Strategyproof Irresolute Social Choice Functions

Markus Brill

LogICCC Final Conference
Berlin, September 2011

- Brandt: Group-Strategyproof Irresolute Social Choice Functions (IJCAI 2011)
- Brandt, Brill: Necessary and Sufficient Conditions for the Strategyproofness of Irresolute Social Choice Functions (TARK 2011)

DSS
Decision Sciences & Systems

Preliminaries

- Finite set of at least three alternatives
  - Each voter *i* has complete preference relation R_i over alternatives
  - For most of this talk, we assume strict preferences (no ties)
- A *social choice function (SCF)* is a function that maps a preference profile to a non-empty subset of alternatives
  - An SCF f is *resolute* if $|f(R)|=1$ for all preference profiles *R*
  - A *Condorcet extension* is an SCF that uniquely chooses the Condorcet winner whenever one exists
- An SCF is *strategyproof* (or non-manipulable) if no voter can obtain a more preferred outcome by misrepresenting his preferences
  - An SCF is *group-strategyproof* if no group of voters can obtain an outcome that all of them prefer to the original one


Allan Gibbard

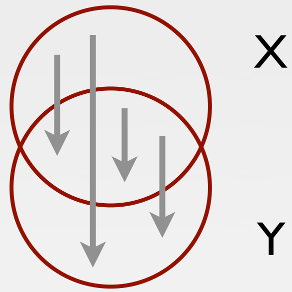
Impossibility


Mark A. Satterthwaite

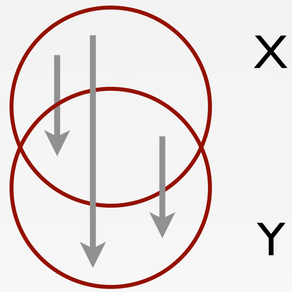
- Theorem (Gibbard, 1973; Satterthwaite, 1975): Every non-imposed, non-dictatorial, resolute SCF is manipulable
    - “*[resoluteness] is a rather restrictive and unnatural assumption*” (Gärdenfors, 1976)
    - “*If there is a weakness to the Gibbard-Satterthwaite theorem, it is the assumption that winners are unique*” (Taylor, 2005)
- Problem: Resolute SCFs have to pick single alternatives based on the individual preferences only
    - incompatible with anonymity and neutrality
- Are there reasonable irresolute strategyproof SCFs?
    - How do voters compare sets of alternatives with each other?

Preference Extensions

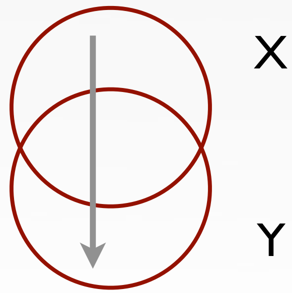
- Kelly's extension (1977):
  - $X \; R_i^K \; Y$ iff $x \; R_i \; y$ for all $x \in X$ and $y \in Y$
  - Example: $a \; R_i \; b \; R_i \; c \Rightarrow \{a\} \; R_i^K \; \{b,c\}$



- Fishburn's extension (1972):
  - $X \; R_i^F \; Y$ iff $x \; R_i \; z \; R_i \; y$ for all $x \in X$, $z \in X \cap Y$, and $y \in Y$
  - Example: $a \; R_i \; b \; R_i \; c \Rightarrow \{a,b\} \; R_i^F \; \{a,b,c\}$

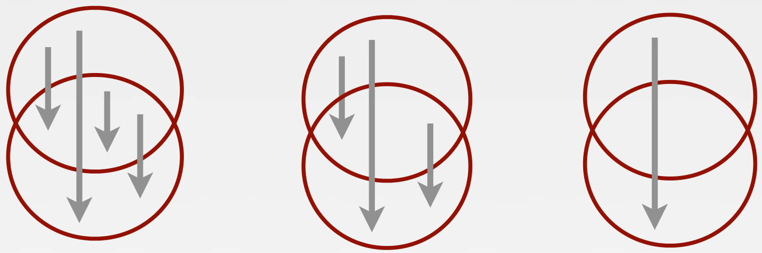


- Gärdenfors' extension (1976):
  - If $X \subseteq Y$ or $Y \subseteq X$, same as Fishburn's extension
  - Otherwise, $X \; R_i^G \; Y$ iff $x \; R_i \; y$ for all $x \in X \setminus Y$ and $y \in Y \setminus X$
  - Example: $a \; R_i \; b \; R_i \; c \Rightarrow \{a,b\} \; R_i^G \; \{a,c\}$



Strategyproofness

- The relations R_i^K, R_i^F, and R_i^G are incomplete and ordered w.r.t. set inclusion:



