that, in the absence of disorder and nonlinearity (zero-order solution), $\hat{z}(t) \simeq \hat{z}(0) - z_{BO}\cos[(Ft/\hbar + k_0)d]$. The full Heisenberg equation for $\hat{k}$ reads $d\hat{k}/dt = F - (\partial V_{dis}/\partial z)(\hat{z}) - (\partial V_{mf}/\partial z)(\hat{z}, t)$. We solve it pertubatively by inserting the zero order solution. Again, assuming a sharp initial momentum distribution we obtain $\hat{z}(t) \simeq \hat{z}(0) - z_{BO}\cos\{\frac{Ftd}{\hbar} + k_0 d - \frac{d}{\hbar}\int_0^t dt'[(\partial V_{dis}/\partial z)(\hat{z}) - (\partial V_{mf}/\partial z)(\hat{z}, t')]\}$. In order to calucalte the dephasing rate the latter expression has to be averaged over the initial spread of $\hat{z}(0)$. A reasonable estimate of the rate is $\gamma^2 \simeq \frac{d^2}{\hbar^2 t^2}\langle\{\int_0^t dt'[(\partial V_{dis}/\partial z)(\hat{z}(0)) + (\partial V_{mf}/\partial z)(\hat{z}(0), t')]\}^2\rangle$. Note that, when acting alone, both disorder and nonlinearity lead to the damping of the Bloch oscillations. However, when acting together, they may compensate each other if the product of the time averaged forces due to disorder and nonlinearity averaged over $\hat{z}(0)$ is negative, qualitatively explaining the dynamical screening of the disorder observed in the lower panel of Figure 2.

IV. FUTURE COLLABORATION

The research on Bloch oscillations in disordered optical lattices serves as a direct guide to the experiments, which are currently performed at the Institute of Quantum Optics, Leibniz Universität Hannover. This provides direct benefit to the researchers home institution at Hannover. The cooperation between the research groups of the Profs. Santos/Ertmer/Arlt at Leibniz Universität Hannover and the host shall be continued. This future cooperation will particularly address the interpretation of the experimental results on damped Bloch oscillations as well as subsequent experimental and theoretical research projects on disordered quantum systems.

V. FURTHER COMMENTS

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