Nash equilibrium in quantitative games played on graphs

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LogICCC Final Conference

GASICS project

Computer programming

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- understand deeply the initial problem;
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- write the program correctly.

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Definition

A software bug is an error, a failure in a computer program or system that induces an incorrect result.

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- human loss,...

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$\Rightarrow$ Real need to **verify** the correctness of a program!

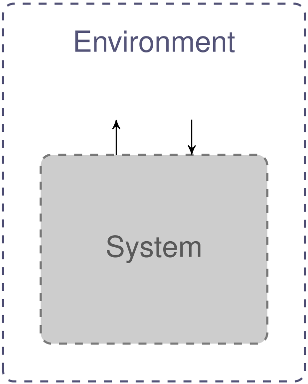
The autopilot case

Requirement: to arrive safe and sound in every weather conditions.

Difficulty: more than 18 million lines of code contained in the computer system of a plane!

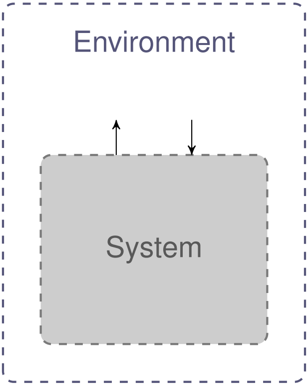
Consequence: for practical reasons (deadlines, costs), it is impossible to test such a system in every situation that it might encounter.

Modelisation



Expected functionality

Modelisation



$\rightsquigarrow$ Mathematical model

? satisfies ?

Expected functionality $\rightsquigarrow$ Logical formula

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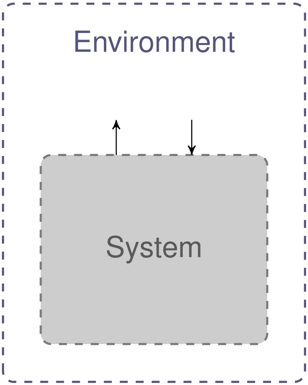

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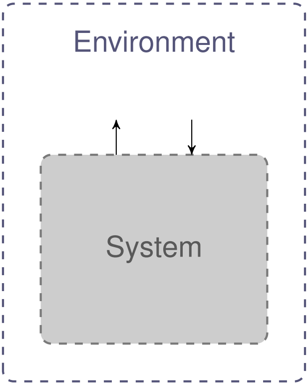

$\rightsquigarrow$ Mathematical model

2-player, zero-sum, qualitative GAME

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The autopilot case

Restrictive model because:

- there are several planes in the air,
- planes do not have antagonist objectives,
- it does not enable to verify a functionality while minimizing the fuel consumption.

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- planes do not have antagonist objectives,
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$\Rightarrow$ Release the constraints of 2-player, zero-sum, qualitative games.

$\Rightarrow$ Consider and study n-player, non zero-sum, quantitative games.

(Part of the GASICS project)

From winning strategies to equilibria

- 2-player, zero-sum, qualitative games:

  $\hookrightarrow$ looking for a winning strategy for one player.

- n-player, non zero-sum, quantitative games:

  $\hookrightarrow$ need for a different concept.

  $\hookrightarrow$ looking for a Nash equilibrium for all players.

A toy example: the printer battle

In a publishing house, **3 persons** intend to print their own document.

Gaston
Office worker

Jeanne
Secretary

Fantasio
Editor-in-chief

(Some characters of the belgian comic strip Gaston)

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2 printers: printer P_1 and printer P_2.

The printer battle as a game...

3 persons	$\hookrightarrow$	3 players
2 printers	$\hookrightarrow$	2 choices for each player

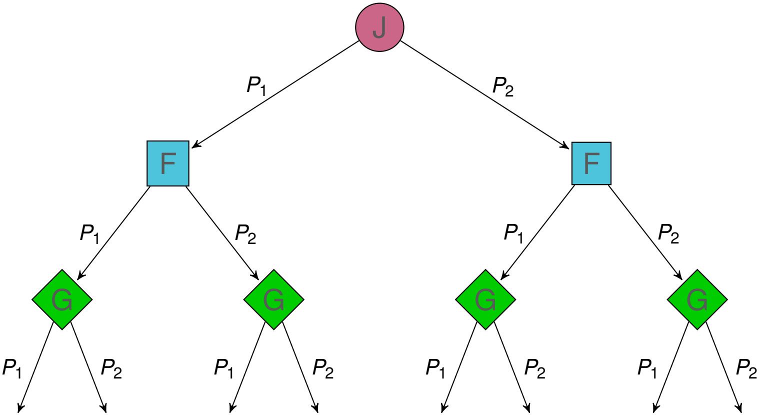
Particularity of printer P_1: paper jam from 2 sent documents.

