

Wednesday Morning

# Noise and information in images and application to automatic image segmentation

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In this tutorial, we shall discuss the main concepts of information theory which are relevant for image processing. One thus will analyze the important notions of Fischer and Shannon information as well as Kolmogorov and stochastic complexities. We shall then show, how these concepts can be applied to segmentation of noisy images such as radar or active optical imagery. More precisely, we shall demonstrate how noise properties can be introduced in information based image processing techniques thus leading to fast and parameter free segmentation algorithms.

# Quantum teleportation and dense coding of optical images

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To date, most theoretically suggested and experimentally realized protocols of quantum information, such as quantum cryptography, quantum teleportation, dense coding etc., are based on spatially single-mode optical schemes. However, it is advantageous to achieve essentially parallel control and processing of quantum information.

In this communication we shall focus on the extension of continuous variable protocols of quantum information onto the broadband in space-time light fields. The basic quantum resource is in our case provided by broadband light fields in entangled EPR state. We discuss the methods of generation of entanglement with the use of the fields in spatially-multimode squeezed state produced, e.g., by broadband OPA's, and find the characteristic spatio-temporal scales of entanglement.

Recently we have proposed the quantum holographic teleportation protocol, which is a direct generalization of earlier described and experimentally realized single-mode scheme, but allows for transportation of quantum state of a broadband in space-time input field (e.g., carrying an optical image) from one place to another. The essential distinctions of the parallel spatially-multimode teleportation protocol from the single-mode version are (i) diffraction of light by propagation in the scheme, the use of multi-pixel detectors and modulators, and important role of relevant spatial scales, and (ii) a possibility to control and optimize the resolving power in space with the use of simple optical elements (lenses).

We demonstrate how the scales of entanglement and of the optical arrangement determine the resolving power of protocol and the fidelity of teleportation [1]. The global fidelity of teleportation associated with all degrees of freedom for large system is always very close to zero. We introduce and discuss the notion of reduced fidelity, which is a quality measure for teleportation of a subset of the image elements of interest. Quantum holographic teleportation can be viewed as a quantum limit of conventional non-stationary holography free of quantum noise.

The dense coding of optical images allows for transmission of two non-stationary optical images (movies) with the use of two entangled spatially-multimode EPR light beams [2]. By contrast to conventional coding, in dense coding protocol both transmitted images are created by sender (Alice) in only one of light beams, the other beam is used by receiver (Bob) as the reference system. Spatially-multimode entanglement between the signal and the reference beams allows for extracting more information from detected images, thus improving the capacity of protocol. We discuss the optical scheme, implementing the protocol, and its information capacity in terms of mutual information between Alice and Bob.

It is worth to mention a close relation between the concept of dense coding and some recent schemes of the entanglement based hi-resolution optical measurements, where the role of entanglement is to provide, in analogy to dense coding, a better reference object for the measured one.

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# Spatial qudits: production, characterisation and their use for quantum bit commitment

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Qubits have been realised in many physical systems; qudits, with their larger Hilbert spaces, have not. Qudits can provide significant improvements over qubits, such as increased channel capacity in quantum communication [1]. Entangled *qutrits* ( $d=3$ ) provide the best known levels of security in quantum bit-commitment (QBC) and coin-flipping protocols, which cannot be matched using qubit-based systems [2]. Entangled qutrits have been realised optically via temporal encoding [3] and via spatial encoding, Fig. 1, but only *indirect* measurements have been made of the entanglement and states of these systems. These include fringe measurements [4, 5] and the violation of a two-qutrit Bell inequality [6]. The ability to completely characterise these entangled qudit states is critical if they are to find application: this is only possible via quantum state tomography.

Using transverse spatial modes of the optical field we produce and measure entangled qubits and qutrits. We use a combination of plane-wave holograms and single-mode fibres to manipulate and quantitatively analyse these qudits: using a quantum state tomography method that only requires two-part superpositions, we achieve the first complete characterisation of entangled qutrits [7]. Plane-wave holograms do not produce/analyse single Gaussian modes, instead yielding infinite superpositions,

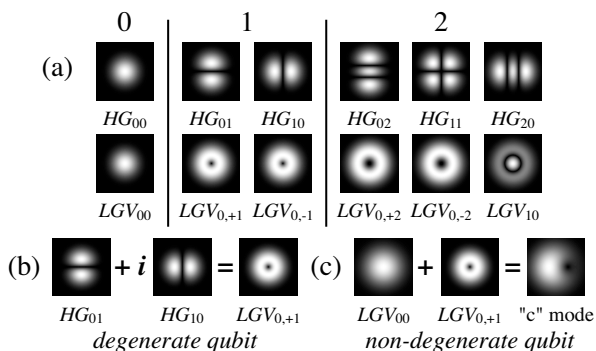


FIG. 1: (a) Lowest three orders of: Hermite-Gauss mode family,  $HG_{rs}$ , with  $r$  horizontal and  $s$  vertical lines of phase discontinuity; and Laguerre-Gauss-Vortex mode family,  $LGV_{pl}$ , with  $p$  ring phase discontinuities and a charge  $l$  phase singularity, or vortex. The mode order is  $r+s$  for  $HG_{rs}$  modes and  $2p+l$  for  $LGV_{pl}$  modes. Superposition states for (b) degenerate and (c) non-degenerate qubits, where the logical modes are respectively of the same and different orders. Non-degenerate qudit superpositions are unstable: the displaced singularities move around the beam centre as they propagate.

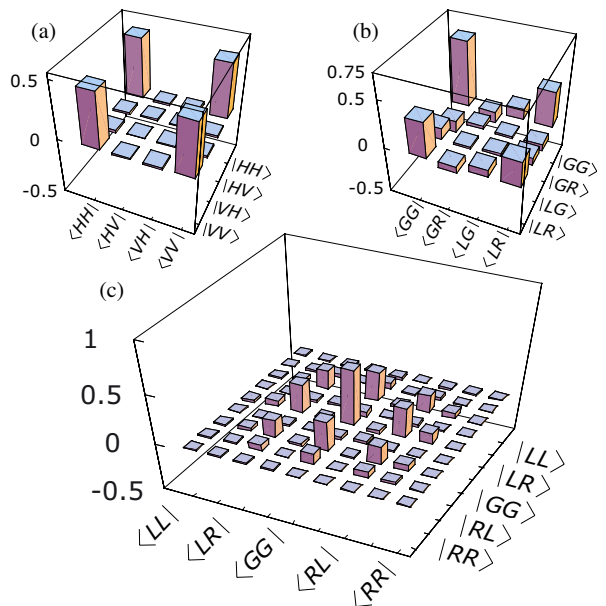


FIG. 2: Measured density matrices for entangled: a) degenerate qubits,  $H \equiv HG_{1,0}$ ,  $V \equiv HG_{0,1}$ . b) & c) non-degenerate qubits and qutrits,  $G \equiv LGV_{0,0}$ ,  $R \equiv LGV_{0,+1}$  &  $L \equiv LGV_{0,-1}$ .

although by appropriate normalisation we approximate single mode behaviour. We have also modelled, produced and measured Gaussian-mode holograms, and discuss the striking differences between these and the plane-wave case. We discuss the implications for higher-dimensions: qudit superpositions have previously been measured by displacing  $|l=2|$  vortex holograms [4, 5] but this is incorrect once  $|l| > 1$  since it analyses a superposition of several orders, not just the desired two-part superposition.

Ideally, entangled qutrits provide better security than qubits in QBC. We model the sensitivity of a purification QBC protocol to mixture, and, using our entangled qutrit state, show experimentally and theoretically that qutrits with even a small amount of decoherence cannot offer increased security over qubit-based protocols [7]. Future theoretical work in this area should consider the critical rôle of even small amounts of mixture.

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# Optical vortices, quantum and classical aspects

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**Abstract:** We review the current understanding and latest developments on the interaction of light beams containing optical vortices and the corresponding applications to classical and quantum phenomena.

