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ESF PESC EXPLORATORY WORKSHOP

Dynamical Systems: from Algebraic to Topological Dynamics

ABSTRACTS



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Alseda, Lluis (Universitat Autonoma Barcelona, Spain) Rotation sets for graph maps of degree 1 (joint work with Sylvie Ruette)

For a continuous map on a topological graph containing a loop S it is possible to define the degree (with respect to the loop S) and, for a map of degree 1, rotation numbers. We study the rotation set and the periods of periodic points having a given rotation number. We show that, if the graph has a single loop S then the set of rotation numbers of points in S has some properties similar to the rotation set of a circle map; in particular it is a compact interval and for every rational α in this interval there exists a periodic point of rotation number α .

Bruin, Henk (University of Surrey, UK) **Unique physical measures for typical unimodal polynomials** (joint work with Weixiao Shen and Sebastian van Strien)

We show that for a one-parameter family of unimodal polynomials f_{λ} with even critical order ≥ 2 , for almost all parameters λ , f_{λ} admits a unique physical measure, being the equidistribution on a nonrepelling periodic orbit, and acip, or a measure supported on a Cantor set $\omega(c)$ where $f|\omega(c)$ is uniquely ergodic. The present text gives an abridged version of a paper called "Existence of unique SRB-measures is typical for unimodal families" by the same authors.

Fayad, Bassam (Universite Paris 13, France) **Ergodicity of disc diffeomorphisms and their rotation number on the boundary** (joint work with M. Saprykina)

We present a construction method providing area preserving weakly mixing diffeomorphisms on a manifold M equal to the 2-torus, 2-annulus or 2-disc. We denote by S_t the elements of (a particular) circle action on M. For any Liouville number α we construct a sequence of

area-preserving diffeomorphisms H_n such that the sequence $H_n \circ S\alpha \circ H^{-1}{}_n$ converges to a smooth weakly mixing diffeomorphism of M. The method is a quantitative version of the approximation by conjugations construction introduced by D. Anosov and A. Katok.

For M = 2-annulus or 2-disc, this result proves the following dichotomy: $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ is Diophantine if and only if there is no ergodic diffeomorphism of M whose rotation number on the boundary equals α . One part of the dichotomy follows from our constructions, the other is an unpublished result of Michael Herman asserting that if α is Diophantine, then any area preserving diffeomorphism with rotation number α on the boundary displays smooth invariant curves arbitrarily close to the boundary which clearly precludes ergodicity or even topological transitivity.

> Avila, Artur (College de France, France) Weak mixing for interval exchange transformations and translation flow (joint work with Giovanni Forni)

We show that a typical interval exchange transformation is either weakly mixing or it is an irrational rotation. We also conclude that a typical translation flow on a surface of genus g \geq 2 (with prescribed singularity types) is weakly mixing. The proof is based on a statistical parameter exclusion scheme which uses the dynamics of the renormalization operator on interval exchange transformations.



Le Calvez, Patrice (Universite Paris 13, France) Time-one maps of identity isotopies on surfaces

We will state some results about homeomorphisms of surfaces which are time one map of an isotopy starting from the identity. A natural example is a diffeomorphism associated to a time dependant vector field, periodic in time. We will see that certain theorems about time one map of flows induced by a time independent vector field may be extended to that situation. This includes for example results about existence of periodic orbits for Hamiltonian homeomorphisms, properties about linking numbers of periodic orbits,

Schmeling, Joerg (Lund University, Sweden & MPI, Germany) Random coverings, shrinking targets and hitting time statistics

In this talk we will discuss some characteristic quantities of dynamical systems. We will explain their role in a concrete question that started in probability theory.

Let M be a Riemannian manifold and x_n a point process. Let also a sequence of radii $r_n > 0$ be given. We consider the following questions:

- 1. What is a.s. the value of dim_H { $x \in M$: #{n: $x \in B(x_n, r_n) = \infty$ }?
- 2. What is a.s. the value of dim_H { $x \in M$: #{ $n: x \in B(x_n, r_n) < \infty$ }?

We address this question when the random process is generated by a dynamical system on the circle with an invariant measure. We discuss the connection to the moving target property and to return time statistics. We use multifractal analysis to analyse linear expanding maps of the circle and Diophantine approximation for rotations.