Aim of each player: manage to print her/his document.

$\hookrightarrow$ A player is winning if her/his document is printed.

$\boxed{\Rightarrow \textbf{3-player, non zero-sum, qualitative game.}}$

Qualitative modelisation of the printer battle



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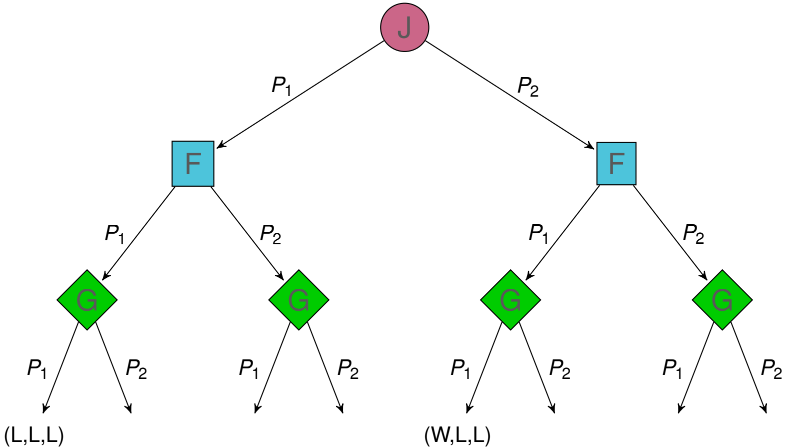

All of them are losing.

Jeanne is winning, men are losing.

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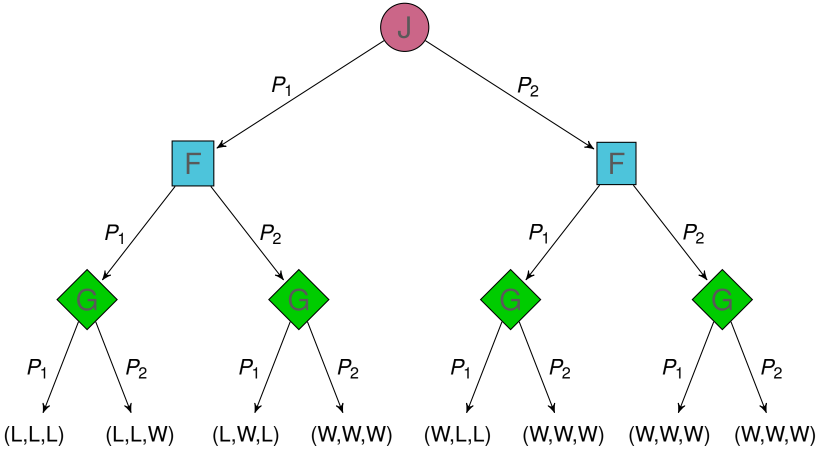

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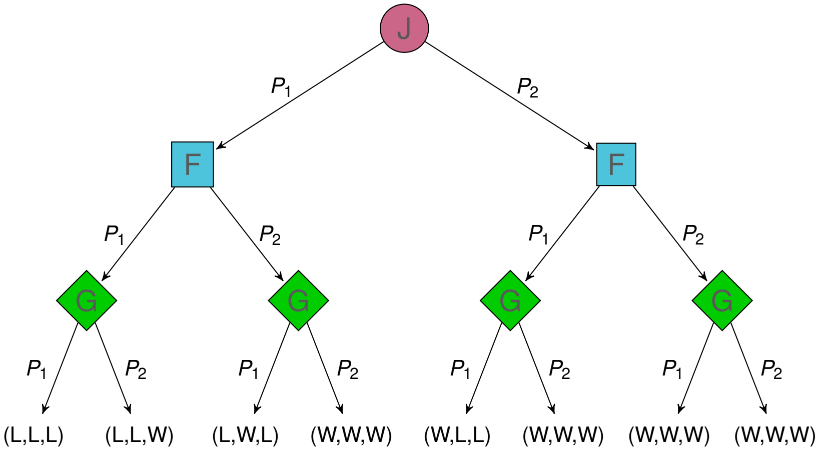