Since screw dislocations were first discussed in general wave fronts [1], the study of these structures has grown to become a field of its own: singular optics [2]. Optical screw dislocations, also called optical vortices, are singularities in the wave front of an optical field, where the amplitude vanishes and the phase twists around the singularity taking all possible values. The number of twists of the phase, modulo  $2\pi$ , is called the topological charge of the vortex, which can be either positive or negative depending on the direction of the twist. Vortices appear spontaneously in several settings, and otherwise can be generated with phase masks, or with astigmatic components. In this context, parametric mixing of multiple waves containing wave front dislocations in quadratic nonlinear media constitutes a fascinating scenario. Because of the nonlinearity of these materials, waves of different frequency become coupled, exchanging not only energy with each other but also wave fronts. In this way the dislocations can be transformed from one wave to another, opening the door to a variety of classical and quantum phenomena with important implications and potential applications. For example, in second harmonic generation with moderate powers and wide beams, light undergoes frequency-doubling together with the generation of a phase dislocation nested in the second-harmonic (SH) beam. The topological charge of the dislocation generated is dictated by the charge of the input light at the Fundamental Frequency (FF). A completely different scenario appears when seeding the process with a charged coaxial SH input beam. In this regime the number, location and charges of the vortices in the output SH beam depend on the features of the pump and seed beams. This effect can be used in the area of optical tweezers and in micromachining, e.g., in wavelength optimized photo-polymerization techniques [3]. A new regime appears when the intensity of the input beams is increased over a certain threshold. In such regime, the output beams contain multicolor solitons, which are localized structures where the FF and SH energy are spatially trapped and locked. Multicolor solitons might have interesting applications in future optical devices. Under certain conditions, clusters of multicolor solitons can be created and controlled by means of the charge of the input beams, in a process where discrete charge information is transformed into controllable patterns of light [4,5].

Singular beams in quadratic nonlinear crystals show also promise in quantum information applications, through the generation of two-photon entangled states [6]. Because the orbital angular momentum carried by the photons defines an infinite-dimensional Hilbert space, it can be employed to generate engineered multidimensional entangled states, or qudits [7]. In this context, we have recently put forward an experimentally feasible technique to generate engineered qudits in parametric down-conversion of photons [8]. The scheme translates the classical topological information imprinted in the light beam, in the form of a vortex pancake, that pumps the two-photon source into quantum information contained in the weights and phases of the quantum entangled two-photon states.

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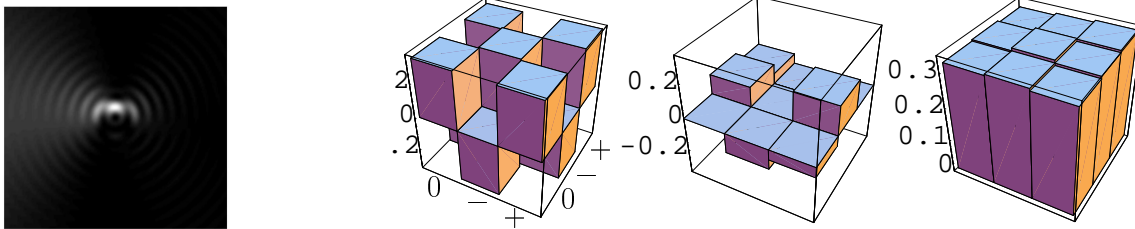
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# Analysis of vortex beams for quantum information

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We report on encoding of information into the spatial structure of vortex patterns. The single optical vortex carries the orbital angular momentum  $m\hbar$  per photon. The integer  $m$  is its topological charge and can be positive or negative. From the classical point of view it determines a pitch of the helical wavefront surrounding the center of the optical vortex. Recently, a variable method enabling efficient creation of the vortex superposition from the single laser beam has been proposed and experimentally verified by means of the spatial light modulator [1]. The structure of computer generated holograms controls the size of the created pattern, vortex number, spatial positions of their centers, and weighting coefficients. This is the basic principle of encoding information into the vortex structure. An example of such a complex structure (superposition of four different on-axis vortices) is shown in the figure, left panel.



Furthermore, the vortex structure is pseudo-nondiffracting so that the length of the region where it remains nearly unchanged can also be controlled. Decoding of information can be based on the vortex separation due to the different topological charges. The complete quantum state tomography of states of the orbital momentum of photon based on the separation of topological charges has recently been reported in [2], see the figure, right panel for the reconstructed real, imaginary and absolute values of a density matrix representing an equal weight superposition of three basic modes. However, other methods of decoding are also possible. Especially promising seems to be the combination of beam shaping and quantum reconstruction methods. As the main result we will show the complete characterization of a simple vortex field from several transversal scans of the field intensity. The quantum state reconstruction of vortex fields will be an important element of future experiments on information processing with propagating fields.

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# Quantum features of arrays of domain walls and cavity solitons

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Spatial structures in extended nonlinear optical devices can display important quantum features. For example, quantum images in Degenerate Optical Parametric Oscillators (DOPO) show quadrature squeezing in the near field and Einstein-Podolsky-Rosen correlations in the far field. We present here new kinds of quantum features in the presence of localised spatial structures in DOPO.

We first show that quantum fluctuations can induce arrays of cavity solitons in a DOPO. We introduce a stochastic model that describes the interaction of pump and signal waves in a  $\chi^{(2)}$  crystal inside an optical cavity with quantum noise entering from the input mirror. Quantum fluctuations introduce random creation and annihilation of pairs of domain walls and, after a transient, an equilibrium between these two processes is found. In the limit of large cavity finesse at the pump frequency with respect to that at the signal frequency, the density of cavity solitons formed by tightly locked domain walls is high because of large amplitude local oscillations in the tails of the domain walls [2]. At equilibrium, arrays of cavity solitons are induced by the quantum fluctuations as clearly shown in the left panel of Figure 1. These noise-induced structures lead to high spatial correlations and a peak in the power-spectrum corresponding to the size of the cavity soliton [2]. Decreasing the cavity finesse of the pump field decreases the average size of the soliton arrays and the density of domain walls. For equal pump and signal cavity finesse, for example, the dynamics reduces to that of domain walls performing random walks and spatio-temporal disorder is restored (see the right panel of Figure 1).

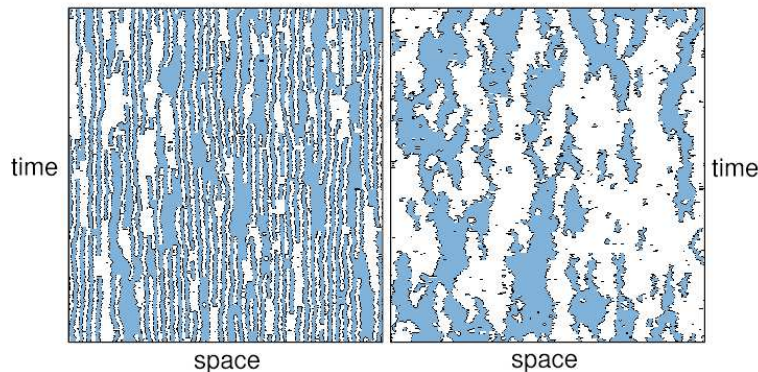


Figure 1: *Left panel: Arrays of cavity solitons for large pump cavity-finesse. Right panel: Domain walls performing random walks for equal pump and signal cavity finesse.*

We then describe quantum fluctuations in arrays of domain walls and cavity solitons in DOPO. In particular we characterise quantum correlation functions of the quadrature components of the signal field inside and outside these localised structures. Since the signal intensity identically vanishes at the core of the wall, high levels of squeezing are found in the positions close to the centre of the walls. Once cavity solitons are formed by the locking of two domain walls, the topological constraint of zero intensity is removed and squeezing is partially decreased in the tails of these structures. Finally, we discuss the importance of pinning the position of the localised structures for information processing, and present calculations of the velocity and direction of motion of cavity solitons in the presence of phase modulated pumps.