> Auslander, Joseph (University of Maryland, US & MPI, Germany) Recurrence in zero-dimensional spaces

Let T be a homeomorphism of the zero dimensional space X. Then it is well known and easily proved that the following are equivalent:

- 1. All points are positively recurrent.
- 2. All points are positively and negatively recurrent.
- 3. All points are almost periodic.

4. All points are almost periodic and the orbit closure relation is closed.

The situation is more complicated for general group actions. For certain classes of groups (and appropriate definitions of recurrence) equivalences as above hold.

Deninger, Christopher (Universität Münster & MPI, Germany) Leafwise cohomology of algebraic Anosov diffeomorphisms

We prove that the maximal Hausdorff quotient of the leafwise cohomology of a nilmanifold with respect to the unstable foliation of an algebraic Anosov diffeomorphism is finite dimensional. The theorem is a special case of a more general result on the leafwise cohomologies of nilmanifolds with respect to certain homogenous foliations. This is a report on joint work with Anton Deitmar.



Akin, Ethan (The City College, CUNY, US & MPI, Germany) Mycielski sets in dynamical systems

Given a residual subset P of the product space X x X we would like to find large subsets A in X such that A x A is a subset of P. Often the nature of P precludes the possibility that a residual subset A can be found. However, if X is a perfect Polish space then Mycielski sets A do always exist satisfying the required condition. This is a dense subset of X which is a countable union of Cantor sets. We describe the associated theory and provide some applications to topological dynamics.

Weiss, Barak (Ben Gurion University, Israel & MPI, Germany) Minimal sets for the horocycle flow and characterization of lattice (joint work with J. Smillie)

Let S be a compact orientable surface, let Q be the moduli space of quadratic differentials on S and let M be a stratum Q. We explicitly describe the minimal sets for the (Teichmüller) horocycle flow on M and on Q, and show that these correspond to horizontal cylindrical decompositions of S. We also complete work of Vorobets giving various necessary and sufficient conditions for $q \in Q$ to be a lattice surface (a.k.a. Veech surface). This involves a strengthening of the Veech alternative.

Boyle, Mike (University of Maryland, US & MPI, Germany) **The entropy theory of symbolic extensions** (joint work with Tomasz Downarowicz)

Extension entropy. Consider a homeomorphism T of a compact metric space X. By a symbolic extension of (X,T) we mean a subshift (Y,S) with a factor map φ onto the system (X,T). Given such a map, we define the symbolic extension entropy function h^{φ}_{ext} on the space M_T of T-invariant Borel probabilities, by setting $h^{\varphi}_{ext}(\mu) = \max\{h(S,\nu): \varphi \ \nu = \mu\}$.

Entropy structure. An entropy structure for (X,T) will be an allowed sequence of u.s.c. (upper semicontinuous) functions h_n on M_T , converging to the entropy function h, with all differences $h_n - h_{n-1}$ also u.s.c. The general determination of "allowed" is a complicated business, but here is one example which gives the right intuition. Suppose the system (X,T) admits a sequence of partitions P_n with *small boundaries* (the boundary of the closure of each partition element has μ measure zero for every $\mu \in M_T$, and with the maximum diameter of elements of P_n going to zero as $n \to \infty$. Then the sequence (h_n) defined by $h_n(\mu) = h(\mu, P_n)$ is an entropy structure for (X,T). There are more complicated constructions which provide entropy structures for any system.

Sex Entropy Theorem. (We use ``sex entropy" to abbreviate ``symbolic extension entropy.) Suppose (h_n) is an entropy structure. A bounded function E on M_T such that every E- h_n is u.s.c. is called a *superenvelope* of the entropy structure. The main result of [BD] is the Sex Entropy Theorem: a bounded function on M_T is a symbolic extension entropy function if and only if it is affine and a superenvelope of the entropy structure.

The function h_{sex}. The infimum of the symbolic extension entropy functions h_{ϕ} above is called the *symbolic extension entropy function* and denoted h_{sex} . This u.s.c. function is a very fine probe into the complexity of the system (X,T), and evidently is intimately connected with the way entropy emerges on ever smaller scales. The Sex Entropy Theorem leads to a variety of results, estimates and examples. For one thing: the maximum of h_{sex} need not be assumed on an ergodic measure. For another: the infimum of the topological entropies of symbolic extensions of (X,T) need not be achieved. For another: the infimum of the maximum of h_{sex} .