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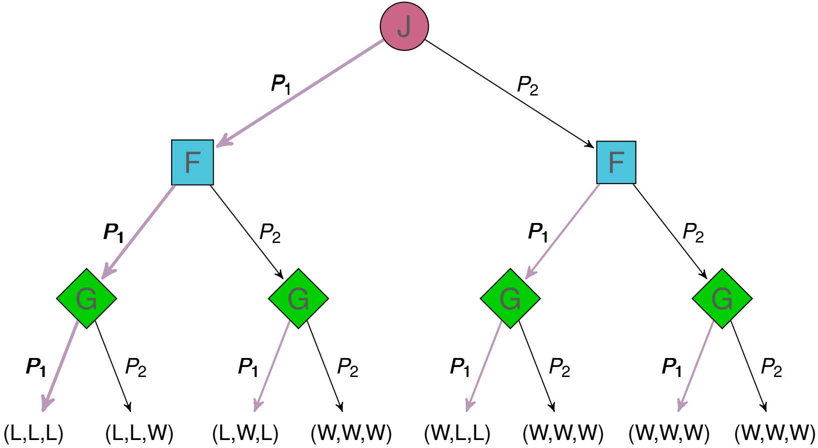

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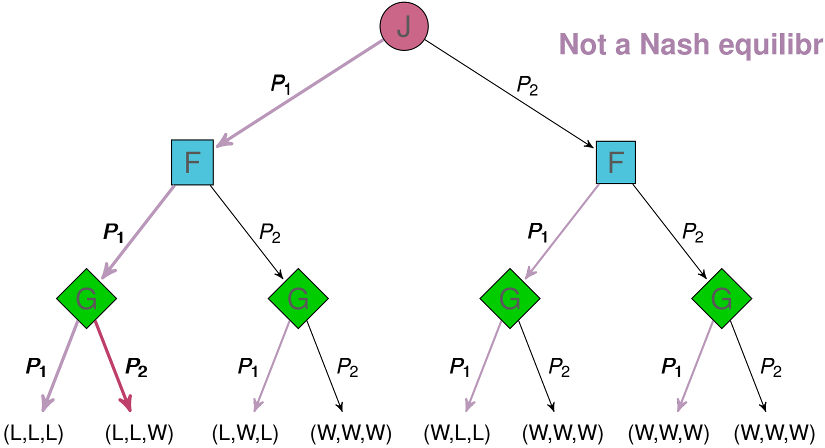

Not a Nash equilibrium!

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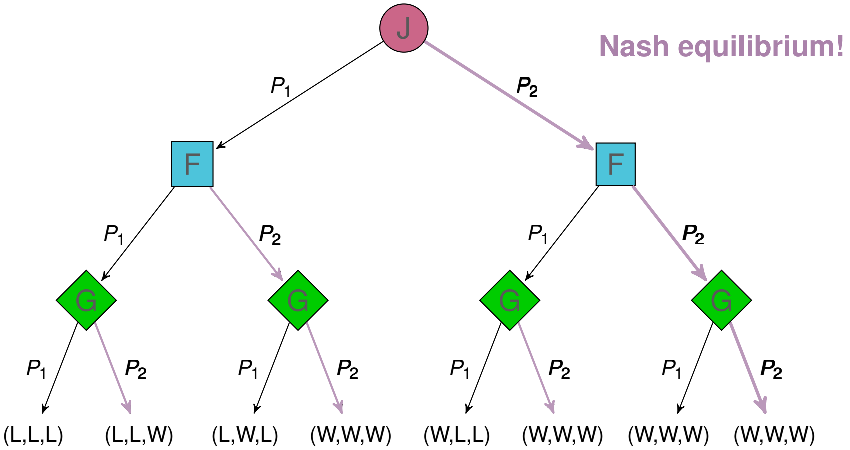