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# Optical Angular Momentum

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It has long been recognised that photons have a spin angular momentum, associated with optical polarisation. It is less well known that a light beam may also carry orbital angular momentum associated with its spatial phase structure. In particular, a light beam having an amplitude of the form  $u(r, \phi, z) = v(r, z)e^{il\phi}$  will carry an orbital angular momentum of about the  $z$ -axis [1]. I will describe how this angular momentum arises and present evidence for its existence [2].

The values of  $l$  are, in principle, unbounded and this means that photons carrying orbital angular momentum are a physical embodiment of “qudits”, that is many-state quantum systems. I will describe how we can measure  $l$  at the single photon level [3] and how the orbital angular momentum can be changed and entangled in nonlinear optics [4].

The angular momentum is conjugate to the angle observable and this leads to an uncertainty relation between them [5]. I will present this uncertainty relation and describe the states that saturate it, together our experimental realisation of the so-called intelligent states [6]. This work led us to develop a free-space communications device based on orbital angular momentum in which security is provided by the uncertainty relation [7].

If there is time, then I will present a discussion of the Abraham-Minkowski debate in which the linear momentum of a photon inside a dielectric is either  $\hbar k/n$  or  $\hbar kn$ . We have devised a new test based on angular momentum inside a dielectric and are planning a new experimental test of these ideas [8].

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# Entanglement induced spatial splitting of light

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We observe a quantum image produced in parametric down-conversion of a pump beam carrying orbital angular momentum. Measuring the spatial location of coincident photons shows that the idler beam splits into two, compared to a single beam when down converting with a Gaussian pump.

**SUMMARY** - It has been shown theoretically [1, 2] that while twin photons entangled in energy and momentum propagate in a plane containing the pump beam ( $\pi$ -plane), twin photons entangled in orbital angular momentum propagate off-plane. This off-plane deviation is dependant on the value of  $l$ , the orbital angular momentum, and, therefore, provides a measure of  $l$  for the pump. Because of the off-plane angle, a spatial pattern is predicted in the form of two coincidence spots instead of one. In this paper, we present the first experimental verification of this entanglement-induced spatial splitting effect. Figure 1 sketches the scattering geometry utilized and the resulting two-spot pattern. This phenomenon is a new type of macroscopic quantum image (an image seen through  $G^{(n)}$  correlation,  $n \geq 2$ ) produced by the twin photons entangled in orbital angular momentum.

In our experiment, the pump is focused into a type-

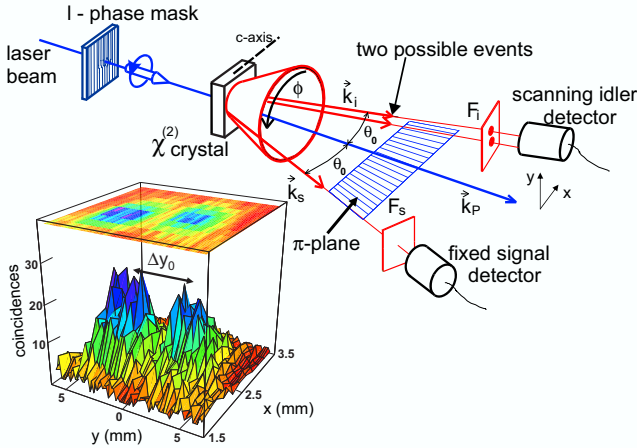


FIG. 1: TOP: Experimental setup sketch. A laser beam with  $\lambda = 0.351\text{nm}$  is prepared holographically with angular momentum  $l = 4$  and focused on a  $\chi^{(2)}$  nonlinear crystal cut for type-I phase matching. A down-converted signal beam is detected by a fixed single-photon detector, while the conjugate idler photons are collected by a scanning detector.  $F_s$  and  $F_i$  are interference filters at  $702\text{nm}$ . BOTTOM: Two coincidence spots are seen. The interference filter bandwidth is  $10\text{nm}$ . The fixed signal detector is placed at  $|x_s| = 3.7\text{cm}$  and  $y_s = 0\text{cm}$  from the pump laser. Pumping with a  $\text{TEM}_{00}$  laser mode produces only a single coincidence spot at  $y = 0$ .

I phase-matched BBO crystal by a  $17.5\text{cm}$  focal-length convergent lens. Once the signal and idler photons are generated, there are no lenses or apertures in their paths besides the  $175\ \mu\text{m}$  diameter detector size. This allows us to obtain a much higher-resolution image at the cost of lower counting rates. The moving detector, which scans the quantum image, is controlled with a minimum step size of  $10\ \mu\text{m}$ , while the single and coincident counts are recorded with a pair of two-channel gated photon counters. One counts events at the fixed detector for  $5\text{ns}$  durations for each trigger from the moving detector while simultaneously counting the number of triggers. The second counter measures the single counts from the fixed detector.

Theory [1] shows that  $l$  can be measured by projecting the scattered wavevectors for signal and idler photons on the plane transverse to the pump wavevector as  $|(k_{sx} + k_{ix})\hat{x} + (k_{sy} + k_{iy})\hat{y}|^2 = k_P l/z_R$ . This measure of  $l$  can be written in terms of the coincidence spot separation as

$$l = \frac{z_R}{k_P} (k_s \sin \theta_0)^2 \left[ \left( 1 - \cos \arcsin \frac{\Delta y_0}{R} \right)^2 + \left( \frac{\Delta y_0}{R \cos \theta_0} \right)^2 \right]$$

where  $R$  is down-converted-ring radius at the detection plane, and  $\theta_0$  is given by Snell's law  $n_s \sin \theta_s = \sin \theta_0$ .

With the experimental parameters  $\theta_s = 0.0698$ ,  $n_s = 1.6648$ ,  $k_s = 1.4897 \times 10^5\ \text{cm}^{-1}$ ,  $R = 3.7\text{cm}$ ,  $z_R = 0.5\text{cm}$ ,  $k_P = 2.9719 \times 10^5\ \text{cm}^{-1}$ , and the fitted value  $\Delta y_0 = 3.3\text{mm}$ , we obtained  $l \simeq 4.0$ . The azimuthal angle from the  $\pi$  plane for one peak is  $|\Delta\phi| = 4.7^\circ$ , which matches the theoretically predicted value.

This experiment provides an independent verification of orbital angular momentum entanglement reported in [3] and predicted in [4].

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# Orbital Angular Momentum Exchange in an Optical Parametric Oscillator

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We present a study of orbital angular momentum transfer from pump to down-converted beams in a type-II Optical Parametric Oscillator. Cavity and anisotropy effects are investigated and demonstrated to play a central role in the transverse mode dynamics.

We study the OAM transfer in a non-degenerate, type-II Optical Parametric Oscillator (OPO). The OPO is pumped by the second harmonic of a Nd:YAG laser. This laser generates a TEM<sub>00</sub> gaussian beam, that is converted to a nearly Hermite-Gauss TEM<sub>01</sub> beam [1]. An astigmatic mode converter is used to produce a Laguerre-Gauss beam [2]. Its phase singularity was evidenced by the self interference pattern obtained in a Michelson interferometer. In the output of the OPO, signal and idler beams are separated by a polarizing beam splitter (PBS). Each down converted beam is sent into a Michelson interferometer, in order to produce interference fringes that can reveal the existence of a phase singularity. Then the two outputs are sent onto a CCD camera, that is used to register either the interference pattern or the intensity profile of the beams. Our results are shown in Fig.(1).