The subtlety of sex entropy. Starting with an entropy structure (h_n), there is an inductive construction of functions u_{α} (with $u_{\alpha} \le u_{\beta}$ when $\alpha < \beta$) such that $h_{sex}=h+u_{\alpha}$ if $u_{\alpha} = u_{\alpha} +1$. The construction always terminates at some countable ordinal α -- but the induction in general is transfinite. This indicates the subtlety of even the topological sex entropy of T, in contrast to the usual topological entropy.

Shrinking scales and entropy structures. As sex entropy is extremely subtle and is still only a reflection of how entropy emerges on shrinking scales, we appreciate the problem of finding appropriate structures to describe "emergence of entropy on shrinking scales". However, a theory strong enough to refine the sex entropy phenomena satisfies a stringent test. Downarowicz [D] has introduced (and will discuss in his talk) an elegant theory of entropy structure, and has developed it to apply to a wide range of approaches to entropy. The entropy structure emerges as a master entropy invariant, determining in particular the entropy function on measures and the set of symbolic entropy functions. It is a previously unseen ghost in the machine, ruling all of entropy; not always accessible, but accessible enough for theorems, examples and sometimes estimates.

Smooth examples. Building from [BD] and Newhouse's work on homoclinic tangencies, for $1 \le \alpha < \infty$, Downarowicz and Newhouse constructed certain generic families of C^{α} diffeomorphisms with topological symbolic extension entropy strictly greater than the topological entropy.

[BD] M. Boyle and T. Downarowicz, *The entropy theory of symbolic extensions*, Inventiones Mathematicae (2004)

[D] T. Downarowicz, *Entropy Structure*, J. d'Analyse, to appear.

[DN] T. Downarowicz and S. Newhouse, *Symbolic extensions in smooth dynamical systems*, preprint (2002).

Downarowicz, Tomasz (Wroclaw U. of Technology, Poland & MPI, Germany) Entropy structure

The investigaion of symbolic extensions reveals that it is important to know how quickly entropy of invariant measures converges to the entropy function as the resolution of measurement improves. We will introduce a meaningful topological invariant, called "entropy structure" which captures the "fault of uniformity" in the above convergence. Most of known entropy invariants, such as entropy itself, the Misiurewicz parameter h*, the symbolic extension entropy, and more, depend completely on the entropy structure. The entropy structure is defined in full generality in topological dynamical systems (actions of a continuous map on a compact metrizable space) and is computable in terms of several notions already existing in the literature.

Przytycki, Feliks (Institute of Mathematics, PAN, Poland) Entropy conjecture on nilmanifolds (joint work with W. Marzantowicz)

In 1974 Michael Shub asked the following question: When the topological entropy of a continuous mapping of a compact manifold into itself is estimated from below by the logarithm of the spectral radius of the linear mapping induced in the cohomologies with real coefficients? This estimate has been called Entropy Conjecture (EC). In 1977 I proved, jointly with Michal Misiurewicz, that EC holds for all continuous mappings of tori. Here we prove EC for all continuous mappings of compact nilmanifolds.



Vorobets, Yaroslav (Pidstryhach Institut, NASU, Ukraine & MPIM, Germany) Periodic geodesics on generic translation surfaces

Let M be a compact connected oriented surface endowed with a flat metric that has a finite number of conical singularities. The flat structure on M can be determined by an atlas of coordinate charts such that all transition functions are rotations or translations of Euclidean plane and chart domains cover the whole surface up to the singularities. If the atlas can be chosen so that all transition functions are translations, then M is called a translation surface. Translation surfaces are closely related to Abelian differentials on compact Riemann surfaces.

The moduli space of translation surfaces of genus p with n singular points is endowed with the natural structure of an affine orbifold of dimension 2(2p+n-1) along with a volume element.

We discuss the properties of periodic geodesics on translation surfaces that hold for almost all elements of the moduli space. These include quadratic asymptotics of various growth functions and uniform distribution of directions of periodic geodesics.

