HARMONIC AND COMPLEX ANALYSIS AND ITS APPLICATIONS

> *We can't solve problems by using the same kind of thinking we used when we created them*
> Albert Einstein

Abstract

The main idea of this project is to establish a fruitful cooperation between two scientific communities: analysts with a broad background in Complex and Harmonic Analysis, and Mathematical Physics, and specialists in Physics and Applied Sciences. Harmonic and Complex Analysis is a well-established area in mathematics. Over the past few years, this area has not only developed in many different directions, but has also evolved in an exciting way at several levels: the exploration of new models in mechanics and mathematical physics and applications has at the same time stimulated a variety of deep mathematical theories. The proposed ESF PESC Programme is a European networking activity aimed at promotion of scientific cooperation at the European and international levels; scientific mobility and integration of the national activities and groups with complementary backgrounds and expertise; and research training of younger scientists by doctoral scholarships and post-doctoral fellowships. Our project is a multidisciplinary programme at the crossroads of mathematics and mathematical physics, mechanics and applications, that proposes a set of co-ordinated actions for advancing in Harmonic and Complex Analysis and for increasing its application to challenging scientific problems. Particular topics which will be considered by this Programme include Conformal and Quasiconformal Mappings, Potential Theory, Banach Spaces of Analytic Functions and their applications to the problems of Fluid Mechanics, Conformal Field Theory, Hamiltonian and Lagrangian Mechanics, and Signal Processing. This project includes scientific groups from Austria, Finland, France, Germany, Ireland, Norway, Spain, Sweden, Switzerland and the UK. The proposed Programme will have partnership with other European and non-European networks, in particular following the scheme ESF-NSF-INTAS. The Programme will have a Steering Committee with a Secretariat based at the University of Bergen, Norway. Activities planned for the period of this Programme include organization and support of conferences, joint seminars and workshops, various visiting programs and fellowships for younger researchers.

Keywords: complex and real analysis, potential theory, mathematical physics, fluid mechanics, conformal and quasiconformal mapping, Laplacian growth, Stokes flow, Riemannian and non-Riemannian geometry, Hamiltonian systems

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1. Introduction

Looking back over the years of the history of physics and mathematics one observes that many important discoveries made in the past have been understood and recognized decades later, and their mutual connections and positioning in modern science have been reasoned and caused by the development of advanced mathematical tools. Three examples of particular importance may be given. The following works carried out at the end of the XIX - beginning of XX century, have influenced the development of modern mathematics and physics and caused a real boom at the end of the XX century. They are (in chronological order) a paper by Diederik Johannes Korteweg and his student Gustav de Vries (published [1] in *Philosophical Magazine, 1895*), a small note [2] placed in *Nature, 1898* by Henry Selby Hele-Shaw, and a seminal work [3] by Albert Einstein, one of the five papers that changed the face of physics, published in *Annalen der Physik, 1905*.

The KdV equation that originally described the "solitary wave", discovered by the Scottish engineer John Scott Russell about half a century earlier, becomes the condition over the potential of the spectral stability of the Schrödinger operator, eigenvalues of which lead to energy quantization for bound systems. The existence of quantized energy levels is determined experimentally by observation of the energy emitted or absorbed when the system makes a transition from one level to another.

Hele-Shaw's experiment describing the evolution of an interface between two immiscible fluids of different viscosity in a narrow channel is nowadays a powerful instrument of investigation of viscous flow in porous media. Recently, it has become clear that its mathematical formulation as the Laplacian growth problem admits a Hamiltonian formulation and, in fact, is equivalent to finding a particular solution to the dispersionless Toda lattice hierarchy that underlies 2D quantum gravity.

Einstein's special, and subsequent general theory of relativity explaining gravity as the curvature of spacetime has the deep and simply formulated Einstein's equation. It is a dynamical equation which describes how matter and energy change the geometry of spacetime, this curved geometry being interpreted as the gravitational field of the matter source. The solutions to this equation are metrics. In the two-dimensional case, Einstein's equation with a cosmological term describes a metric of constant negative Gaussian curvature. Recent advances in the theory of gravitation include Polyakov's discovery [4] that first-quantized bosonic strings propagating in $\boldsymbol{R}^d$ can be described as the theory of d free bosons coupled with with the 2-D quantum gravity, the theory formulated as a quantum geometry of Riemann surfaces, where the Liouville theory plays the principal role.