In images 1 and 4, the output intensity in the idler is the one of a Laguerre-Gauss beam. The corresponding interference patterns show the topological defects in the center of the Laguerre-Gauss beam characteristic of phase singularities. In this situation, the idler beam carries the orbital angular momentum of the down converted pump photons. In image 2, the shape of the idler beam is intermediary between a first order Laguerre-Gauss and a diagonal first order Hermite-Gauss modes. A vortex can still be observed through the interference fringes. In both cases, the signal beam remains in the fundamental gaussian mode. Following the Poincaré-sphere representation proposed in Ref.[3], we can look at the idler mode shown in image 2 as an orbital equivalent of an elliptical polarization.

Image 3 the signal beam oscillates in the transverse mode with higher order, but with no angular momentum. The output is a pure Hermite-Gauss TEM<sub>01</sub> mode, vertically oriented. Therefore, the orbital angular momentum is not conserved due to the crystal anisotropy, which induces an astigmatic propagation of the signal beam, thus preventing the transfer of orbital angular momentum.

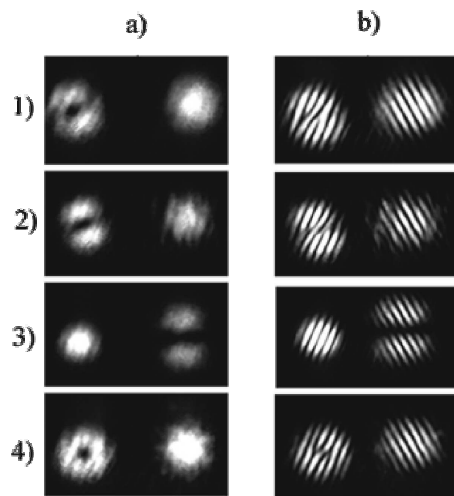


FIG. 1: a-) Intensity and b-) interference patterns of signal (right) and idler (left) beams.

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# Entanglement and conservation of orbital angular momentum in spontaneous parametric down-conversion

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It is known that if the paraxial approximation is valid, the two-photon detection amplitude, which can be regarded as a two-photon wave function, is given by

$$\Psi(\boldsymbol{\rho}_s, \boldsymbol{\rho}_i) = \mathcal{U}\left(\frac{\boldsymbol{\rho}_s + \boldsymbol{\rho}_i}{\sqrt{2}}\right) \mathcal{F}\left(\frac{\boldsymbol{\rho}_s - \boldsymbol{\rho}_i}{\sqrt{2}}\right)$$

where  $\mathcal{U}(\boldsymbol{\rho}, z)$  is a field with the same angular spectrum as the pump beam and  $\mathcal{F}(\boldsymbol{\rho}, z) = \frac{\sqrt{KL}}{2\pi z} \text{sinc}\left(\frac{KL}{8z^2}\rho^2\right)$ .

Let us now suppose that the down-converter is pumped by a LG beam whose orbital angular momentum is  $l\hbar$  per photon, described by the amplitude  $\mathcal{E}_p^l(\boldsymbol{\rho}; \lambda_o, w_o)$ . Here,  $p$  is the radial index. In order to study the conservation of angular momentum in SPDC, we will expand the two-photon wave function  $\Psi(\boldsymbol{\rho}_s, \boldsymbol{\rho}_i)$  in terms of the LG basis functions  $\mathcal{U}_{p_s}^{l_s}(\boldsymbol{\rho}_s)\mathcal{U}_{p_i}^{l_i}(\boldsymbol{\rho}_i)$ . That is,

$$\Psi(\boldsymbol{\rho}_s, \boldsymbol{\rho}_i) = \sum_{l_s, p_s} \sum_{l_i, p_i} C_{p_s p_i}^{l_s l_i} \mathcal{U}_{p_s}^{l_s}(\boldsymbol{\rho}_s) \mathcal{U}_{p_i}^{l_i}(\boldsymbol{\rho}_i)$$

From the orthogonality of the LG basis,  $C_{p_s p_i}^{l_s l_i}$  is given by

$$C_{p_s p_i}^{l_s l_i} = \iint d\boldsymbol{\rho}_s d\boldsymbol{\rho}_i \mathcal{U}_p^l\left(\frac{\boldsymbol{\rho}_s + \boldsymbol{\rho}_i}{\sqrt{2}}\right) \mathcal{F}\left(\frac{\boldsymbol{\rho}_s - \boldsymbol{\rho}_i}{\sqrt{2}}\right) \times \mathcal{U}_{p_s}^{*l_s}(\boldsymbol{\rho}_s) \mathcal{U}_{p_i}^{*l_i}(\boldsymbol{\rho}_i).$$

Based on the above expression, it is straightforward to show that orbital angular momentum is conserved in the SPDC process. In principle, this conservation could be satisfied by fields exhibiting either a classical or quantum correlation (entanglement) of orbital angular momentum. We will now show that the conservation leads to entanglement of orbital angular momentum of the down-converted fields.

If the crystal is thin enough, we can set  $\mathcal{F} = 1$ . Then, the biphoton wave function is

$$\Psi(\boldsymbol{\rho}_s, \boldsymbol{\rho}_i) = \mathcal{U}_p^l\left(\frac{\boldsymbol{\rho}_s + \boldsymbol{\rho}_i}{\sqrt{2}}\right).$$

It is evident that  $\Psi(\boldsymbol{\rho}_s + \boldsymbol{\Delta}, \boldsymbol{\rho}_i - \boldsymbol{\Delta}) = \Psi(\boldsymbol{\rho}_s, \boldsymbol{\rho}_i)$ , which leads to a detection probability satisfying  $\mathcal{P}(\boldsymbol{\rho}_s + \boldsymbol{\Delta}, \boldsymbol{\rho}_i - \boldsymbol{\Delta}) = \mathcal{P}(\boldsymbol{\rho}_s, \boldsymbol{\rho}_i)$ . This property of the detection probability is not compatible with a classical correlation that conserves orbital angular momentum, that is,

$$\mathcal{P}_{\text{cc}}(\boldsymbol{\rho}_s, \boldsymbol{\rho}_i) = \sum_{l_i=-\infty}^{\infty} P_{l_i} |F_{l-l_i}(\boldsymbol{\rho}_s)|^2 |G_{l_i}(\boldsymbol{\rho}_i)|^2,$$

where  $F_{l_s}(\boldsymbol{\rho}_s)$  and  $G_{l_i}(\boldsymbol{\rho}_i)$  represent down-converted signal and idler fields with orbital angular momentum  $l_s\hbar$  and  $l_i\hbar$  per photon, respectively. Here the coefficients  $P_{l_i}$

satisfy  $\sum_{l_i=-\infty}^{\infty} P_{l_i} = 1$  and  $P_{l_i} \geq 0$ . So, the two-photon state generated by SPDC must be entangled in orbital angular momentum.

By means of a simple experiment, we prove that the above wave function describes accurately the state generated by SPDC within the assumed approximations. Although direct coincidence detection provides information about the modulus of  $\Psi(\boldsymbol{\rho}_s, \boldsymbol{\rho}_i)$ , its phase structure can only be revealed by some sort of interference measurement. We do this with the help of the Hong-Ou-Mandel (HOM) interferometer. When the interferometer is balanced, that is, when paths  $s$  and  $i$  are equal, we have fourth-order interference, which reveals the phase structure of the wave function. When the path length difference is much greater than the coherence length of the down-converted fields, the interferometer plays essentially no role, and the modulus of the wave function is measured.

If  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are the positions of detectors  $D_1$  and  $D_2$ , each located at one output of the HOM interferometer, the coincidence detection amplitude is given by

$$\Psi = \Psi_{tt}(\mathbf{r}_1, \mathbf{r}_2) + \Psi_{rr}(\mathbf{r}_1, \mathbf{r}_2),$$

where the indices  $tt$  and  $rr$  refer to the cases when both photons are transmitted or reflected by the beam splitter, respectively.