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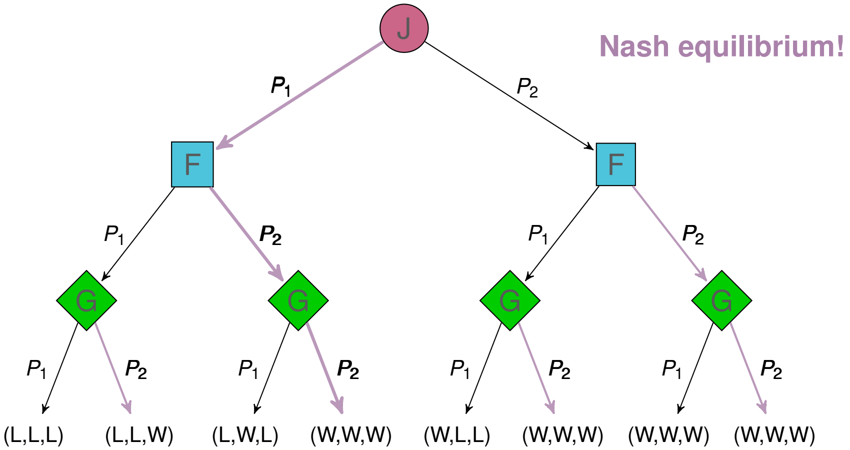


Let us add time constraint...

Jeanne, Fantasio and Gaston are all in a hurry.

Other particularity of printer P_1: two times faster than printer P_2.

Printer P_1 $\hookrightarrow$ 1 second/document

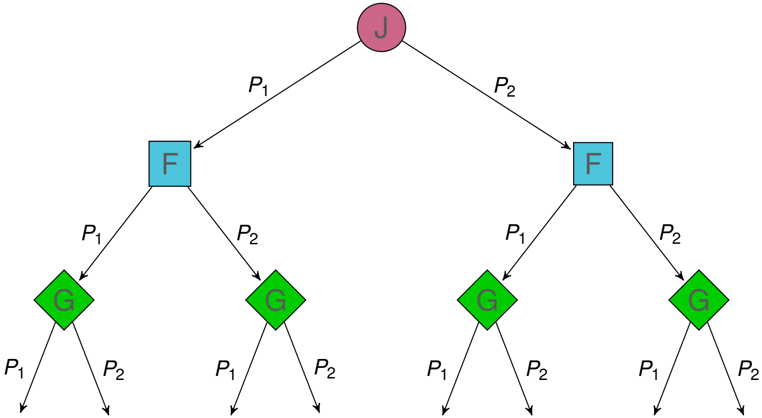
Printer P_2 $\hookrightarrow$ 2 seconds/document

Aim of each player: minimize the time to print one's document.

$\hookrightarrow$ The less time it takes, the happier a player is.

$\Rightarrow$ **3-player, non zero-sum, quantitative game.**

Quantitative modelisation of the printer battle



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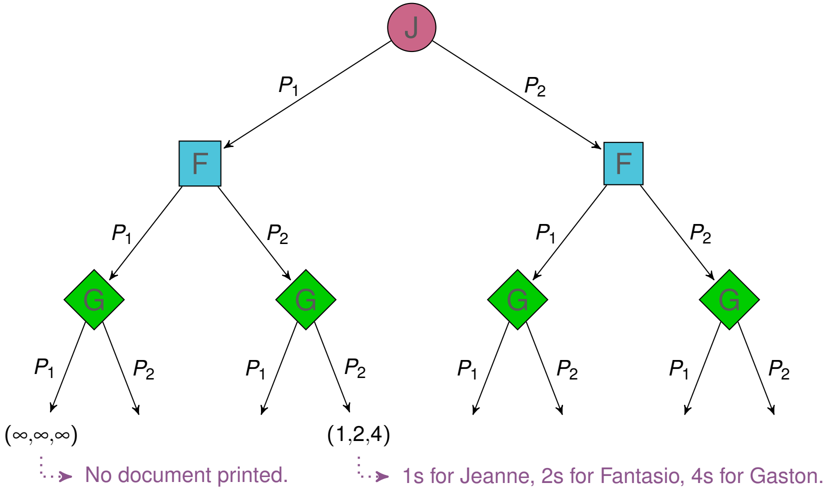