All these great results and discoveries are related intrinsically and they could not be made without the development and the progress in mathematical techniques and models, in particular, in Harmonic and Complex Analysis in a broad sense. General approaches of Harmonic Analysis provide deep insight into a vast array of modern topics of Real, Complex and Geometric Analysis and adjacent topics of Mathematical Physics and Mechanics. A wide variety of problems of Mathematical Physics and Fluid Mechanics can be intrinsically modeled in terms of geometric and analytic methods, in particular, when viewed under the Hamiltonian formalism. A deep interplay between Partial Differential Equations, Integrable Systems, manifold structures (Riemannian and non-Riemannian), Field Theory from one side, and Complex and Harmonic Analysis and Geometric Function Theory from the other one, allows us to concentrate our efforts in this project dedicated to creating new approaches and general methods of considering both these spectra of problems as well as to applications of these methods in Physics and Mechanics. Particular topics which will be considered by this programme include **Conformal and Quasiconformal Mappings, Potential Theory, Banach Spaces of Analytic Functions** and their applications to the problems of **Fluid Mechanics, Conformal Field Theory, Hamiltonian and Lagrangian Mechanics and Signal Processing**.

2. Objectives and Scope of the Programme

Preliminaries

Harmonic analysis and its influence on differential equations, complex and potential analysis has a long and distinguished history starting with the Laplacian equation. While the main ideas of this proposal are inspired by three seminal works by Korteweg and de Vries [1], by Hele-Shaw [2], and by Einstein [3], the main development and understanding of the topic has taken place only in the last couple of decades. It arises from certain classes of nonlinear differential equations known as **evolution equations.**

An important evolution equation characterizing Laplacian Growth is concerned with the problem that arises from the analysis of viscous flow in plane parallel shells, which one can consider as motion of petroleum in sections of porous medium. The problem of the boundary deformation in two dimensions is known as the free boundary problem (Laplacian Growth) in Hele-Shaw cells. The usual Dirichlet problem possesses new and unexpected features under the supposition of a moving boundary. Hele-Shaw first described his famous cell [2], that became a subject of deep investigation only more than 50 years later. Probably the most important characteristic of flows in such a cell is that when the Reynolds number based on gap width is sufficiently small, the Navier-Stokes equations averaged over the gap reduce to a linear relation analogous to Darcy's law, and hence to a Laplace equation for pressure. Through the similarity in the governing equations, Hele-Shaw flows are particularly useful for visualization of saturated flows in porous media, assuming they are slow enough to be governed by Darcy's law. The next important step after Hele-Shaw's pioneering note was made independently by Polubarinova-Kochina [5] and Galin [6] who developed a complex variable approach to the Hele-Shaw problem. Another important contribution has been made by Saffman and Taylor in 1958 [7] who discovered the long time existence of a continuum set of long bubbles between two parallel walls in a Hele-Shaw cell that have since been called the Saffman-Taylor fingers. A real burst of interest in this problem arose recently, as the great number of bibliographical references [8] testifies. Nowadays, the Hele-Shaw cell provides a powerful tool in several fields of natural sciences and engineering, in particular, matter physics, materials science, crystal growth and, of course, fluid mechanics.

The article [1] was based on De Vries' Ph.D. thesis, in which he proved that the "solitary wave", discovered and described by the Scottish engineer John Scott Russell [9] about half a century earlier, really could keep its form. The KdV-equation remained in obscurity until 1965, when Zabusky and Kruskal [10] discovered that two such solitary waves, or "solitons", emerge unchanged from a collision. The discovery of this remarkable stability property caused a - still ongoing - tide of research (sometimes called the "soliton revolution"). Two years later, Gardner, Greene, Kruskal and Miura invented the Inverse Spectral Transform to solve the Cauchy problem for KdV, and with this method Zakharov and Faddeev proved in 1971 [11] the complete integrability of KdV. Also, in 1970, Kadomtsev and Petviashvili [12] introduced the more general KP-equation to study transversal stability of the soliton solutions to the KdV equation. Many applications (including Laplacian Growth) can be thought of as the dispersionless limit of this or of the integrable Toda hierarchy constrained by a string equation. Whereas many applications of KP theory are in fluid mechanics, also vast areas of mathematics (algebraic curves, theta functions, vector bundles, Jacobian and Prym varieties, infinite-dimensional Grassmanians, commuting partial differential operators, algebraic and symplectic geometry, Lie group theory, differential geometry) and theoretical physics (quantum field theory, string and conformal field theory, quantum gravity, classical general relativity) opened up as a consequence of the basic research into the KP-equation. It is worth remarkering that the inverse scattering problem for dispersionless KP remains open.