It is straightforward to show that, for  $t = r = 1/\sqrt{2}$ , apart from a common factor,

$$\Psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \propto \frac{1}{2} \left[ \mathcal{U}_p^l\left(\frac{x_1 + x_2}{\sqrt{2}}, \frac{y_1 + y_2}{\sqrt{2}}\right) - \mathcal{U}_p^l\left(\frac{x_1 + x_2}{\sqrt{2}}, \frac{-y_1 - y_2}{\sqrt{2}}\right) \right].$$

Since  $\mathcal{U}_p^l$  has the form  $\mathcal{U}_p^l(\boldsymbol{\rho}) = u_p^l(\rho)e^{il\phi}$ ,

$$\Psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \Psi(R, \theta) \propto u_p^l(R) \sin l\theta,$$

where  $R = \frac{1}{\sqrt{2}}|\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2|$  and  $\theta$  is defined by the relations

$$\sin \theta = \frac{\rho_1 \sin \phi_1 + \rho_2 \sin \phi_2}{R}$$

$$\cos \theta = \frac{\rho_1 \cos \phi_1 + \rho_2 \cos \phi_2}{R}.$$

The coincidence detection probability is therefore

$$\mathcal{P}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \propto |u_p^l(R)|^2 \sin^2 l\theta.$$

Experimental data have confirmed our model for SPDC pumped by LG beams.

# Manipulating quantum entanglement with multi-mode photonic devices

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Optical devices based upon multi-mode interference play an important role in classical optics (e.g. in integrated optics and fiber optics). We consider the effect of multi-mode devices in a quantum context, more specifically, when dealing with twin-photon entanglement. The multi-mode devices are inserted in the beam lines of a spontaneous parametric down-conversion (SPDC) set-up. As we will show, this may either degrade or upgrade the photonic entanglement. We address two examples in detail.

## *Subwavelength metal hole arrays*

The optical transmission of such an array is strongly enhanced by resonant excitation of surface plasmons [1]. The propagating nature of these (polarized) surface plasmons implies that a single photonic input mode is coupled into an ensemble of photonic output modes. Tracing over the latter, by using a bucket detector, leads to degradation of the twin-photon polarization entanglement [2]. The corresponding phenomenon at the classical level is depolarization, i.e. fully polarized input light is transformed, by the hole array, into partly polarized output light. This is due, as is polarization disentanglement in the quantum case, to coupling of the polarization degrees of freedom to spatial degrees of freedom [3].

## *Non-integer spiral phase plates*

A spiral phase plate with integer step is a well-known device to produce a light beam with integer orbital angular momentum (OAM) per photon [4]. We have recently extended this concept to the case of a non-integer step, allowing us to study entanglement of non-integer OAM [5]. Input photons, when passing through such a device, are scattered into a very-high-dimensional Hilbert space of integer OAM eigenmodes. As a consequence, the Bell parameter  $S$ , which has as maximum value  $2\sqrt{2}$  for two entangled qubits, is now predicted to have 3.2 as maximum value, for the case of two half-integer OAM photons. In order to verify this, we have developed high-quality half-integer spiral phase plates [6]. These have allowed us, very recently, to measure  $S=3.04 \pm 0.06$ , which demonstrates the predicted beyond-Bell pairing of two photons.

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# Quantum Imaging of Fractional Orbital Angular Momentum States

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We perform Fourier-optical quantum imaging of twin photons carrying fractional orbital angular momentum.

Orbital angular momentum (OAM) of light [1] draws increasing interest in the field of quantum information processing since it provides infinitely many spatial degrees of freedom. Particularly, high-dimensional entanglement is expected to be robust against decoherence and to show stronger quantum correlations as compared to low-dimensional systems such as qubits. [2]

In this experimental work, we study “ghost” images [3] of correlated twin photons with *fractional* OAM. These states are selected by using mode filters consisting of a spiral phase plate (SPP) in combination with a single-mode fiber. The SPPs used in this experiment have been manufactured with a step height,  $h$ , such as to convert a fundamental Gaussian beam with zero OAM per photon into a beam with OAM per photon equal to  $l = 3.5$  (in units of the Planck constant), or vice versa. [4] Fig. 1(a) displays a schematic of the SPP. The calculated far-field intensity distribution for a fundamental Gaussian beam after passing such a SPP is shown in Fig. 1(b).

Ghost images are formed in coincidence counting of twin photons from spontaneous parametric down-conversion, where the signal wave,  $s$ , is filtered by a combination of a SPP and a single-mode optical fiber. The idler wave,  $i$ , is transversally scanned by a pinhole, see Fig. 1(c). Lenses of focal lengths  $f$  and  $f'$  form a Fourier-optical scheme with the pinhole residing in a far-field image plane of the SPP. As a result of the (quantum) correlations the coincidence counts of signal and idler should reveal a distribution similar to that in Fig. 1(b). So far, the transversal walk-off from the birefringent crystal (2 mm thick  $\beta$ -BBO) blurs the ghost image. This prevents us from clearly resolving the optical vortex structure. We present our investigation of this walk-off and discuss possible remedies.

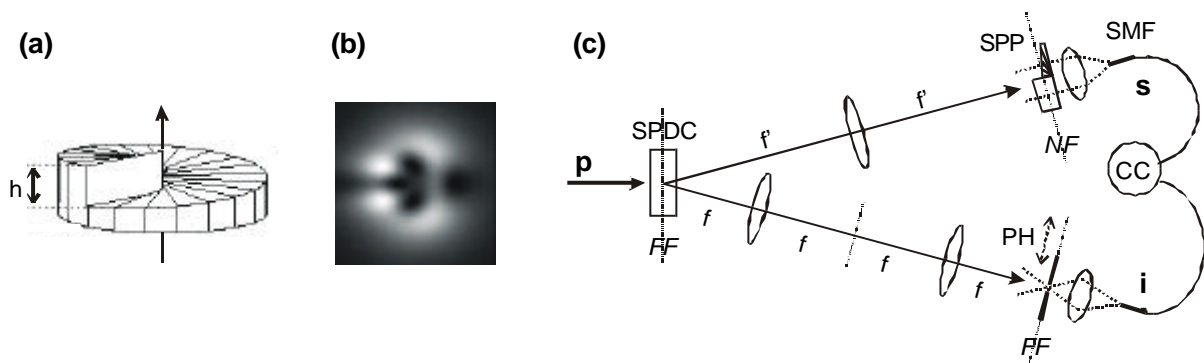


FIG. 1: (a) Spiral phase plate. (b) Far field of a Gaussian beam after passing a SPP with  $l = 3.5$ . (c) Ghost imaging scheme with plane pump wave ( $p$ ), non-linear crystal (SPDC, type II phase matching), signal path ( $s$ ) with Fourier transforming lens, idler path ( $i$ ) with 1:1 imaging telescope, mode filter with SPP and single-mode fiber (SMF), scanning pinhole (PH), coincidence photon counting (CC). Near-field (NF) and far-field (FF) planes of the SPP are indicated with dashed lines. A vertical linear polarizer and a narrow-band interference filter in the idler arm are not shown.

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Thursday Afternoon

## Synthesizing spatial squeezing and entanglement: generation and applications

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We can now synthesize spatial multi-mode laser beams which contain quantum correlations suitable for the detection of spatial modulations and signal below the standard quantum noise limit. This was achieved through a close collaboration between the Australian National University in Canberra (ANU) and the Laboratoire Kastler Brossel in Paris (LKB). Our technology is based on generation of reliable and strong noise suppression using optical parametric oscillators (OPO). These devices can produce beams with squeezing of 5 dB or larger, which allows already the detection of modulation signals below the quantum noise limit in the time domain, but with single mode, TEM<sub>00</sub>, spatial beams.

Spatial measurements with the TEM<sub>00</sub> mode result in the equivalent quantum noise limitation for the signal to noise ratio [1]. In order to measure the position of the beam, or any other spatial information, we have to use detectors with several segments, such as quadrant photo diodes, and we should use the matching multi-mode spatial beams. This is a special set of modes with different properties to the conventional hermitian TEM<sub>xy</sub> modes. The simplest are the so called flip modes, which are modes with a Gaussian amplitude, but a 180 degree phase shift between two sides [2]. The properties of a whole system of such modes has been explored [3].