Fisher, David (Lehman College – CUNY, US & MPI, Germany) Effective estimates on invariant metrics and Zimmer's conjecture

Zimmer has made several conjectures concerning actions of large groups on low dimensional manifolds. His main motivation for the conjectures was that he could produce measurable Riemannian metrics on the manifold, invariant under the group action. The conjecture is that these invariant metrics are smooth. A major difficulty in the problem is that the existence of the metrics is proven in a highly non-constructive manner. I will discuss how I can produce L^2 invariant metrics for some cases of conjecture in an effective manner starting from an arbitrary metric. I will then discuss the reasons why this may lead to a proof of Zimmer's conjecture whenever the effective method is available.

Durand, Fabien (Université de Picardie Jules Verne, France & MPI, Germany) Eigenvalues of linearly recurrent dynamical systems

Linearly recurrent systems are minimal dynamical systems defined on Cantor sets with a recurrence behaviour "at most linear". Examples of such systems are minimal substitutive dynamical systems, some Sturmian systems, some "Cantor" interval exchanges, some Töplitz systems, etc We are interested in their continuous and L² eigenfunctions. We will give a characterization of the eigenvalues with a continuous eigenfunction.

McMullen, Curtis (Harvard University, US & MPI, Germany) Dynamics of SL_2(R) over moduli space

The closure of a complete geodesic in a finite-volume hyperbolic manifold M can take on a multitude of exotic shapes: for example, it can resemble a spider's web, with its local cross-section a Cantor set. On the other hand, if $f : H^2 \to M$ is a totally geodesic plane, then the structure of the closure of $f(H^2)$ is much simpler: it is always an immersed hyperbolic k-manifold of finite volume, $2 \le k \le n$. In this talk we present similar results for certain hyperbolic planes in the moduli space of Riemann surfaces \mathcal{M}_g , when g = 2.



Einsiedler, Manfred (University of Washington, US & University of Vienna, Austria & MPI, Germany) Invariant measures on SL(3,ℝ)/SL(3,ℤ) and Littlewood's conjecture (joint work with A.Katok and E. Lindenstrauss)

Dynamics on locally homogeneous spaces have interesting connections to number theory and diophantine approximations. By Ratner's work the case of unipotent actions is completely understood. Recent progress on the partially hyperbolic case has lead to more applications: The set of exceptions to Littlewood's conjecture in the theory of multidimensional Diophantine approximations has Hausdorff dimension zero.

Mirzakhani, Maryam (Harvard University, US & MPI, Germany) Ergodic theory of the earthquake flow on the moduli space of curves

In this talk we study the ergodic properties of the earthquake flow on the bundle of geodesic measured laminations by using a relationship between the earthquake flow and the Teichmüller horocycle flow. We use these results to find the growth of the number of simple closed geodesics on a hyperbolic surface.

Glasner, Eli(Tel Aviv University, Israel & MPI, Germany) Spatial and non-spatial actions of Polish groups

This talk presents results from papers by B. Tsirelson, B. Weiss and myself. Classical ergodic theory deals with measure (or measure class) preserving actions of locally compact groups on Lebesgue spaces. An important tool in this setting is a theorem of Mackey which provides spatial models for Boolean G-actions. We show that in full generality this theorem does not hold for actions of Polish groups. In particular there is no Borel model for the Polish automorphism group of a Gaussian measure. In fact, we show that this group as well as many other Polish groups do not admit any nontrivial Borel measure preserving actions.

Schmidt, Klaus (ESI & University of Vienna, Austria & MPI, Germany) Symbolic representations of toral automorphisms, Mahler measure and equivalence relations on sequence spaces

Following an idea originally due to Vershik one can construct, for every hyperbolic automorphism A of the n-torus X=Tⁿ, a continuous shift-equivariant group homomorphism from the space ℓ^{∞} of bounded two-sided integer sequences to X. By analyzing this homomorphism one can construct certain symbolic representations of A in terms of bounded subshifts of ℓ^{∞} .

In the special case where A is the companion matrix of a Pisot number \beta, the associated two-sided β -shift is such a symbolic representation, and if A is an arbitrary hyperbolic matrix one can find a symbolic representation which is a "generalized" β -shift, arising as a cross-section of a subrelation of the Gibbs-equivalence relation on ℓ^{∞} .

If A is nonhyperbolic but ergodic there cannot exist any reasonable symbolic representation of A. Nevertheless one can extend the above ideas to find a close connection between A and a (generalized) β -shift which allows, for example, the construction of A-invariant measures on X.