The Liouville action and the Liouville equation play a key role in two-dimensional gravity. They are based on a Lagrangian given on the Riemannian geometry of the underlying space. Liouville action describes a highly non-trivial dynamics in quantum field theory and appears in Feynman's path integral that represents the transition amplitude between two quantum states of a system expressed as a sum over contributions from possible classical histories of that system. Complex transition functions appear naturally in the theory of evolution equations. The original formulation of quantum Liouville theory through a path integral has been given by Polyakov [4] in 1981. A thorough mathematical treatment appeared later in the works by Takhtajan and Zograf [19].

It is not very surprising that several quantum features appear in non-linear problems of hydrodynamics. The ubiquitous nature of integrable systems and evolution equations, in particular, those implied by the KdV, KP (or analogous), Liouville theory, and Laplacian Growth (Hele-Shaw problem), is by now well established. Nevertheless, the search for their common origin has not yet been successful. The present project will bring together and generalize a number of concepts of PDE, Harmonic, Complex, and Geometric Analysis that may have been implicitly known and applied by practitioners in the field of integrable systems and evolution equations (Laplacian Growth, in particular) but wich have not yet been placed on a broader and sound basis. The main idea of this project is to establish a fruitful cooperation between two scientific communities: analysts with a broad background in Complex and Harmonic Analysis and Mathematical Physics, and specialists in Physics and Applied Sciences.

Research Topics

Complex Analysis and Hamiltonian Systems. One of the main aims of this topic is to elaborate general approaches to evolution equations in Complex Analysis and Mathematical Physics. The crucial idea is to put ahead the **Hamiltonian formalism** as a general point of view that has great value for many methods of mechanics and mathematical physics.

After the discovery, in the early 1990's, of the fundamental role played by the KdV and the KP equations in the matrix and topological models of two-dimensional gravity, and, in particular, after the solution (Witten and Kontsevich [14,15]) of the old problem of the topology of the moduli spaces of algebraic curves in terms of KdV hierarchy, and the solution by (Sato and others [34]), in terms of the KP hierarchy, of the Schottky problem of characterizing the moduli space of curves inside the moduli space of abelian varieties, the theory of integrable systems and evolution equations are becoming one of the unifying principles in 21st century mathematics.

Recent works by Krichever, Zabrodin, Wiegmann, Mineev-Weinstein (see, e.g., [20-23]) established that the Laplacian Growth problem was equivalent to finding a particular solution to the dispersionless Toda lattice hierarchy- an analogue of the classical KdV hierarchy. The basic Polubarinova-Galin equation in the Hele-Shaw problem, in this language, becomes a version of the string equation that selects the solution to the hierarchy. **We plan to study deeply the connections between various formulations of the Hele-Shaw problem and the construction of a Whitham-Toda hierarchy.** In particular, **we shall investigate the problem of harmonic moments for simply and multiply connected phase domains in Hele-Shaw cells**, the connections with the weak formulation of the Hele-Shaw problem, and the problem of backward uniqueness and the branching of weak solutions. Another important class of evolutionary equations consists of systems admitting a Hamiltonian formulation on an infinite dimensional symplectic manifold. The classical Hele-Shaw problem can be described by means of parametric univalent maps smooth on the boundary, leading to the so-called Kirillov infinite dimensional space of diffeomorphisms of the unit circle which are closely connected with the Virasoro algebra. We will study the Laplacian evolution on the Kirillov space as on a general parametric space. The Taylor coefficients of these evolution maps generate another Hamiltonian system that can be treated by means, e.g., of the Pontryagin principle, or the Loewner equation and its generalizations. **We plan to establish the correspondence between these Hamiltonian systems.** Some initial results have been obtained in [16].