$R_i^K \subseteq R_i^F \subseteq R_i^G$

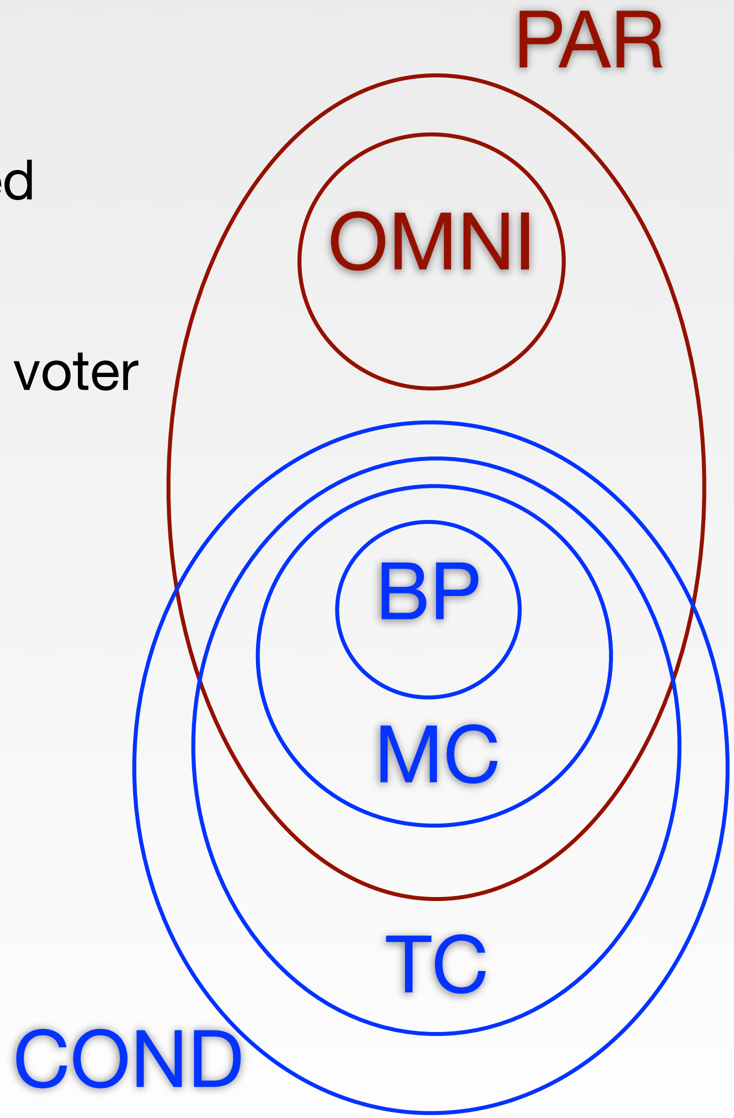
- Given $E \in \{K, F, G\}$, an SCF is *E-strategyproof* if no voter can obtain a more preferred outcome (according to R_i^ε) by misrepresenting his preferences
  - G-strategyproofness ⇒ F-strategyproofness ⇒ K-strategyproofness

Kelly-Strategyproofness

- If individual preferences are not required to be strict, every Condorcet extension is K-manipulable
  - strengthening of results by Gärdenfors (1976) and Taylor (2005)
- New axiom: An SCF satisfies *set-monotonicity* if weakening unchosen alternatives has no effect on the choice set
- Theorem: Every SCF that satisfies set-monotonicity is K-strategyproof

Set-Monotonic SCFs

- Pareto rule (PAR)
  - all alternatives that are not Pareto-dominated
- Omninomination rule (OMNI)
  - all alternatives that are ranked first by some voter
- Condorcet rule (COND)
  - Condorcet winner if it exists;
    all alternatives otherwise
- Top Cycle (TC) [Good, 1971; Smith, 1973]
  - maximal elements of the transitive closure of the weak majority relation
- Minimal Covering Set (MC) [Dutta, 1988]
- Bipartisan Set (BP) [Laffond et al., 1993]



Pairwise SCFs

- An SCF is *pairwise* if it only depends on pairwise comparisons (Young, 1974)
  - Examples: Kemeny, Borda, Maximin, ranked pairs, all tournament solutions (Slater set, uncovered set, Banks set, TEQ etc.)
- Theorem: Every K-strategyproof, pairwise SCF satisfies set-monotonicity
  - most standard SCFs violate set-monotonicity
- Corollary: A pairwise SCF is K-strategyproof if and only if it satisfies set-monotonicity
  - generalization of Muller-Satterthwaite theorem to irresolute SCFs

Fishburn-Strategyproofness

- An SCF f satisfies *exclusive independence of chosen alternatives* (EICA) if $f(R') \subseteq f(R)$ for all pairs of preference profiles R and R' that differ only on alternatives in $f(R)$
  - f satisfies *weak EICA* if $f(R')$ is not a strict superset of $f(R)$
- Theorem: Every SCF that satisfies set-monotonicity and EICA is F-strategyproof
  - Corollary: PAR, OMNI, COND, TC are F-strategyproof
- Theorem: Every pairwise SCF that is F-strategyproof satisfies set-monotonicity and weak EICA
  - Corollary: MC and BP are *not* F-strategyproof

	K-str.proof	F-str.proof	G-str.proof
Pareto rule	✔	✔ (Feldman 1979)	✖
Omninomination rule	✔	✔	✔ (Gärdenfors 1976)
Condorcet rule	✔	✔	✔ (Gärdenfors 1976)
Top cycle	✔	✔ (Sanver & Zwicker 2010)	✔
Minimal covering set	✔	✖	✖
Bipartisan set	✔	✖	✖
"everything else"	✖	✖	✖

Conclusion

- Irresolute SCFs circumvent the GS impossibility
  - strategyproofness depends on choice of preference extension
- Our axiomatic approach yields
  - new results and unified proofs of known results
  - error in the literature: COND∩PAR is not G-strategyproof
- All results extend to group-strategyproofness
- Future work
  - is there a Pareto-optimal pairwise SCF that is G-strategyproof?
  - other types of manipulation
    - Moulin's no-show paradox only applies to resolute SCFs
  - justification of Kelly's preference extension