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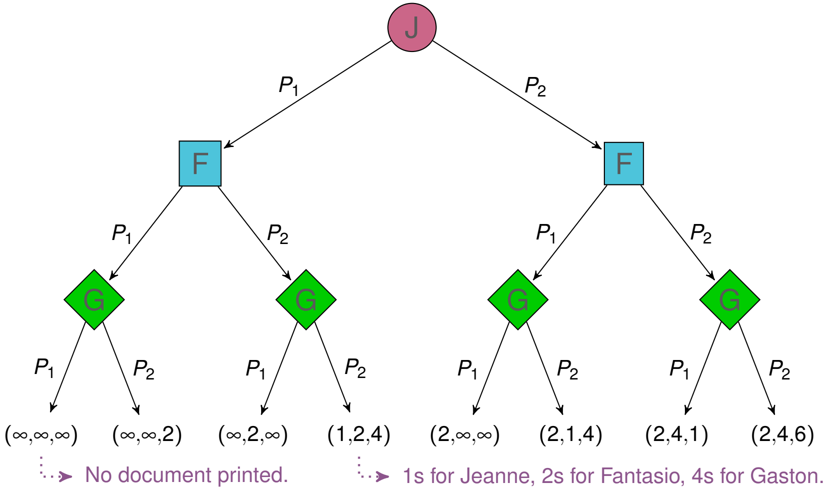

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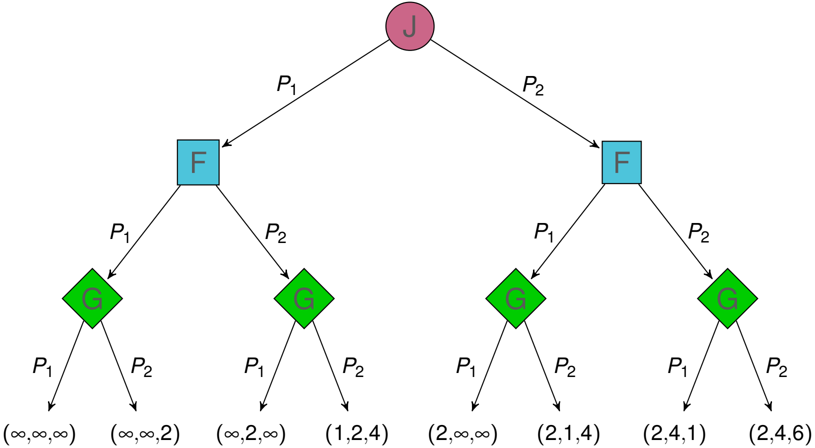

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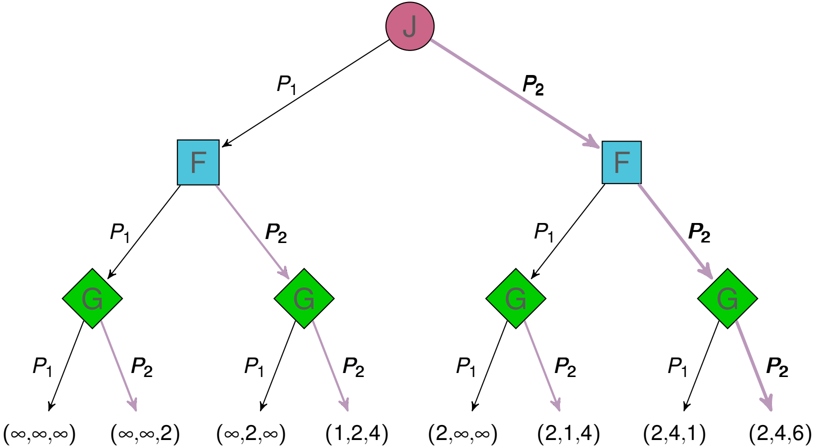

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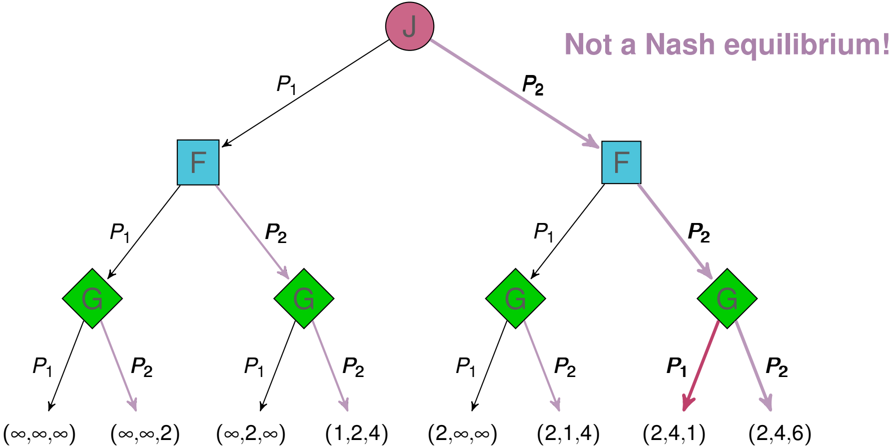