Conversely to the Laplacian growth problem, an important family of dispersionless integrable systems is obtained by looking at the case when solutions of the dispersionless KP equation are described by only finitely many Riemann invariants. Such solutions then depend on an N-parameter family of univalent conformal mappings of a domain, typically the upper half-plane, into the same domain with N non-intersecting slits running in along fixed non-intersecting Jordan arcs from its boundary [27]. The moving end points of these slits are the Riemann invariants. The construction of such families of conformal maps, at least with polygonal boundaries, has been carried out explicitly in some cases. In general, the governing equations are compatibility conditions between the N Loewner equations which govern the growth of each slit. **It remains unclear whether these governing equations themselves form an integrable system.**

The Liouville theory plays a key role in two-dimensional gravity, see e.g., [13]. It is based on a Lagrangian given on the base of Riemannian geometry of the underlying space. A Liouville action describes a highly non-trivial dynamics in quantum field theory and appears in connection with Feynman's path integral that represents the transition amplitude between two quantum states of a system expressed as a sum over contributions from possible classical histories of that system. Complex transition functions appear naturally in the theory of evolution equations. The original formulation of quantum Liouville theory through a path integral has been obtained by Polyakov [4] in 1981. A thorough mathematical treatment was made later by Takhtajan and Zograf [17-19]. It is not very surprising that several “quantum features” appear in non-linear problems of hydrodynamics, in KdV and integrable hierarchies, and in the Laplacian Growth problem, in particular. **We plan to construct and study various actions based on different Lagrangians giving energy characteristics of the Laplacian growth and of general subordination evolutions.** The principal tools for studying energy characteristics for the problems of electro-magnetic and gravity fields are the modulus of families of curves, and the capacity. **We plan to widen our results on Lagrangians and Liouville theory onto moduli and capacities.**

Tools from complex analysis have turned out to be useful in a more general set-up of metric measure spaces than in classical Riemannian manifolds. Metric measure spaces provide a proper platform for a new type of extremal problem where extremality is replaced by being close to an extremal function. Quasiminimizers and related concepts play an important role in these problems. Perturbations of the classical extremality theory and material behavior provide applications to this theory which is still its infancy. **We plan to create a potential theoretic approach to the theory of quasiminimizers**.

Evolution equations are naturally connected to the general theory of **dynamical systems**, which are of continuing interest to us. In particular, reversible systems will be considered and we are excited by the opportunities for exploiting dynamical methods in problems that are not originally dynamical, see, e.g. [33]. The bigger project is to find out as much as possible about the discrete dynamical systems that are conjugate to their inverses.

General problems in **joint approximation** will be discussed and new results will be obtained for the Dirichlet space and other spaces of analytic functions. Recently, Khavinson [31] suggested several open problems in **harmonic approximation** closely connected with the problems for quadrature domains discussed in the next section. Applications of harmonic approximation to classical problems of the boundary behaviour of harmonic and superharmonic functions will be considered. One such problem is to characterise the real-valued functions on the unit sphere that can arise as the radial limit function of a harmonic function on the unit ball. In the two-dimensional case the answer was provided by Gardiner in [32]. Another problem is to reveal a truly constructive solution (involving Newtonian capacity) to the problem of uniform harmonic approximation on compact sets in Euclidean space.

The interplay between **non-linear partial differential equations** and harmonic and complex analysis will be discussed. In particular, the present project will be focused on new trends in the development of KdV and larger hierarchies and integrable systems and their connections with problems of Mathematical Physics and Fluid Mechanics, as well as the connections between harmonic analysis and complex analysis of spaces of analytic functions.

The **probabilistic viewpoint** to problems in complex analysis and potential theory has proved to be very fruitful in the context of the classical theory. The applicability of probabilistic techniques in the present context will be fully investigated. In particular, this is related with quantum Laplacian growth which will be discussed in the next research topic.

Our research in complex differential equations aims to create interplay between complex analysis and mathematical physics via continuous and discrete Painlevé equations. Another aspect here is to apply complex function space theory to investigations of **complex differential equations in the unit disc**.

In geometric function theory, the present knowledge of quasi-conformal and quasi-regular mappings will be extended to a theory of **multivalued quasi-regular mappings**. Local structure considerations needed here will be applied to finding extremal metrics and moduli in non-orientable Riemannian manifolds. Moreover, similar extremal problem ideas in univalent functions will be applied to approximation theory.

We remark here that Geometrical and Complex Analytic viewpoints give complementary views of many problems in Integrable Systems Theory, geometrical questions of which are addressed by the ESF PESC Programme MISGAM.