The theoretical work at LKB has shown that by combining squeezed states of the correct flip mode with a TEM<sub>00</sub> laser beam we can synthesize laser beams with the correct spatial correlations. Experiments at ANU have shown that we can demonstrate an improvement of the sensitivity for the detection of small periodic oscillation of the beam position [4]. We have found a way of combining two squeezed beams with a third coherent beam with minimum losses. This technique uses the properties of the transmitted and reflected beams of resonant cavities. The result, named a quantum laser pointer, is a light source which can improve the spatial sensitivity simultaneously in both transverse directions by a factor of about two below the quantum noise limit [5].

We can now look for applications of these beams and exploit the quantum correlations in such situations as microscopy, data storage or the measurement of the small oscillations of the cantilever in an atomic force microscope. At the same time we are investigating the options for spatial entanglement within one multi-mode beam which can be used in quantum information protocols. This synthesis is complementary to the techniques in progress in Paris to generate multimode beams with quantum correlations in degenerate OPOs.

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# Optimal strategies in image processing : a quantum study

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**Abstract:** We consider the problem of extracting information from an optical image. The modes responsible for the quantum noise affecting the extraction are identified, which leads to a strategy optimizing the information read out.

We consider the general problem of extracting information out of an optical image with an array of detectors or a CCD camera. The extracted information consists of a given linear combination of the signals recorded on the different pixels of the detector, which allows us to measure one or several parameters which are responsible for the image variation : It can be a global motion of the image (translation[1,2] or rotation), a distortion, or also the determination of the spatial Fourier transform. When the sources of technical noise have been removed, one is faced with the quantum noise in the system which limits the accuracy with which the information can be extracted.

We have made the full quantum analysis of this measurement process, and identified the origin of quantum noise : for each measurement, a single transverse mode of specific shape is involved and carries the whole noise affecting the measurement. We show also that it is possible to go beyond the shot noise limit for this measurement and therefore improve the information read-out sensitivity beyond the standard quantum limit by injecting a squeezed vacuum state in this mode.

We then investigate the various ways permitting us to enhance simultaneously the signal to noise ratio in different measurements of the kind described above, and show how to adjust the linear combination of pixel intensities in order to optimize the information extraction from the image at, and then below, the standard quantum limit.

We finally apply these considerations to the problem of read-out of optical memories having more than one bit of information in the focal spot, which would allow us to increase optical data storage densities beyond the Diffraction limit of one optical bit per  $\lambda^2$ .

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Friday Morning

## Optical data storage: an overview, capacity limits and new options

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We present a brief overview of the state of the art in the field of optical data storage. The past trend (reduction of the diffraction unit  $\lambda/NA$  with  $NA$  the numerical aperture of the scanning objective) has attained its practical limits. A further reduction of this unit by using UV light or ‘immersion’ techniques or optical tunneling seems to pose practical problems for a mass-produced consumer product.

Higher capacities on a storage medium ask for a higher data retrieval rate. Via a numerical example we will discuss the various noise sources that are encountered in a real system. It turns out that some basic noise sources are getting close to limiting the performance and the data rate of a next-generation optical disc player.

A new option for disk capacity increase will be presented that is based on the use of specially designed optical effects imparting orbital angular momentum to the reflected light beam. This option is a spin-off of the European project SLAM (IST-2000-26479) on density increase in optical recording that was carried out in the years 2001-2004. Initial simulations and experiments on disc structures producing orbital angular momentum are described and further research activities in this field and the expected gain in spatial density is discussed.

## Angular momentum as information channel in optical data storage

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### Abstract

*We present a way to encode multiple information levels in one location on an optical disc that can affect or introduce angular momentum on a scanning light beam.*

In the field of optical recording, the increase in spatial density has primarily been achieved by reducing the quantity  $\lambda/NA$ , the diffraction unit. A further reduction of the spot size on the disc along the wavelength path implies the development of low cost UV sources. The  $NA$  increase beyond the unit value requires immersion read-out or optical tunnelling. Both techniques are not practical, the first one because of handling problems, the second because of system and medium robustness. From the very beginning, together with removability, system robustness has been the most attractive feature of optical recording for the customer and any compromise in this direction would severely limit the use of optical discs for the cheap storage of massive amounts of information.

In the present optical recording systems mainly the information on the amplitude of the beam that is reflected from the structure is used as bit recognition channel. There are however other quantities that can be used in order to increase the information density, namely the phase and polarisation. Also, these quantities are more suitable for the introduction of using several detection levels in contrast to the single intensity level currently used in optical recording. The combination of both quantities can be studied in the form of angular momentum as an independent detectable quantity. Angular momentum may be a robust factor for the encoding process since this is a quantity which is conserved, therefore, noise and defects will have a limited influence on the angular momentum detection.

In this paper we analyse the angular momentum that is introduced on a beam of light via reflection from a specially designed optical surface on an optical disc, namely a four-quadrant structure with an increasing depth for each quadrant. The interferometrically measured height profile of such a structure, made with a lithographic process, is shown in Fig. 1. A quadrant-type detector can be used to readout these multiplexed effects. It should be possible to adequately detect both the sign of the angular momentum and several stages in its amplitude, making this an interesting alternative for high density optical recording. The experimental verification is in progress.

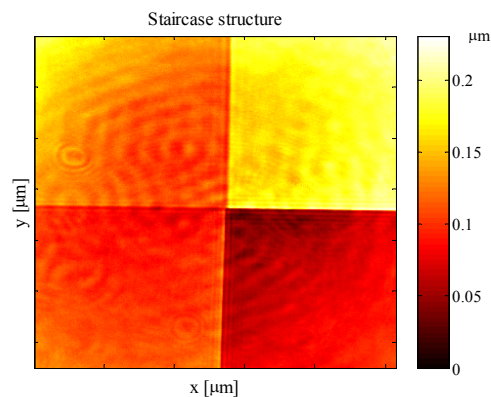


Fig. 1. Interferometrically measured height profile of the staircase structure.

## Noiseless amplification of images, quantum and classical

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An image carries an amount of spatial information that can be assessed as the product of the number of resolution cells by the useful dynamics. I will first recall the link between phase matching conditions and the size of the resolution cell in a parametrically amplified image, then I will show the equivalence between the notion of resolution cell and the notion of spatial mode.

Regarding the dynamics, spatial information is degraded by spatial noise. For very weak images, spatial fluctuations of quantum origin become predominant. Information theory precludes any improvement of the signal to noise ratio (SNR) by amplification. However, an amplified image is less sensitive to degradation of SNR due to detection and therefore the SNR after detection can actually be improved by image amplification. If spontaneous down conversion (SPDC) is neglected, phase-sensitive amplification is noiseless, while phase insensitive amplification adds 3 dB on the quantum noise level. I will compare theoretical and experimental results in both configurations and discuss how to assess purely spatial fluctuations on one-shot images. Another important point is taking into account SPDC, which is in fact non negligible for weak images. Fortunately, subtracting the mean level of SPDC will be shown to be useful in a large range of practical situations.

Last, I will present some potential applications of parametric image amplification in general connected to time-gating, like imaging through diffusing media, fluorescence life-time measurement or active imaging, and I will discuss advantages and drawbacks of parametric image amplification versus other schemes ensuring time gating, like for example image up-conversion.