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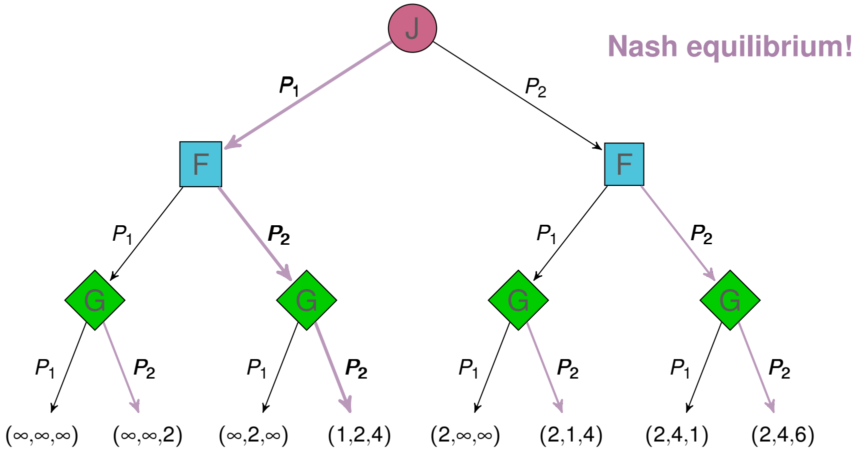

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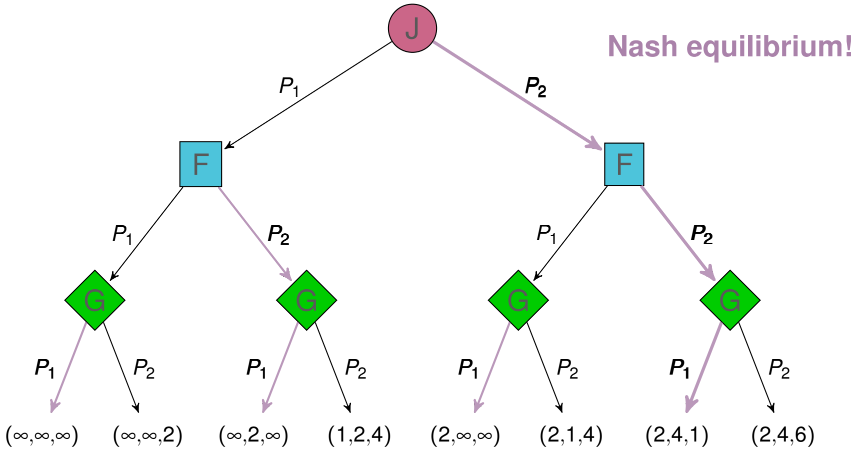

Some of our results in reachability games

Theorem ([BBD10])

There exists a finite-memory Nash equilibrium in every n-player quantitative reachability game.

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Given a NE in a n-player quantitative reachability game, there exists a finite-memory NE with the same set of "winning" players.

[BBD10] T. Brihaye, V. Bruyère, J. De Pril, Equilibria in Quantitative Reachability Games, CSR, vol. 6072 of *LNCS*, 72-83. Springer 2010.

More recent work

In *n*-player quantitative reachability games [BBDG11] :

- Existence of a subgame perfect equilibrium.
- Algorithm to decide existence of a secure equilibrium.

In *concurrent* *n*-player quantitative reachability games [KLST11] :

- Algorithm to extract strategies of Nash equilibria.
- Complexity of the decision variant of the problem.

[BBDG11] Ongoing work of T. Brihaye, V. Bruyère, J. De Pril and H. Gimbert.

[KLST11] M. Klimos, K. G. Larsen, F. Stefanak, J. Thaarup, Finding Nash Equilibria in Concurrent Priced Games.

Thank you for your attention!