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Free boundary problems. The preceding topic has already treated the Laplacian Growth (Hele-Shaw) problem. We will study **quadrature domains** (q.d.) and the **Schwarz function** approach that play an important role in the study of free boundary problems of Fluid Mechanics, in particular, Laplacian growth. An interesting feature of q.d. is that they unite nice ideas from complex analysis, potential theory, partial differential equations and moment problems, and operator theory in Hilbert space. On the one hand, a q.d. is characterized as a uniform lamina having very remarkable "gravitational" problems: it attracts bodies far away just as if it were a finite collection of point masses. On the other hand, it is characterized by the solvability of a certain overdetermined Cauchy problem (for the Laplace operator), so the subject at once led to new problems involving Newtonian gravitation and overdetermined PDEs. We will study Laplacian growth on curved surfaces to reveal new connections between complex analysis and Riemannian geometry. The differential geometry problem of finding a smooth surface with given curvature form having minimal area under a normalizing condition also leads to deep complex analysis questions regarding the possible appearance of extraneous zeros in the Bergman kernel function.

The Schwarz function approach will be used to treat **selection problems** in Hele-Shaw flow. It was recently conjectured [29] that the generic solution breakdown for Hele-Shaw suction problems with vanishingly small surface tension is via the free boundary entering the point sink as a wedge of uniquely selected angle. The hypothesised criterion for wedge-angle selection is that the Schwarz function of the associated zero-surface tension solution with this breakdown has a repeated zero in its derivative. We plan to put this hypothesis on a sounder footing, using methods of asymptotics beyond all orders in the complex equation governing Schwarz function evolution in Hele-Shaw flow. In addition, growing circular bubbles in a Hele-Shaw cell undergo fingering instabilities, analogous to Saffman-Taylor fingers (see, e.g., [30]). Such fingers widen as they grow radially, undergoing further tip-splitting instabilities. We plan to investigate the large-time structure of this radial fingering pattern. We shall also study non-Newtonian Hele-Shaw flows governed by the p-Laplacian equation that have many industrial applications, e.g., plastic injection-moulding processes.

Another object to be studied is **quantum Laplacian growth** which is a random measure process in the complex plane and that arises in the theory of electronic droplets confined to a plane under a strong magnetic field.

There are many striking parallels between the Laplacian growth and two-dimensional **Stokes flows** with a free boundary which is governed by the biharmonic equation. Many of the same techniques, including complex variable methods and quadrature domains, can be used to approach both. We shall be interested in the problem of doubly-connected Stokes flows. It is known that, when the pressures external to each free boundary are the same, and when two constants of integration are set to be equal, exact solutions exist to the free boundary problem that are represented by loxodromic conformal maps from an annulus [28]. We plan to use complex variable methods, including a variant of the Baiocchi transform, to investigate the structure of solutions in the more general case, when the external pressures and constants are not equal.

Researchers: A.Björn, J.Björn, M.Ben Amar, D.Crowdy, L.Cummings, Ch.Elliott, J.Escher, J.Gibbons, B.Gustafsson, H.Hedenmalm, S.Howison J.King, A.Lacey, J.-P.Lohéac I.Markina, M.Oliver, J.Ockendon, G.Richardson, A.Sebbar, H.Shahgholian, H.Shapiro, S.Shimorin, A.Vasil'ev.

Hamiltonian Mechanics on non-Riemannian structures. Riemannian manifolds and Riemann surfaces, in particular, are the geometric medium for the problems in classical mechanics. The corresponding medium for quantum mechanics is sub-Riemannian manifolds. Recently the Hamiltonian formalism came into play in the study of sub-Riemannian manifolds and, in particular, has been successfully applied for Heisenberg groups [24]. The geometry of sub-Riemannian manifolds is defined by the properties of the underlying subspace of the tangent space [25]. **We plan to investigate the uniqueness problem and the regularity of geodesics on sub-Riemannian manifolds.** In particular, we shall be interested in the so-called singular geodesics that do not appear as solutions to the appropriate Euler-Lagrange equations. Note that geodesic curves, being the projection of the solutions of a Hamiltonian system to the space of generalized coordinates, do not coincide, in general, with the shortest curves in the problem of distance. We shall use the Hamilton – Jacobi theory of bicharacteristics to study the geometry induced by the sub-Laplacian, which is the sum of squares of underlying basis vector fields of sub-Riemannian manifold. We aim to find the heat kernel for the heat equation with potentials. We shall write the Euler - Lagrange system of equations for the Lagrangian and try to characterize the system qualitatively from the conservation laws point of view. In general, these systems cannot be solved explicitly. However, for simple equations, one may characterize the solutions by finding the first integral of motions. We shall consider some examples of geodesics on such sub-Riemannian manifolds as Grusin manifolds, Heisenberg and Carnot groups of H-type [24-26]. **Geodesics on Heisenberg groups describe the trajectories of particles defined by the Lorentz equation, so new interactions with particle physics are expected to be found.** Other major questions in this topic are related to geometric measure theory: sharp isoperimetric and Sobolev inequalities, symmetrization, parameterization of surfaces and rectifiability of higher codimension submanifolds.