## Experimental demonstration of noiseless parametric amplification of images.

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Optical noiseless amplification property of a phase sensitive parametric interaction is wellknown and has been investigated theoretically in space [1] and [2] time domains and experimentally in the time domain [3]. To our knowledge, this work presents the first experimental demonstration of purely spatial noiseless amplification of images. A low level image limited by the shot-noise in its transverse plane is obtained by illuminating a binary transparency with a pulsed plane wave ( $T=1$  ps @527.50 nm). This signal is amplified by means of a pump pulse ( $T=0.9$  ps @263.75 nm) in a type I BBO crystal designed for a collinear phase matching at the parametric degeneracy. Finally, the image is recorded by a cooled CCD camera (Si thin array, back illuminated, dark-noise=0.05 e/pix/s @-40°C, readout-noise=3.4 e.rms @100 kHz, QE=0.9 @527 nm). Both, phase sensitive (PSA) and phase insensitive amplification (PIA) are investigated. PIA is performed with a non-collinear phase matched interaction and by spatially filtering the amplified signal, contrary to what is done for PSA which corresponds to a completely degenerated interaction. Statistics on the photoelectrons fluctuations in the transverse plane of images are calculated over all the pixels in the considered areas. Noise figures (NF) after detection, for both PSA and PIA interactions, are found to be less than the expected values (figure1), as it was theoretically predicted [4] when the elementary area of detection (pixel) is smaller than the resolution cell of the optical system. From experimental data (magnification of the imaging system, pixel size, spatial frequencies bandwidth of the amplifier) the size of a resolution cell is evaluated to an area of 5x5 pixels on the CCD. Thus, when statistics are performed on binned pixels, NF's increase and remain constant when the size of the elementary area of detection becomes greater than the resolution cell [4]. Experimental NF's are in good agreement with the values calculated from equations that integrate the effective QE of the detection system and the amplification gain.

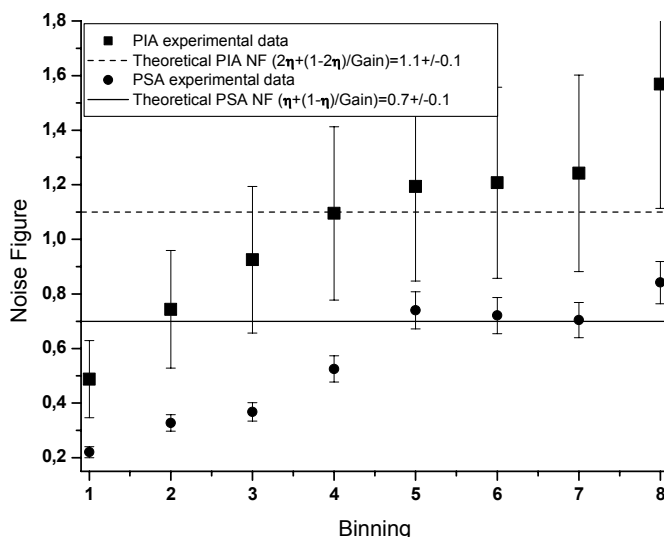


Figure 1: NF's vs binning.

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## Noiseless image amplification in a transverse multimode sub-threshold OPO

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In the last two decades, the OPO has been widely used in the quantum optics and quantum information domains for producing single-mode non-classical cw beams, such as squeezed beams, twin beams and more recently EPR beams, in which only the temporal quantum fluctuations of the light are appropriately tailored. But when trying to use non-classical light to enhance the resolution of optical images, one needs to consider the spatial aspects of the light<sup>1</sup>. Spatially multimode quantum light, unlike single mode light, has been shown to be able to exhibit striking spatial features such as local squeezing, spatial quantum correlations between different parts of the beam. High-resolution imaging made by very sensitive detectors is ultimately limited by the quantum noise of the light and such quantum multimode light could be used to enhance the resolution of optical images.

We have developed an experiment to perform noiseless image amplification and to produce spatially multimode cw bright non-classical beams of light. It consists of a frequency degenerate Optical Parametric Oscillator (OPO) using an optical cavity resonating simultaneously on an infinite set of transverse modes (“degenerate cavity”). In this system, pairs of quantum-correlated photons, generated in the crystal by the down-conversion of the pump, produce, thanks to the degenerate geometry of the cavity, multimode non-classical beams at the output of the OPO.

Pumped below threshold, the OPO does not emit bright beams, but is predicted to produce a multimode squeezed vacuum. Furthermore, if injected, the OPO below threshold can act as an Optical Amplifier for injected images. In this configuration it is well-known that it can perform a phase-sensitive amplification or de-amplification. This amplification is predicted to be noiseless, i.e. it does not degrade the signal to noise ratio, which means that the amplifier does not have a excess vacuum noise input.

In our experiment, using a semi-confocal cavity as the degenerate cavity, we have performed phase sensitive amplification of simple images through the triply resonant dual-cavity, type II OPO. We have characterized the spatial gain and the spatial frequency bandwidth of the system, the fidelity of the output image, and demonstrated that the observed image amplification was truly multimode. We are currently investigating the noise figure of the device, i.e. the degradation of the signal to noise ratio through the amplifier, to determine whether the amplifier acts in the noiseless amplification regime, and we are also studying the quantum properties of the pair of images produced on the ordinary and extraordinary polarizations. These images should exhibit intensity correlations beyond the standard quantum limit, not only on the whole beam, but also locally.

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## Untill which limits is the optical amplification of a weak image possible?

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Optical amplification of a signal or an image is always accompanied by the amplification of quantum noise with a mean rate corresponding, in the limit of high gains, to an input noise of a photon per spatio-temporal mode, or coherence area. Hence, it seems difficult to amplify an image bearing much less than one photon per mode, like, for example, black-body radiation of astronomical origin<sup>1</sup>. We have shown however<sup>2</sup> that an image of dye fluorescence can be retrieved with a good contrast after parametric amplification even if the level of the input image corresponds to significantly less than a photon per mode. Fig.1 shows the dye image after amplification, that can be considered as the addition of the dye amplified image and of the amplified quantum noise (or spontaneous down conversion, SPDC). Fig.2 shows the image of SPDC without any input image and fig.3 is obtained by subtraction of the SPDC image to the output dye image. The number of photons per mode  $m$  of the input dye image can be estimated either by comparison between the image after subtraction and the level of the SPDC or by counting the number of modes in the temporal and spatial bandwidth of the amplifier<sup>3</sup>. Both methods give a result of about  $m=0.15$  photons per mode.

After subtraction of its mean value, the level of the noise induced by SPDC, given by its standard deviation, must be smaller than the mean of the dye image to allow a correct retrieval of this image. Because images of the  $N$  different temporal modes add incoherently on the C.C.D. sensor and single mode SPDC has a thermal statistics, this condition can be expressed as :  $N*m *G > \sqrt{N} G \Rightarrow m > (1/\sqrt{N})$ , where  $G$  is the high amplifier gain.  $N$  can be increased by increasing the recording time, i.e. by recording the addition of several one-shot images. Hence, detecting images with  $10^{-2}$  photons per mode should be possible, leading to the possibility of amplifying black-body radiation in the optical domain.

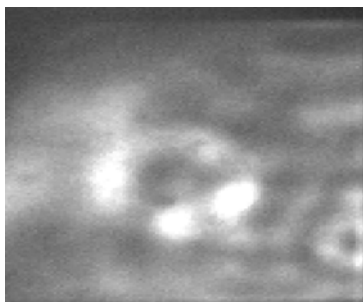


fig.1 : dye experimental image

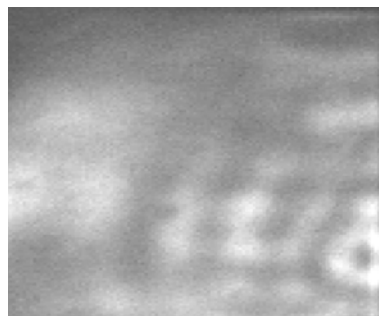


fig.2 : image of parametric fluorescence

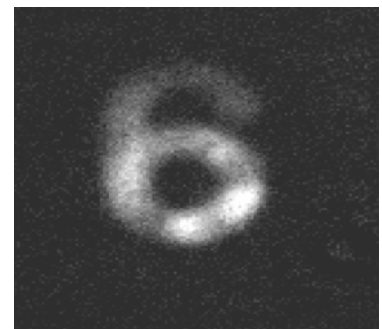


fig.3: image with subtraction of parametric fluorescence

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Friday Afternoon

# Resolution limits in three-dimensional fluorescence microscopy and some methods to overcome the classical Abbe criterion

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## ABSTRACT

The optical microscope has become the instrument of choice for the comprehension of cellular mechanisms, thanks to its unique properties of 3-D imaging into living specimens, the recording of temporal evolutions being also possible.