Much of this talk is based on joint work with Elon Lindenstrauss.



Pollicot, Mark (University of Manchester, UK) Pair correlations for closed geodesics

It is a classical problem to study statistical properties of hyperbolic dynamical systems (e.g., distribution of closed orbits, Birkhoff sums etc.). One of the standard examples is that of geodesic flows on negatively curved surfaces. We shall prove various distribution results involving shrinking intervals. This is illustrated by a pair correlation result, in which we count the asymptotic number of pairs of closed geodesics the difference of whose lengths lies in intervals, which are allowed to shrink at a sub-exponential rate.

Lemanczyk, Mariusz (Nicholas Copernicus U., Poland & MPI, Germany) Dynamical systems of probability origin and flows on surfaces

A flow $(T_t)_{t \in \mathbb{R}}$ of a standard probability Borel space (X, \mathcal{B}, μ) is said to have the ELF property if the weak closure of its times considered as Markov operators of L²(X, \mathcal{B}, μ) consists solely of indecomposable Markov operators. We will show that the ELF property can be related to the fact that a flow is of "probability origin". More precisely, Gaussian flows, flows coming from symmetric α -stable processes, Poisson suspension flows have the ELF property. On the other hand we will show that some classical smooth flows on surfaces have "sufficiently" decomposable Markov operators in the weak closure of its times, and in fact they turn out to be disjoint (in Furstenberg's sense) with ELF flows. The talk will be based on some joint works by Y. Derriennic, K. Fraczek, M. Lemanczyk and F. Parreau.

> **Denker, Manfred** (Georg-August-Universität, Goettingen, Germany) **Sierpinski walks**

The Sierpinski gasket is one of the best known examples of a self-similar fractal set and can be obtained as the limit set of a semigroup action generated by finitely many affine contractions. Considering this space as a Martin boundary of Markov chains one obtains a familiy of Martin metrics, different from the Euclidean metric.

Besides this representation we discuss fractal-geometric aspects of these Martin metrics on the Sierpinski gasket. In particular, we find metrics for which the Hausdorf dimension is not $\log N / \log 2$.

M.Denker and H.Sato: Sierpinski gasket as a Martin boundary I: Martin kernels. Potential Anal. 14 (2001), 211--232.

M.Denker and H.Sato: Sierpinski gasket as a Martin boundary II: The intrinsic metric. Publ. Res. Inst. Math. Sci. 35 (1999), 769--794.

E.B. Dynkin: Boundary theory of Markov processes (the discrete case). Russian Math. Surveys 24 (1969), 1--42.

A. Imai, Y. Kawasaki, H. Sato: Martin metrics on the Sierpinski gasket. Stoch. Dyn. 3 (2003), 267--277.

V.A. Kaimanovich: Random walk on Sierpinski graphs: Hyperbolicity and stochastic homogenization. Fractals in Graz 2001, Birkhauser, Basel, 2002, 145–183.



Hoffman, Christopher (University of Washington, US & MPI, Germany) Multiple invariant measures for C¹-expanding maps and gfunctions

Let f be an expanding map of the unit circle which is in C^{1+}_{ϵ} . It is well known that f has a unique invariant absolutely continuous measure and the natural extension of this system is isomorphic to a Bernoulli shift. However Quas has shown that if f is only assumed to be in C^1 then f may have multiple absolutely continuous invariant measures with a variety of ergodic theoretic properties.

To study C¹ expanding maps I consider the stationary processes generated by regular and continuous g-functions. We show that if the g-function has a certain modulus of continuity, then there is a unique stationary measure which is consistent with the g-function. This measure also has "nice" ergodic theoretic properties. But if it does not then it is possible that there are multiple invariant measures which are consistent with the g-function. These measures may have many possible ergodic theoretic properties.

This talk includes joint work with Noam Berger, Anthony Quas, and Vladas Sidoravicius.

Lind, Douglas (University of Washington, US & MPI, Germany) Amoebas and algebraic dynamics

This talk will survey some recent results in algebraic dynamics, emphasizing the role that amoebas, both complex and p-adic, play. It is intended to be introductory and accessible.