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Inverse scattering problems for non-linear configurations. Inverse scattering problems are (now) a classical topic in spectral analysis of differential operators. They also provide one of the main tools for the study of integrable systems, for one of the main steps of the solution is related to reconstruction of (mechanical, say) characteristics of a system from its spectral characteristics. Classical tools from complex and real analysis (such as Hilbert transform and spaces of analytic functions) are widely used for studying this problem and in many cases the inverse scattering problem serves as a motivation for setting new questions in these areas. Inverse scattering problems are studied in detail for linear configurations such as infinite and semi-infinite strings (though a lot of important and interesting questions remain open in this case) while for systems having more complicated geometry (for example a "standard web": several semi-infinite strings connected through a system of strings of finite length) even the basic questions are not answered yet. Here are some questions of this kind: **how do the equations of inverse scattering problems look in this case; what geometrical information can be recovered from the scattering data, how can the boundary conditions depend upon spectral parameters?** We are planning to consider these questions as well as some related ones. Together with the standard tools related to inverse scattering problems we expect to use techniques related to rational matrix functions, and in particular realization of Nevanlinna matrix functions, and also some other techniques from the classical complex analysis.

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Spectral problems for Landau equations with impurities. The classical Landau equation describes motion of an electron through an ideal crystal. It is known to have a purely discrete spectrum, each eigenvalue corresponding to an infinite-dimensional eigenspace. In the case when the crystal contains some randomly located impurities the structure of the spectrum may be different. The difference depends upon the density of impurities. Physicists expect that there exists an increasing sequence of critical levels such that, when the density of impurities exceeds the current level, the point spectrum corresponding to this level is replaced by an absolutely continuous one. For the lowest energy level (that is for the smallest eigenvalue) this effect will be described. The corresponding techniques include uniqueness theorems for functions from the Bargman-Fock space as well as the study of entire functions with randomly located zeros. Investigation of the similar problem related to higher levels leads to a very interesting uniqueness problem for functions from the Bargman-Fock space: in this problem the expression which vanishes in a system of points includes combination of a function, its derivatives and also some polynomials of z conjugated. Amazingly enough the same problem appeared in recent studies, motivated by needs of Gabor analysis, namely the frame properties of Hermitian polynomials in $L^2(R)$ were studied. A similar problem (in the Bergman space setting) appears in the study of self-commutators of cyclic subnormal operators. We are planning to study this uniqueness problem and apply the results to the questions, mentioned above.

Researchers: A.Borichev, H.Hedenmalm, Yu.Lyubarskii, E.Malinnikova, N.Nikolski, F. Pérez-González, K.Seip, A.Stray.

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<u>*Previous Results*</u>

The participants of the present proposal have published numerous papers and books on topics related to the proposal. In particular, the principal investigators have, during 5 last years, published 13 monographs and 676 papers, and edited 28 collections of papers. Below is the list of some (only a few) representative works published by the principal investigators during 5 last years:

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- J.Heinonen, T. Kilpeläinen, *BLD-mappings in $W_{2,2}$ are locally invertible*, Math. Ann. **318** (2000), no. 2, 391-396.
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<u>*Principal Investigators*</u>

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Linköping University: Gunnar Aronsson, Anders Björn, Jana Björn, Lars Hedberg, Vladimir Kozlov
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University of Geneve: Stanislav Smirnov
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University of Nottingham: Linda Cummings, James Langley, John King, Giles Richardson
University of Oxford: Sam Howison, John Ockendon
Loughborough University: Rod Halburd
Imperial College London: J.Milne Anderson, Darren Crowdy, John Gibbons, Walter Hayman
Heriot-Watt University: Andrew Lacey
University of Sussex: Charles Elliott

There is also a number of younger researchers, doctoral students and post-docs. The Programme will have associated researchers, who are involved partially in its work, e.g., Christian Pommerenke, emeritus professor of the Technische Universität Berlin.