Many optical techniques (transmission, phase contrast, differential phase contrast) do exist. However, fluorescence microscopy has often the preference of biologists, because specially developed fluorescent tags (chemical fluorophores, tagged antibodies, quantum dots) permit to specifically study precisely selected structures inside the cell, as well as their associated biological functions.

However, compared to other techniques like electron microscopy or scanning near-field optical microscopy (SNOM), the optical microscope has a limited resolution. Indeed, the diffraction phenomenon, associated to the focusing of an electromagnetic wave, which explains the image formation process, limits the smallest accessible details to about 150 nm laterally, and no better than 350 nm along the optical axis for a conventional microscope (these values being purely theoretical).

First, the classical theory for image formation in a fluorescence microscope is presented, taking into account the stratified structure composed of the immersion medium, the coverslip and the specimen, which is characteristic of biological observations.

Then, techniques to overcome the classical Abbe limit in fluorescence microscopy will be presented. Among these techniques, structured illumination, 4Pi microscopy and STED microscopy are among the most promising. Their principle and some of their limitations are presented.

These techniques promise an ultimate resolution in the few nanometer range in far-field. Obtaining these results in a biological specimen will however require mastering the complete chain of data acquisition and processing, from the phenomena of photon emission into the specimen and the transmission of this fluorescence light through the specimen and the instrument, to image restoration techniques.

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# Scattering imaging: from near field to 3-D

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Resolution in optical microscopy is limited by diffraction to 200-300 nm. Scanning probe microscopes such as the scanning tunneling microscopes (STM), the atomic force microscopes (AFM), and the scanning near-field optical microscopes (SNOM) have been developed to overcome the diffraction limit. Scanning probe microscopes provide high resolution images by scanning a tip across the sample surface.

Most of the instruments built for Scanning Optical Near Field Microscopy (SNOM) use a metal coated tapered optical fiber with a nanometric hole at the tip apex. Such systems squeeze the electromagnetic field in the tapered region and act like metallic wave-guides above their cutoff frequency. There is so a trade-off between resolution and efficiency. The resolution is higher for small holes and small angles of the tapered part.

The *use of a tip* such as the apex of a near field probe (Atomic Force or Tunneling) to *scatter* the local field (propagating and evanescent waves) has proven to be very powerful in terms of both resolution and efficiency. To understand the physical origin of these performances let us recall that in the very near field of the tip electromagnetic fields behave like electrostatic ones. Moreover, with a metallic tip, the smaller is the curvature radius, the larger the field enhancement. We will describe a few experimental schemes performing imaging below the diffraction limit of classical microscopy. An emphasis will be made on the apertureless near-field microscopy approach which, in our opinion, offers a number of advantages such as: good spatial resolution (typically 10 nm), easy tip fabrication, large spectral domain (from near UV to mid IR ) and high efficiency because of local field enhancements. We will show how the signal is generated and what kind of local information can be deduced from the light scattered by this nanometric size antenna. We will then give some experimental examples illustrating the ability of this near-field microscope to reveal local physical properties associated to absorption, refractive indexes, reflection, localization ... as well as polarization effects and magneto-optics.

The strong and precise connection of the probe and the sample with the microscope *prevents probing three-dimensional objects*. Recently, it has been proposed to replace the physical connection by optical tweezers that trap and stabilize a particle, which then serves as a probe [1]. Three-dimensional images of polymer network structures were reconstructed from the histogram of thermal position fluctuations of a single particle probe. We propose a new technique using *multiple nanometric beads* as probes dispersed and fixed in the volume of the sample. The bead positions are determined with a precision better than the diffraction limit. We have two experimental goals : *gels rheology and 3-D imaging of hollow structures*.

The measurement of the displacement of the beads when a mechanical constraint is applied to the sample enables one to discover the local deformations of the sample and provides information about its three-dimensional structure. One of the applications is precise measurement of plastic flow heterogeneities in gels under simple shear strain. It is known that fluids exhibiting a yield stress have very heterogeneous flow. Although the mechanical properties of gels

have been measurable for several decades, the nature of the flow is a subject of intensive research in the field of soft matter physics (jamming and rheology). Therefore we aim, with our optical device, to investigate the flow of concentrated colloidal particle suspension at small scales with a three-dimensional resolution, in order to gain insight into this field of physics.

We will describe the experimental set-up. We calculate the sensitivity required to image the beads. We explain how the beads are localized in three dimensions with high resolution and report on experimental measurements of the resolution.

Then we will discuss the use of such probes for 3-D imaging and in particular the main constraints of the setup in terms of speed and signal-to-noise ratio.

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# Enhancing the axial resolution of quantum optical coherence tomography by nonperiodic quasi-phase-matching

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**Abstract:** We demonstrate theoretically that a properly designed nonperiodic quasi-phase-matched nonlinear crystal structure can serve to enhance the axial resolution of quantum optical coherence tomography, permitting it to attain sub-micron imaging. We show how the pump pulse duration affects the resolution that is attainable.

Optical coherence tomography (OCT) is a versatile, noninvasive imaging technique with a host of applications in biology and medicine [1,2]. Because of its interferometric nature, OCT can achieve high axial resolution, independent of focusing conditions, since axial and transverse resolution are determined independently. A quantum version of this technique, called quantum optical coherence tomography (QOCT), has recently been set forth [3]. It has been experimentally demonstrated that in the presence of dispersion in the sample under test, axial resolution can be improved relative to classical OCT [4,5] by virtue of the quantum cancellation of even-order dispersion [6]. QOCT makes use of a pair of entangled photons generated via the nonlinear optical process of spontaneous parametric down-conversion. The frequency anti-correlation inherent in the two-photon state leads to the dispersion cancellation, while the spectral content, or bandwidth, of the down converted photons determines the axial resolution of QOCT. Resolution improves as the bandwidth increases.

In this paper we suggest that quasi-phase-matching (QPM) can be used to enhance the spectral bandwidth of the entangled two-photon state produced in the down-conversion process, while simultaneously maintaining the frequency anti-correlation. QPM provides a serious alternative to conventional phase matching for many optical-parametric-process applications, and QPM engineering opens up a variety of new possibilities. In particular, we show that down-conversion via longitudinally-chirped QPM can substantially enhance the axial resolution of QOCT. The merit of this particular modulation scheme is that it permits many different signal and idler photon wavelengths to be phase-matched at different positions inside the nonlinear crystal, thus broadening the spectral content of the two-photon state. QPM down-conversion with a cw pump offers dispersion-cancelled QOCT and is thereby superior to conventional bandwidth enhancement techniques that make use of ultrashort pump pulses.

The sketch of a particular nonperiodic QPM sample is displayed in Figure 1. Writing a chirp in the QPM grating such that its period is linearly swept, for example, leads to a broadening in the spectrum of the signal and idler photons. This bandwidth broadening is directly translated into an improvement in resolution of the QOCT technique. The model of the sample under consideration comprised two membranes separated by a distance of 5  $\mu\text{m}$  and filled with water. Using simulations, we have shown that sub-micron resolution can be achieved by using properly designed QPM structures. The influence of the pump pulse duration on the attainable resolution has also been studied for different nonperiodic quasi-phase-matching gratings and a discussion of this issue will be provided.

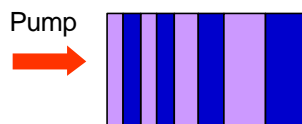


Fig. 1. Sketch of a nonperiodic QPM structure for spontaneous parametric down conversion.

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