Keller, Gerhard (Universität Erlangen, Germany & MPI, Germany) **Sharkovsky-type theorem for minimally forced interval maps** (joint work with R. Fabbri, R. Johnson and T. Jäger)

We state and prove a version of Sharkovsky's theorem for forced interval maps in which the forcing flow is minimal (Birkhoff recurrent). This setup includes quasiperiodically forced interval maps as a special case. We find that it is natural to substitute the concept of 'fixed point' with that of 'core strips'. These last are frequently of almost automorphic type.

Liverani, Carlangelo (University of Rome Tor Vergata, Italy) Banach spaces adapted to Anosov systems

I present a unified approach to study the statistical properties of dynamical systems alternative to the one based on Markov partition. Such an approach yields stronger results in many cases and it is hoped to have a larger realm of applicability.

Mayer, Dieter (Institut für Theoretische Physik, Clausthal, Germany) Transfer operators for geodesic flows on modular surfaces and Hecke operators for period functions

There is a close relation between the eigenfunctions with eigenvalue $\lambda = 1$ of the transfer operator L_{β} on the line Re $\beta = 1/2$ for the geodesic flow on a modular surface $\Gamma \setminus H$, Γ a subgroup of finite index of the full modular group SL(2,Z), and the Maass wave forms for this group Γ . These eigenfunctions, the so called period functions of Lewis and Zagier, generalize the Eichler-Manin theory of period polynomials for holomorphic cusp forms for these groups. On the Maass wave forms one has the well known action of the Hecke operators describing some internal symmetry of the Laplace-Beltrami operator for these groups. Manin, Zagier and Choie extended the theory of Hecke operators to period



polynomials. By using basically the same method Mühlenbruch was able to derive these operators also for the period functions for the full modular group SL(2, \mathbb{Z}). Independently we determined these Hecke operators for the same group by a completely different method by using only the transfer operators for the congruence subgroups $\Gamma_0(n)$. This work has been extended by us in the meantime to all Hecke congruence subgroups $\Gamma_0(n)$. Hecke-like operators are thereby derived by constructing for any eigenfunction of the transfer operator with eigenvalue $\lambda = 1$ for $\Gamma_0(n)$ a whole family of new special eigenfunctions for the transfer operators for the groups $\Gamma_0(nm)$, m = 2,3,.... These eigenfunctions on the other hand determine again eigenfunctions for the original group $\Gamma_0(n)$ which depend linearly on the original eigenfunction. The linear operators constructed this way surprizingly are closely related to the Hecke operators. This has been shown quite recently by Mühlenbruch who constructed these operators for $\Gamma_0(n)$ by an explicit integral transform going basically back to Martin in his work on period functions for holomorphic automorphic forms of weight 1.

Baladi, Viviane (Institut de Math. de Jussieu, France) Anisotropic Sobolev spaces and transfer operators for Anosov diffeomorphisms

We consider the transfer operator associated to an Anosov diffeomorphism with $C^{1+\alpha}$ unstable foliation, and show good bounds on its essential spectral radius when acting on suitable Sobolev spaces. (If the foliation is C^{∞} , these are just standard anisotropic Sobolev spaces, otherwise we use Alinac's conormal distributions involving paradifferential operators.)

Ward, Thomas (University of East Anglia, UK & MPI, Germany) Orbit counting without hyperbolicity

Analogues of the prime number theorem and Merten's theorem are well-known for dynamical systems with hyperbolic behaviour. We describe work by Victoria Stangoe, in which a 3-adic extension of the circle doubling map is studied. The map has a 3-adic eigendirection in which it behaves like an isometry, and the loss of hyperbolicity leads to weaker asymptotic results on orbit counting than those obtained for hyperbolic maps.

Vdovina, Alina (Universität Bonn, Germany) Shift operators on buildings and noncommutative spaces (joint work with Matilde Marcolli)

We study dynamical systems associated with group actions on two-dimensional buildings. We construct an operator algebra coming from a two-dimensional subshift induced by the group action.

Sarig, Omri (Pennsylvania State University, US & MPI, Germany) Invariant Radon measures for horocycle flows on regular covers (joint work with F. Ledrappier)

The horocycle flow of a geometrically infinite hyperbolicsurface may preserve more than one non-trivial ergodic invariant Radon measure. We classify these measures in case the surface is a regular cover of a compact hyperbolic surface, by constructing a bijection between these measures and the minimal positive eigenfunctions of the Laplacian of the cover. This is the first classification result for the horocycle flow on a geometrically infinite surface.