Scope of the Programme

There exist traditionally strong groups in Harmonic and Complex Analysis, Mathematical Physics and Fluid Mechanics in European countries. The main scope of the Programme is to unify the efforts of these groups as well as of individual researchers on this exciting interdisciplinary project and to create an encouraging environment for young researchers, doctoral students and post-docs. An important goal of the proposed Programme is to carry out an interdisciplinary research jointly with specialists in Physics, Mechanics, Applied and Industrial Mathematics, creating methods and results that will have application in adjacent sciences, and possibly, in industry. The proposed Programme will provide an environment for fruitful and quick exchange of techniques, methods and results with the ultimate goal to elaborate unifying methods and a common vision of problems of Harmonic and Complex Analysis inspired by the problems of Mechanics and Mathematical Physics.

4. European Added Value

Harmonic and Complex Analysis is an established area of Mathematics. However recently, inspired by several problems of Mathematical Physics and Mechanics, a new rapidly developing direction of this area has been seen, which has relevant overlaps with other important areas of Mathematics, Mechanics and Theoretical Physics, as well as several industrial applications (oil recovery and chemistry in particular). This project promotes the dialogue and collaboration between theoretical mathematicians and specialists from applied sciences. The Programme aims to be an important element of European science within this stream. Currently, more than two hundred scientists permanently work within European institutions in this area as well as around three hundred doctoral students. Quick exchange of the latest results, new ideas and the interdisciplinary training and collaboration at a competitive level are absolutely necessary to maintain the leadership of European science in this area that will yield European progress in fundamental and applicable research and the development of new methods and technologies.

5. International Cooperation and Interaction

The proposed Programme will develop actively interaction and partnership with other European and non-European programmes, networks and organizations on a reciprocity basis. In particular, the Programme will have partnership with other networks such as a European partner network (a NordForsk network proposal “Complex Analysis and Potential Theory: new methods and applications”, a network for Nordic countries), and the NEST project GALA: “Geometric analysis in Lie groups and applications”, (coordinators: Giovanna Citti, Bologna). The present proposal will follow the scheme ESF-NSF-INTAS: the existing Focus Research Group NSF “Geometric Function Theory: From complex functions to quasiconformal geometry and nonlinear analysis”, (coordinators: Mario Bonk and Juha Heinonen (University of Michigan, Ann Arbor, USA); the USA partners (who are applying at the same time for a NSF DMS network “Connections”, coordinator Der-Chen Chang, Georgetown University, Washington DC); former SU countries (two INTAS grant proposals “Hard problems of complex analysis and related topics”, coordinator prof. Håkan Hedenmalm, KTH, Stockholm, Sweden, and “Analysis on metric spaces and PDE”, coordinator prof. Olli Martio, Finnish Academy of Sciences, Helsinki, Finland)..

6. Students, Post-docs and Visitors

Visiting senior researchers of high scientific level will strengthen all parts of our project. We plan to pursue collaborative research where junior staff will play an important role. In particular, our project proposes grants and scholarships for younger researchers in the form of post-docs and Ph.D. fellowships.

7. Participation of Women, Minorities and Equal Rights

Presently, women form a part of our research groups at a senior level. There are also various female doctoral students and other women junior researchers associated with our group. However, the lack of female staff has been observed within the Programme. We adhere to a policy of equal opportunities. Therefore, women as well as minorities are especially encouraged to apply for the studentships and post-doc fellowships.

8. Programme Activities

The Programme includes the following planned activities:

- Short-term individual visits by the senior Programme participants and long-term visits (fellowships) by younger researchers, doctoral students and post-docs;
- Joint workshops, seminars and round-tables of the Programme participants with possible invitation of external scientists and experts, in particular, from applied sciences and industry;
- Organization of at least two larger conferences (the second and the last year of execution);
- Co-sponsoring relevant conferences and study centers;
- Establishing joint activities with related networks and international organizations;
- Establishing and maintaining of a WEB server that presents the European activities on Harmonic and Complex Analysis. It should become a standard link to related servers and it will have mirrors in the countries where the related networks have their centers;
- An annual committee meeting in conjunction with an ESF conference or workshop. This committee will coordinate the Programme activities and organization, management of grants and fellowships, and the preparation of an annual report to ESF.

9. Programme Budget

The Programme budget is prepared to reach the objectives and to operate the above activities. All figures are given in kEuros

Activity	2007	2008	2009	2010	2011
Workshops, seminars	50	20	50	50	20
Conferences	-	80	-	-	80
Fellowships	60	60	60	60	60
Short-term visits	40	30	40	40	30
Coordinators	5	5	5	5	5
TOTAL	155	195	155	155	195

APPENDIX 1: Principal Proposers

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APPENDIX 2: Short CV

Alexander Vasiliev, born 1 Apr 1962, graduated in 1984 from the Saratov State University (Russia), earned his Ph.D. (candidate of sciences) in 1987 from the same university, had a post-doctoral stay at the Fudan University, Shanghai (China), earned his D.Sci. degree (Habilitation) from the Novosibirsk State University (Russia), Docent in Mathematics (Moscow, 1993). He was appointed at the Saratov State University, Russia (1991-1998), the University Los Andes, Colombia (1998-2000), Technology University Federico Santa Maria, Chile (2000-2004), University of Bergen, Norway (2004-present). He is an author of about 50 research papers and two monographs (one is submitted). His current research interests are Conformal and Quasiconformal maps, Mathematical Physics, Potential Theory, Free Boundary Problems. 5 recent publications on the proposal theme are:

1. B.Gustafsson, A. Vasil'ev: *Nonbranching weak and starshaped strong solutions for Hele-Shaw dynamics.*-Arch. Math. (Basel) **84** (2005), 551-558.
2. B.Gustafsson, D.Prokhorov, A.Vasil'ev: *Infinite lifetime for the starlike dynamics in Hele-Shaw cells.*- Proc. Amer. Math. Soc. **9** (2004), no. 9, 2661-2669.
3. A.Vasil'ev: *Univalent functions in two-dimensional free boundary problems*.- Acta Applic. Math. **79** (2003), no. 3, 249-280.
4. A.Vasil'ev: *Moduli of families of curves for conformal and quasiconformal mappings.* - Lecture Notes in Mathematics, vol. 1788, Springer-Verlag, Berlin- New York, 2002, 212 pp.
5. Ch.Pommerenke, A.Vasil'ev: *Angular derivatives of bounded univalent functions and extremal partitions of the unit disk.*- Pacific. J. Math. **206** (2002), no.2, 425-450.

Björn Gustafsson, born 30 Oct 1947, graduated in 1972 from the Royal Technology Institute (Sweden), earned his Ph.D. in 1981 from the same institute, Docent in Mathematics (Stockholm, 1987). He has been appointed at the Royal Technology Institute (Sweden) since 1972. He is an author of about 60 research papers and one monograph is submitted. His current research interests are Potential Analysis, Free Boundary Problems, Conformal Mappings, Partial Differential Equations. 5 recent publications (different from those co-authoring with A.Vasiliev) on the proposal theme are:

1. B.Gustafsson, H.S.Shapiro: What is a quadrature domain?- Quadrature Domains and Applications (eds. P.Ebenfelt, B.Gustafsson, D.Khavinson, M.Putinar), Birkhäuser, 2005, 1-25.
2. B.Gustafsson, M.Sakai: *On the curvature of the free boundary for the obstacle problem in two dimensions.*-Monatsh. Math. **142** (2004), 1-5.
3. B.Gustafsson, M.Putinar: *The exponential transform: a renormalized Riesz potential at critical exponent.*- Indiana Univ. Math. J. **52** (2003), 527-568.
4. B.Gustafsson, M.Putinar, H.S.Shapiro: *Restriction operators and doubly orthogonal systems of analytic functions*.- J. Funct. Anal. **199** (2003), 123-142.
5. B.Gustafsson, M.Putinar: *Linear analysis of quadrature domains II.* – Israel J. Math. **119** (2000), 187-216.

APPENDIX 3: Steering Committee

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No further application on this or a similar topic has been proposed to the ESF
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